Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 15, Number 6 (2019), pp. 1045-1054

http://www.ripublication.com/gjpam.htm

Cognition of Nano Binary Topological Spaces

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Abstract

The purpose of this paper, we introduce and study the nano binary topological spaces. The notions of nano binary open sets, nano binary closed sets, nano binary interior and nano binary closure are introduced and their basic properties are discussed with the suitable examples and also introduced nano binary extremally disconnected with some properties.

KeyWords: N_B α - open, N_B semi - open, N_B pre – open, and nano binary extremally disconnected.

1. INTRODUCTION

M. Lellis Thivagar **[2]** introduced the concept of nano topological space with respect to a subset X of a universe U and in **[1]** nano was developed by using α^{s*}-open. S. Nithyanantha Jothi and P. Thangavelu **[3]** introduce the concept of binary topological spaces. We introduce the concept nano binary topological spaces with respect to a subset (X_1, X_2) of a universe (U_1, U_2). We investigated the relationship between some weak forms of nano binary open sets in nano binary topological spaces. To enrich our paper we are here to introduce and study about nano binary extremally disconnected with some properties.

2. NANO BINARY TOPOLOGICAL SPACES

Throughout this paper $(U_1, U_2, \tau_R(X_1, X_2))$ represent nano binary topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset (H_1, H_2) of a space$(U_1, U_2, \tau_R(X_1, X_2))$, $N_B{}^{\circ}(H_1, H_2)$ and $\overline{N_B}(H_1, H_2)$ denote the nano binary interior of (H_1, H_2) and the nano binary closure of (H_1, H_2) respectively.

Definition 2.1: Let (U_1, U_2) be a non-empty finite set of objects called the universe and R be an equivalence relation on (U_1, U_2) named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U_1, U_2, R) is said to be the approximation space. Let $(X_1, X_2) \subseteq (U_1, U_2)$

1. The lower approximation of (X_1, X_2) with respect to R is the set of all objects, which can be for certain classified as (X_1, X_2) with respect to R and it is denoted by $L_R(X_1, X_2)$

That is, $L_R(X_1, X_2) = \bigcup_{(x_1,x_2)\epsilon(U_1,U_2)}\{R\,(x_1, x_2): R(x_1, x_2) \subseteq (X_1, X_2)\}$

where $R(x_1, x_2)$ denotes the equivalence class determined by (x_1, x_2)

2. The upper approximation of (X_1, X_2) with respect to R is the set of all objects, which can be possibly classified as (X_1, X_2) with respect to R and it is denoted by $U_R(X_1, X_2)$

That is, $U_R(X_1, X_2) = \bigcup_{(x_1,x_2)\epsilon(U_1,U_2)}\{R\,(x_1, x_2): R(x_1, x_2) \cap (X_1, X_2) \neq \phi\}$

3. The boundary region of (X_1, X_2) with respect to R is the set of all objects, which can be classified neither as (X_1, X_2) nor as not $-(X_1, X_2)$ with respect to R and it is denoted by $B_R(X_1, X_2)$.

That is, $B_R(X_1, X_2) = U_R(X_1, X_2) - L_R(X_1, X_2)$

Proposition 2.2: If (U_1, U_2, R) is an approximation space and(X_1, X_2), $(Y_1, Y_2) \subseteq (U_1, U_2)$; then

1. $L_R(X_1, X_2) \subseteq (X_1, X_2) \subseteq U_R(X_1, X_2)$

2. $L_R(\phi, \phi) = U_R(\phi, \phi) = (\phi, \phi)$ and $L_R(U_1, U_2) = U_R(U_1, U_2) = (U_1, U_2)$

3. $U_R\big((X_1, X_2) \cup (Y_1, Y_2)\big) = U_R(X_1, X_2) \cup U_R\,(Y_1, Y_2)$

4. $U_R\big((X_1, X_2) \cap (Y_1, Y_2)\big) \subseteq U_R(X_1, X_2) \cap U_R(Y_1, Y_2)$

