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Behaviour of Kuratowski Operators on Some New Sets in Topology

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Abstract

The purpose of this paper is to introduce q_k – sets using Kuratowski's operator and to study its basic properties. In Topology, the Kuratowski's operator plays a pivotal role to define a topological structures on a set. As the closure and interior operator plays a major role in rough set theory, here we introduce q_k-closure and q_k– interior some of its properties are discussed.

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1. Introduction

In Topology, The Kuratowski's operator has a great role to define a topological structure on a set. Chandrasekhara Rao and P. Thangavelu introduce q-set and studied its basic properties. As an application of Kuratowski operator on q set, we introduce qk sets, the basic properties of the q_k sets, interior and closure operations are introduced . We introduce q_k interior, $q_k$closure and its properties are investigated. The concepts of q sets are studied in [9]. Throughout this paper X is a topological space and A, B are the subsets of X. The cl A and Int A are the notations which represents closure of A and Interior of A respectively . The following concepts bring backs the basics which are used in this paper to our memory. A subset of a topological spaces X is clopen if it is both open and closed. Let A be the subset of X then cl^*A is the intersection of all g-closed set containing A and then int^*A is the union of all g-open set contained in A.

2. q_k– Sets and Its Basic Properties.

Definition 2.1:

A set A of $(X, \ )$ is said to be q_k– set if $cl^*(int\ A) \supseteq int^*(cl\ A)$.

Definition 2.2 :

A set A of $(X, \ )$ is said to be p_k– set if $cl^*(int\ A) \subseteq int^*(cl\ A)$.

Result 2.3 :

If A is g- clopen then A is a p_k – set.

Result 2.4 :

If A is clopen then A is both q_k and p_k

Result 2.5 :

Union of q_k– sets need not be q_k . The result is described with the help of the following example. Let $X = \{a, b, c, d\}\ and\ \tau = \{\ \varphi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ be a topology on X. Here {a} and {b,c} are q_k – sets but their union {a}∪ {b,c} ={a,b,c} which is not a q_k -set.

Result 2.6:

Intersection of q_k- sets need not be q_k-set . The result is explained in the following example. Let $X = \{a, b, c, d, e\}\ and\ \tau = \{\ \varphi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$ be a topology on $X.$ Here {a,b,c,e} and {a,b,d} are two q_k – sets but their intersection {a,b,c,e} ∩ {a,b,d} = {a,b} which is not a q_k- set.

Result 2.7:

Union of p_k – sets need not be p_k – set. The result is discussed using the example shown below. Let $X = \{a, b, c, d, e\}\ and\ \tau = \{\ \varphi, \{a, b\}, \{\ c, d\}, \{a, b, c, d\}, X\}$ be the topology on X. Here {a} and {b} are p_k- set but their union {a} ∪ {b} = {a,b} which is not a p_k – set.

Result 2.8:

Intersection of p_k- sets need not be p_k set. The result is explained by the following example. Let $X = \{a, b, c, d\}\ and\ \tau = \{\ \varphi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ be the topology on X. Here {a,c} and {a,b} are p_k sets but their intersection is {a} which is not a p_k set.

Theorem 2.9 :

Every q_k– sets is a q set .

Proof : Let A be a q_k– set . Clearly, we say that $cl\ (int\ A) \supseteq cl^*(int\ A) \supseteq int^*(cl\ A) \supseteq int\ (cl\ A)$. Since A is a q – set.

Remark 2.10:

The converse of the above theorem is not true that is "Every q – set need not be q_k– set . It is shown in the following example. Let $X = \{a, b, c, d\}\ and\ \tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ be the topology on X. Here {a,c} and {a,d} are q -sets but they are not q_{k} – sets .

Theorem 2.11 :

A is a q_k– set if and only if X-A is a q_k -set.

Proof : Suppose A is a q_k – set .

Then , $cl^*(int\ X - A) \subseteq X - int^*(cl\ A)$

$\supseteq X - cl^*(int\ A)$

$= int^* cl\ (X - A)$

$\Rightarrow X - A$ is a q_k – set .

Conversely, We assume that $X - A$ is a q_k – set

$$\Rightarrow cl^*(int\ A) = cl^*(int\ (X - (X - A))) = X - int^*\ cl(X - A)$$

$$\supseteq X - cl^* int\ (X - A)$$

$$= int^* cl\ (X - (X - A))$$

$$= int^* cl\ A$$

Hence, A is a q_k – set .

Theorem 2.12:

Every p set is a p_k – set .

Proof :

Suppose A be a p – set.

$$\Rightarrow cl\ (int\ A) \subseteq int\ (clA)$$

Now, $cl^*(int\ A) \subseteq cl\ (int\ A) \subseteq int(cl\ A) \subseteq int^*(cl\ A)$

$$\Rightarrow cl^*(int\ A) \subseteq int^*(cl\ A)$$

Hence, A is a p_k - set.

Remark 2.13:

The Converse of the above theorem is not true .It is shown in the following example:

If $X = \{a, b, c, d\}$ and $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ be a topology on X.Here , every subset of X i.e. P(X) is a p_k- set which are behaving like q sets. Hence, none of them are p sets.

Result 2.14:

The proper sets of X is neither p_k setsnor q_k sets. It is shown in the following example:

Let $X = \{a, b, c, d\}$ and $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ be a topology on X .