5. $L_R\big((X_1, X_2) \cup (Y_1, Y_2)\big) \supseteq L_R(X_1, X_2) \cup L_R\,(Y_1, Y_2)$

6. $L_R\big((X_1, X_2) \cap (Y_1, Y_2)\big) \subseteq L_R(X_1, X_2) \cap L_R(Y_1, Y_2)$

7. $L_R(X_1, X_2) \subseteq L_R(Y_1, Y_2)$ and $U_R(X_1, X_2) \subseteq U_R(Y_1, Y_2)$ whenever $(X_1, X_2) \subseteq (Y_1, Y_2)$

8. $U_R(X_1, X_2)^C = [L_R(X_1, X_2)]^C$ *and* $L_R(X_1, X_2)^C = [U_R(X_1, X_2)]^C$

9. $U_R U_R(X_1, X_2) = L_R U_R(X_1, X_2) = U_R(X_1, X_2)$

10. $L_R L_R(X_1, X_2) = U_R L_R(X_1, X_2) = L_R(X_1, X_2)$

Definition 2.3: Let (U_1, U_2) be the universe, R be an equivalence on(U_1, U_2) and

$\tau_R(X_1, X_2) = \{(U_1, U_2), (\phi, \phi), L_R(X_1, X_2), U_R(X_1, X_2), B_R(X_1, X_2)\}$ where $(X_1, X_2) \subseteq (U_1, U_2)$. Then by the property R$(X_1, X_2)$ satisfies the following axioms

1. (U_1, U_2) and $(\phi, \phi) \epsilon (X_1, X_2)$

2. The union of the elements of any sub collection of $\tau_R(X_1, X_2)$ is in $\tau_R(X_1, X_2)$

3. The intersection of the elements of any finite sub collection of $\tau_R(X_1, X_2)$ is in $\tau_R(X_1, X_2)$.

That is, $\tau_R(X_1, X_2)$ is a topology on (U_1, U_2) called the nano binary topology on (U_1, U_2) with respect to (X_1, X_2). We call $(U_1, U_2, \tau_R(X_1, X_2))$ as the nano binary topological spaces. The elements of $\tau_R(X_1, X_2)$ are called as nano binary open sets.

Examples 2.4:

Let $U_1 = \{a, b, c\}$ and $U_2 = \{1,2\}$ with $(U_1, U_2)/R = \{ (\{a, b\}, \{1\}), (\{c\}, \{2\}) \}$ and

$(X_1, X_2) = \{(\{a, c\}, \{2\})\}$. Then the nano binary topology

$$\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), (\{c\}, \{2\})\}$$

Definition 2.5: If $\big(U_1, U_2, \tau_R(X_1, X_2)\big)$ is a nano binary topological spaces with respect to (X_1, X_2) and if $(H_1, H_2) \subseteq (U_1, U_2)$, then the nano binary interior of (H_1, H_2) is defined as the union of all nano binary open subsets of (A_1, A_2) and it is defined by $N_B{}^\circ(H_1, H_2)$

That is, $N_B{}^\circ\ (H_1, H_2)$ is the largest nano binary open subset of (H_1, H_2). The nano binary closure of (H_1, H_2) is defined as the intersection of all nano binary closed sets containing (H_1, H_2) and it is denoted by $\overline{N_B}(H_1, H_2)$.

That is, $\overline{N_B}(H_1, H_2)$ is the smallest nano binary closed set containing (H_1, H_2)

Proposition 2.6: Let $(U_1, U_2, \tau_R(X_1, X_2))$ be a nano binary topological space and $(A_1, A_2), (B_1, B_2) \in P(X_1) \times P(X_2)$ then

i) $N_B{}^\circ\ (\Phi, \Phi) = (\Phi, \Phi)$

$\overline{N_B}\ (\Phi, \Phi) = (\Phi, \Phi)$

ii) $N_B{}^\circ\ (U_1, U_2) = (U_1, U_2)$

$\overline{N_B}\ (U_1, U_2) = (U_1, U_2)$

iii) $N_B{}^\circ\ (A_1, A_2) \subseteq (A_1, A_2) \subseteq \overline{N_B}\ (A_1, A_2)$

iv) $(A_1, A_2) \subseteq (B_1, B_2)$ implies $N_B{}^\circ\ (A_1, A_2) \subseteq N_B{}^\circ\ (B_1, B_2)$ and

$\overline{N_B}\ (A_1, A_2) \subseteq \overline{N_B}\ (B_1, B_2)$

v) $N_B{}^\circ\ ((A_1, A_2) \cap (B_1, B_2)) \subseteq N_B{}^\circ\ (A_1, A_2) \cap N_B{}^\circ\ (B_1, B_2)$

vi) $\overline{N_B}\ ((A_1, A_2) \cap (B_1, B_2)) \subseteq \overline{N_B}\ (A_1, A_2) \cap \overline{N_B}\ (B_1, B_2)$

vii) $N_B{}^\circ\ ((A_1, A_2) \cup (B_1, B_2)) \supseteq N_B{}^\circ\ (A_1, A_2) \cup N_B{}^\circ\ (B_1, B_2)$

viii) $\overline{N_B}\ ((A_1, A_2) \cup (B_1, B_2)) \supseteq \overline{N_B}\ (A_1, A_2) \cup \overline{N_B}\ (B_1, B_2)$

ix) $N_B{}^\circ\ (N_B{}^\circ\ (A_1, A_2)) \subseteq N_B{}^\circ\ (A_1, A_2)$

x) $\overline{N_B}\ (\overline{N_B}\ (A_1, A_2)) \supseteq \overline{N_B}\ (A_1, A_2)$

xi) $N_B{}^\circ\ (\overline{N_B}\ (A_1, A_2)) \supseteq N_B{}^\circ\ (A_1, A_2)$

xii) $\overline{N_B}\ (N_B{}^\circ\ (A_1, A_2)) \subseteq \overline{N_B}\ (A_1, A_2)$

Proof: The proof of (i) and (ii) is obvious.

iii) We assume $(X_1, X_2) \notin (A_1, A_2)$ which implies that (X_1, X_2) does not belongs to any nano binary open set contained in (A_1, A_2). Hence $(X_1, X_2) \notin N_B{}^\circ\ (A_1, A_2)$ which implies

$N_B{}^\circ\ (A_1, A_2) \subseteq (A_1, A_2)$….. (1). Also suppose $(X_1, X_2) \notin \overline{N_B}(A_1, A_2)$. Then (X_1, X_2) does not belongs to any nano binary closed sets containing(A_1, A_2). Hence $(X_1, X_2) \notin (A_1, A_2)$ which implies $(A_1, A_2) \subseteq \overline{N_B}\ (A_1, A_2)$….. (2).From (1) and (2) we say $N_B{}^\circ$ $(A_1, A_2) \subseteq (A_1, A_2) \subseteq \overline{N_B}\ (A_1, A_2)$

iv) The result is obviously true

v) $(A_1, A_2) \cap (B_1, B_2) \subseteq (A_1, A_2)$ and $(A_1, A_2) \cap (B_1, B_2) \subseteq (B_1, B_2)$

Therefore, $N_B{}^\circ\ ((A_1, A_2) \cap (B_1, B_2)) \subseteq N_B{}^\circ\ (A_1, A_2)$ and

$N_B{}^\circ\ ((A_1, A_2) \cap (B_1, B_2)) \subseteq N_B{}^\circ\ (B_1, B_2)$.

Hence $N_B{}^\circ((A_1, A_2) \cap (B_1, B_2)) \subseteq N_B{}^\circ\ (A_1, A_2) \cap N_B{}^\circ\ (B_1, B_2)$.

Similarly we get (vi)

(vii) $(A_1, A_2) \subseteq (A_1, A_2) \cup (B_1, B_2)$ and $(B_1, B_2) \subseteq (A_1, A_2) \cup (B_1, B_2)$

Therefore, $N_B{}^\circ\ (A_1, A_2) \subseteq N_B{}^\circ\ ((A_1, A_2) \cup (B_1, B_2))$ and

$N_B{}^\circ\ (B_1, B_2) \subseteq N_B{}^\circ\ ((A_1, A_2) \cup (B_1, B_2))$

Hence $N_B{}^\circ((A_1, A_2) \cup (B_1, B_2)) \supseteq N_B{}^\circ\ (A_1, A_2) \cup N_B{}^\circ\ (B_1, B_2)$.

Similarly we get (viii)

(ix) Since ${N_B}^{\circ}\ (A_1, A_2) \subseteq (A_1, A_2)$ and $\overline{N_B}\ (A_1, A_2) \supseteq (A_1, A_2)$ it follows that

${N_B}^{\circ}\ ({N_B}^{\circ}\ (A_1, A_2)) \subseteq {N_B}^{\circ}\ (A_1, A_2)$ and

$\overline{N_B}\ (\overline{N_B}\ (A_1, A_2)) \supseteq \overline{N_B}\ (A_1, A_2)$.

This proves (xi) and (xii).

Hence the proof of the proposition.

3. COGNITION OF NANO BINARY OPEN SETS

Definition 3.1: A subset (H_1, H_2) of a nano binary topological spaces $(U_1, U_2, \tau_R(X_1, X_2))$ is called

1. N_B Pre - open set if $(H_1, H_2) \subseteq\ {N_B}^{\circ}\ (\overline{N_B}\ (H_1, H_2))$
2. N_B semi- open set if $(H_1, H_2) \subseteq\ \overline{N_B}\ ({N_B}^{\circ}\ (H_1, H_2))$
3. N_B α-open if $(H_1, H_2) \subseteq {N_B}^{\circ}\ (\overline{N_B}\ ({N_B}^{\circ}\ (H_1, H_2)))$.

The complements of the above mentioned sets are called their respective nano binary closed sets.

Results 3.2:

1. Every nano binary open sets is N_B α-open.
2. Every N_B α-open is N_B semi -open.
3. Every N_B α-open is N_B pre -open.
4. Every nano binary open set is N_B semi -open.
5. Every nano binary open set is N_B pre -open.

Proof: 1. If (H_1, H_2) is a nano binary open set, then ${N_B}^{\circ}\ (A_1, A_2) = (A_1, A_2)$

Now, ${N_B}^{\circ}\ (\overline{N_B}\ ({N_B}^{\circ}\ (A_1, A_2))) = {N_B}^{\circ}\ (\overline{N_B}(A_1, A_2)) \supseteq {N_B}^{\circ}\ (A_1, A_2)$, since $(A_1, A_2) \subseteq \overline{N_B}(A_1, A_2)$ implies ${N_B}^{\circ}\ (A_1, A_2) \subseteq {N_B}^{\circ}\ (\overline{N_B}\ (A_1, A_2)) = (A_1, A_2)$. Therefore, ${N_B}^{\circ}\ (\overline{N_B}\ ({N_B}^{\circ}\ (A_1, A_2))) \supseteq (A_1, A_2)$. hence every nano binary open set is N_B α-open.

2. If (H_1, H_2) is a N_B α - open, then

$(H_1, H_2) \subseteq {N_B}^{\circ}\ (\overline{N_B}\ ({N_B}^{\circ}\ (H_1, H_2))) \subseteq \overline{N_B}\ ({N_B}^{\circ}\ (H_1, H_2))$.

Therefore, (H_1, H_2) is a N_B semi -open. Hence every N_B α-open is N_B semi- open.

3. If (H_1, H_2) is a N_B α - open, then

$(H_1, H_2) \subseteq N_B{}^\circ(\overline{N_B}(N_B{}^\circ(H_1, H_2))) \subseteq N_B{}^\circ(\overline{N_B}(H_1, H_2))$. Therefore, (H_1, H_2) is a N_B pre -open. Hence every N_B α-open is N_B pre -open.

4. If (H_1, H_2) is a nano binary open set, then $(A_1, A_2) = N_B{}^\circ(A_1, A_2)$

$\subseteq \overline{N_B}(N_B(H_1, H_2))$. Therefore, (H_1, H_2) is a N_B semi -open. Hence every nano binary open set is N_B semi -open.

5. If (H_1, H_2) is a nano binary open set, then $(H_1, H_2) \subseteq \overline{N_B}(H_1, H_2)$

$N_B{}^\circ(H_1, H_2) \subseteq N_B{}^\circ(\overline{N_B}(H_1, H_2)) \Rightarrow (H_1, H_2) \subseteq N_B{}^\circ(\overline{N_B}(H_1, H_2))$.Therefore, (H_1, H_2) is a N_B pre- open hence every nano binary open set is N_B pre- open.

Note 3.3: The converse of the above result is need not true by the following example.

Example 3.4:

Let $U_1 = \{a, b, c, d, e\}$ and $U_2 = \{1,2,3,4\}$ with $(U_1, U_2)/R = \{(\{a, b\}, \{2\}), (\{c\}, \{4\}), (\{d\}, \{3\}), (\{e\}, \{1\})\}$ and

$(X_1, X_2) = \{(\{a, c, d\}, \{2,3,4\})\}$. Then the nano binary topology

$\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), (\{c, d\}, \{3,4\}), (\{a, b, c, d\}, \{2,3,4\}), (\{a, b\}, \{2\})\}$. The nano binary closed sets $= \{(\Phi, \Phi), (U_1, U_2), (\{a, b, e\}, \{1,2\}), (\{e\}, \{1\}), (\{c, d, e\}, \{1,3,4\})\}$. In this example, 1. $(\{a, b, c, d, e\}, \{2,3,4\})$ is N_B α - open but not nano binary open.

2. $(\{a, b, e\}, \{2\})$ is N_B semi- open but not N_B α – open.

3. $(\{a, b, e\}, \{2,3\})$ is N_B pre- open but not N_B α – open.

4. $(\{a, b, e\}, \{2\})$ is N_B semi- open but not nano binary open.

5. $(\{a, b, e\}, \{2,3\})$ is N_B pre- open but not nano binary open.

Proposition 3.5: Let (U_1, U_2) be a non-empty finite universe and $(X_1, X_2) \subseteq (U_1, U_2)$

1. If $L_R(X_1, X_2) = (\Phi, \Phi) and\ U_R(X_1, X_2) = (U_1, U_2)$ then $\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2)\}$,the indiscrete nano binary topology on (U_1, U_2).

2. If $L_R(X_1, X_2) = U_R(X_1, X_2) = (X_1, X_2)$, then the nano binary topology,

$\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), L_R(X_1, X_2)\}$.

3. If $L_R(X_1, X_2) = (\Phi, \Phi)\ and\ U_R(X_1, X_2) \neq (U_1, U_2)$ then

$\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), U_R(X_1, X_2)\}$.

4. If $L_R(X_1, X_2) \neq (\Phi, \Phi)$ and $U_R(X_1, X_2) = (U_1, U_2)$, then

$\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), L_R(X_1, X_2), B_R(X_1, X_2)\}$.

5. If $L_R(X_1, X_2) \neq U_R(X_1, X_2)$ where $L_R(X_1, X_2) \neq (\Phi, \Phi) and\ U_R(X_1, X_2) \neq (U_1, U_2)$, then

$\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), L_R(X_1, X_2), U_R(X_1, X_2), B_R(X_1, X_2)\}$ is the discrete nano binary topology on (U_1, U_2)

4. NANO BINARY EXTREMALLY DISCONNECTED

Definition 4.1: A nano binary topological spaces $(U_1, U_2, \tau_R(X_1, X_2))$ is nano binary extremally disconnected, if the nano binary closure of each nano binary open set is nano binary open in (U_1, U_2).

Example 4.2:

Let $U_1 = \{a, b, c\}, U_2 = \{1,2\}$ with $(U_1, U_2)/_R = \{(\{a\}, \{1\}), (\{b, c\}, \{2\})\}$ and

$(X_1, X_2) = (\{a, b, c\}, \{2\})$. Then the nano binary topology

$\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), (\{a\}, \{1\}), (\{b, c\}, \{2\})\}$. Then the nano binary closed sets in (U_1, U_2) are $(\Phi, \Phi), (U_1, U_2), (\{a\}, \{1\})$ *and* $(\{b, c\}, \{2\})$. Then $\overline{N_B}\ (U_1, U_2) = (U_1, U_2)$,

$\overline{N_B}\ (\Phi, \Phi) = (\Phi, \Phi)$, $\overline{N_B}\ (\{a\}, \{1\}) = (\{a\}, \{1\})$, $\overline{N_B}\ (\{b, c\}, \{2\}) = (\{b, c\}, \{2\})$.

That is, nano binary closure of each nano binary open set in (U_1, U_2) is nano binary open. Therefore, $(U_1, U_2, \tau_R(X_1, X_2))$ is nano binary extremally disconnected.

Theorem 4.3: A nano binary topological spaces $(U_1, U_2, \tau_R(X_1, X_2))$ is nano binary extremally disconnected if and only if $U_R(X_1, X_2) = (U_1, U_2)$.

Proof: Let $(U_1, U_2, \tau_R(X_1, X_2))$ be nano binary extremally disconnected. That is, nano binary closure of each nano binary open set is nano binary open. That is, (U_1, U_2), (Φ, Φ), $[B_R(X_1, X_2)]^C$ and $[L_R(X_1, X_2)]^C$ are nano binary open. That is, (U_1, U_2), (Φ, Φ), $B_R(X_1, X_2), L_R(X_1, X_2)$ are nano binary closed. Therefore, $\overline{N_B}\ (B_R(X_1, X_2)) = B_R(X_1, X_2)$ and $\overline{N_B}\ (L_R(X_1, X_2) = L_R(X_1, X_2))$. That is, $[L_R(X_1, X_2)]^C = B_R(X_1, X_2)$ and

$[B_R(X_1, X_2)]^C = L_R(X_1, X_2)$.That is, $[L_R(X_1, X_2)]^C = U_R(X_1, X_2) - L_R(X_1, X_2)$

$= U_R(X_1, X_2) \cap [L_R(X_1, X_2)]^C \subseteq U_R(X_1, X_2)$. Therefore, $[L_R(X_1, X_2)]^C \subseteq U_R(X_1, X_2)$. Since $L_R(X_1, X_2) \subseteq U_R(X_1, X_2)$, $[U_R(X_1, X_2)]^C \subseteq [L_R(X_1, X_2)]^C$. Hence $[U_R(X_1, X_2)]^C \subseteq [L_R(X_1, X_2)]^C \subseteq U_R(X_1, X_2)$. This is possible when $[U_R(X_1, X_2)]^C = (\Phi, \Phi)$. This implies $U_R(X_1, X_2) = (U_1, U_2)$.

Conversely, let if $U_R(X_1, X_2) = (U_1, U_2)$ and $L_R(X_1, X_2) = (\Phi, \Phi)$, then

$\tau_R(X_1, X_2)$ ={(U_1, U_2), (Φ, Φ)}, where nano binary closure of each nano binary open set is obviously nano binary open. If $L_R(X_1, X_2) \neq (\Phi, \Phi)$, then

$\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), L_R(X_1, X_2), B_R(X_1, X_2)\}$ where $B_R(X_1, X_2) = [L_R(X_1, X_2)]^C$ and hence $\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), L_R(X_1, X_2), [L_R(X_1, X_2)]^C\}$. Therefore, each set in $\tau_R(X_1, X_2)$ is both nano binary open and nano binary closed. Hence nano binary closure of each nano binary open is nano binary open. Therefore, $(U_1, U_2, \tau_R(X_1, X_2))$ is nano binary extremally disconnected. Thus (U_1, U_2) is nano binary extremally disconnected if and only if $U_R(X_1, X_2) = (U_1, U_2)$.

Theorem 4.4: If $L_R(X_1, X_2) = U_R(X_1, X_2)$ where $(X_1, X_2) \subseteq (U_1, U_2)$, then $(U_1, U_2, \tau_R(X_1, X_2))$ is nano binary extremally disconnected.

Proof: Since$L_R(X_1, X_2) = U_R(X_1, X_2)$, $B_R(X_1, X_2) = (\Phi, \Phi)$ and hence $\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), L_R(X_1, X_2)\}$ where $\overline{N_B}\,(U_1, U_2) = (U_1, U_2)$, $\overline{N_B}\,(\Phi, \Phi) = (\Phi, \Phi)$and

$\overline{N_B}\,(L_R(X_1, X_2)) = [B_R(X_1, X_2)]^C = [(\Phi, \Phi)]^C = (U_1, U_2)$. That is, the nano binary closure of each nano binary open set in (U_1, U_2) is nano binary open. Therefore, $(U_1, U_2, \tau_R(X_1, X_2))$ is nano binary extremally disconnected. The converse of the above theorem is not true by the following example.

Example 4.5:

Let $U_1 = \{a, b\}, U_2 = \{1,2,3\}$ with $(U_1, U_2)/_R = \{(\{a\}, \{1,2\}), (\{b\}, \{3\})\}$ and

$(X_1, X_2) = (\{a, b\}, \{3\})$. Then $L_R(X_1, X_2) = (\{b\}, \{3\})$, $U_R(X_1, X_2) = (U_1, U_2)$ and

$B_R(X_1, X_2) = (\{a\}, \{1,2\})$. Therefore, $L_R(X_1, X_2) \neq U_R(X_1, X_2)$. Then the nano binary topology

$\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), (\{a\}, \{1,2\}), (\{b\}, \{3\})\}$, where $\overline{N_B}\,(U_1, U_2) = (U_1, U_2)$,

$\overline{N_B}\,(\Phi, \Phi) = (\Phi, \Phi)$, $\overline{N_B}\,(\{a\}, \{1,2\}) = (\{a\}, \{1,2\})$, $\overline{N_B}(\{b\}, \{3\}) = (\{b\}, \{3\})$.

That is, nano binary closure of each nano binary open set in (U_1, U_2) is nano binary open. Therefore, $(U_1, U_2, \tau_R(X_1, X_2))$ is nano binary extremally disconnected but $L_R(X_1, X_2) \neq U_R(X_1, X_2)$.

Theorem 4.6: If $L_R(X_1, X_2) = (\Phi, \Phi)$, then $(U_1, U_2, \tau_R(X_1, X_2))$ is nano binary extremally disconnected.

Proof: Since$L_R(X_1, X_2) = (\Phi, \Phi)$, $B_R(X_1, X_2) = U_R(X_1, X_2)$, we have

$\tau_R(X_1, X_2) = \{(\Phi, \Phi), (U_1, U_2), U_R(X_1, X_2)\}$ where $\overline{N_B}\,(U_1, U_2) = (U_1, U_2)$, $\overline{N_B}\,(\Phi, \Phi) = (\Phi, \Phi)$ and $\overline{N_B}\,(U_R(X_1, X_2)) = (U_1, U_2)$. That is, the nano binary closure of each nano binary open set is nano binary open is (U_1, U_2). Hence (U_1, U_2) is nano binary extremally disconnected.

Theorem 4.7: If $L_R(X_1,X_2) \neq (\Phi,\Phi)$ and $U_R(X_1,X_2) = (U_1,U_2)$, then $(U_1,U_2,\tau_R(X_1,X_2))$ is nano binary extremally disconnected.

Proof: Since $L_R(X_1,X_2) \neq (\Phi,\Phi)$ and $U_R(X_1,X_2) = (U_1,U_2)$ we have $\tau_R(X_1,X_2) = \{(\Phi,\Phi),(U_1,U_2),L_R(X_1,X_2),B_R(X_1,X_2)\}$ where $\overline{N_B}$ (U_1,U_2) $=(U_1,U_2)$, $\overline{N_B}$ (Φ,Φ) $=$ (Φ,Φ) and $\overline{N_B}$ $(U_R(X_1,X_2))$ $=(U_1,U_2)$. That is, the nano binary closure of each nano binary open set is nano binary open is (U_1,U_2). Hence (U_1,U_2) is nano binary extremally disconnected.

Remark 4.8: If $L_R(X_1,X_2) \neq U_R(X_1,X_2)$ where $L_R(X_1,X_2) \neq \Phi$ and $U_R(X_1,X_2) \neq (U_1,U_2)$ then $(U_1,U_2,\tau_R(X_1,X_2))$ need not be nano binary extremally disconnected.

Example 4.9:

Let $U_1 = \{a,b,c\}, U_2 = \{1,2,3\}$ with $(U_1,U_2)/_R = \{(\{a\},\{2\}),(\{b\},\{1\}),(\{c\},\{3\})\}$ and

$(X_1,X_2) = (\{a\},\{2,3\})$. Then $L_R(X_1,X_2) = (\{a\},\{2\})$, $U_R(X_1,X_2) = (\{a,c\},\{2,3\})$ and

$B_R(X_1,X_2) = (\{c\},\{3\})$ thus $\tau_R(X_1,X_2) = \{(\Phi,\Phi),(U_1,U_2),(\{a\},\{2\}),(\{a,c\},\{2,3\}),(\{c\},\{3\})\}$. Then the nano binary closed sets in (U_1,U_2) are $(\Phi,\Phi),(U_1,U_2),(\{b,c\},\{1,3\}),(\{b\},\{1\})$ and $(\{a,b\},\{1,2\})$. Now, $\overline{N_B}$ (U_1,U_2) $=(U_1,U_2)$,

$\overline{N_B}$ $(\Phi,\Phi) = (\Phi,\Phi)$, $\overline{N_B}$ $(\{a\},\{2\}) = (\{a,b\},\{1,2\}) \notin \tau_R(X_1,X_2)$,

$\overline{N_B}$ $(\{a,c\},\{2,3\}) = (U_1,U_2)$ and $\overline{N_B}$ $(\{c\},\{3\}) =(\{b,c\},\{1,3\}) \notin \tau_R(X_1,X_2)$,

That is, nano binary closure of each nano binary open set in (U_1,U_2) is not nano binary open hence it is not nano binary extremally disconnected

5. NANO BINARY GENERALIZED CLOSED SET

Definition 5.1: Let $(U_1,U_2,\tau_R(X_1,X_2))$ be a nano binary topological spaces. A subset (H_1,H_2) of $(U_1,U_2,\tau_R(X_1,X_2))$ is called nano binary generalized closed set if $\overline{N_B}$ $(H_1,H_2) \subseteq (X_1,X_2)$ whenever $(H_1,H_2) \subseteq (X_1,X_2)$ and (X_1,X_2) is nano binary open.

Definition 5.2: A subset (H_1,H_2) of (U_1,U_2) is said to be nano binary generalized open if its complement is nano binary generalized closed.

Definition 5.3: The intersection of all nano binary generalized closed sets containing (H_1,H_2) is said to be nano binary generalized closure of (H_1,H_2).

Definition 5.4: The union of all nano binary generalized open sets contained in (H_1, H_2) is said to be nano binary generalized interior of (H_1, H_2).

Definition 5.5: A nano binary generalized closed set is denoted by $\overline{N_B}^{*}$ and nano binary generalized open set is denoted by ${N_B}^{\circ *}$.

Result 5.6: 1. Every nano binary open set is nano binary generalized open.

2. Every nano binary closed set is nano binary generalized closed.

CONCLUSION

Thus, the nano binary topological spaces were introduced and studied. The further results of nano binary topological spaces are under discussion.

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