Let A = { a} and B = {b} which are neither p_k set nor $q_k$set. Since, $cl^*(int\ A) =$ {a,d} and $int^*(cl\ A)$= {a,c}. Similarly, $cl^*(int\ B) =$ {b,d} and $int^*(cl\ A) =$ {b,c} from this we conclude that the proper sets of X neither p_k sets nor q_k sets.

Lemma 2**. 15** :

If A is a subset of the topological space X then $\alpha^{s*}cl\ \alpha^{s*}\ int\ A = cl\ int\ A$ Where α^{s*} interior of A is denoted by $\alpha^{s*}int\ A$ and α^{s*}closure of A is denoted by $\alpha^{s*}cl\ A$ respectively.

Proof : Suppose x $\in cl\ int\ A$ $\Rightarrow$ x $\in \cap F$, F is closed set such that $F \subseteq int\ A \subseteq \alpha^{s*}\ int\ A$. Since every closed set is α^{s*} closed , x belongs to instersection of all α^{s*} closed set F such that all α^{s*} closed set F contained in $\alpha^{s*}\ int\ A$ which implies us that x $\in \alpha^{s*}cl\ \alpha^{s*}\ int\ A$ Hence , $cl\ int\ A \subseteq \alpha^{s*}cl\ \alpha^{s*}\ int\ A$. Now , suppose x $\notin cl\ int\ A$ $\Rightarrow$ x $\notin \cap F$, F is the closed set such that $F \subseteq int\ A$ $\Rightarrow$ x $\notin F$, for some F is α^{s*}-closed such that $F \subseteq int\ A \subseteq \alpha^{s*}\ int\ A$. This implies that x $\notin \alpha^{s*}cl\ \alpha^{s*}\ int\ A$. Hence , $\alpha^{s*}cl\ \alpha^{s*}\ int\ A = cl\ int\ A$.

Theorem 2.16:

If A is a p_k – set if and only if X -A is a p_k – set.

Proof :

Suppose A is a p_k – set

Then , $cl^*(int\ X - A) = X - int^*(cl\ A)$

$\subseteq X - cl^*(int\ A)$

$= int^*cl\ (X - A)$

$\Rightarrow X - A$ is a p_k – set .

Conversely, We assume that $X - A$ is a p_{k-} set

$\Rightarrow$ $cl^*(int\ A) = cl^*(int\ (X - (X - A)))= X - int^*\ cl(X - A) \subseteq X - cl^* int\ (X - A)$

$= int^* cl\ (X - (X - A))$

$= int^* cl\ A$

Hence, A is a p_{k-} set .

Proposition 2.17 :

If A is clopen , A and B is a p_- set then A∩B is also a p_{k-} set.

3. q_k– Interior and q_k– Closure:

Definition 3.1:

Let A be a subset of a topological space X .The q_k– Interior of A is denoted by q_k– int and is defined as the union of all q_k– sets contained in A .

Let A be a subset of a topological space X .The q_k– Closure of A is denoted by q_k– cl and is defined as the intersection of all q_k– sets containing A .

As the collection of q_k- sets is not closed under union and intersection it follows that q_k–$int\ A$ need not be a q_k – set and also q_k– $cl\ A$ need not be a q_k – set. But ,q_k–$int\ A \subseteq A \subseteq$ q_k – $cl\ A$ is always true for any subset A of a topological space.

Proposition 3.2:

(i) q_k–$int\ \varphi = \varphi$; q_k–$cl\ \varphi = \varphi$.

(ii) q_k–$int\ X = X$; q_k–$cl\ X = X$.

(iii) q_k–$int\ A \subseteq A \subseteq$ q_k – $cl\ A$

(iv) A ⊆ B ⇒ q_k–$int\ A \subseteq$ q_k–$int\ B$ & A ⊆ B ⇒ q_k–$cl\ A \subseteq$ q_k–$cl\ B$

(v) q_k–$int\ (A \cap B) \subseteq$ q_k–$int\ A \cap$ q_k–$int\ B$

(vi) q_k–$cl(A \cap B) \subseteq$ q_k–$cl\ A \cap$ q_k–$cl\ B$

(vii) q_k–$cl(A \cup B) \supseteq$ q_k–$cl\ A \cup$ q_k–$cl\ B$

(viii) q_k–$int(A \cup B) \supseteq$ q_k–$int\ A \cup$ q_k–$int\ B$

(ix) q_k–int (q_k–$int\ A$) $\subseteq$ q_k–$int\ A$

(x) q_k–cl (q_k–$cl\ A$) $\subseteq$ q_k–$int\ A$

(xi) q_k–int (q_k–$cl\ A$) $\supseteq$ q_k–$int\ A$

(xii) q_k–cl (q_k–$int\ A$) $\subseteq$ q_k–$cl\ A$

Proof:

The proof of (i) and (ii) are obvious.In order to prove (iii) we assume $x \notin A$ whichimplies that x does not belongs to any q_k- sets contained in A.Hence, $x \notin q_k - int\ A$ which gives us $q_k - int\ A \subseteq A$ and suppose , $x \notin q_k - cl\ A$ then x does not belongs to any q_k- sets containing A.Hence we get that $A \subseteq q_k - cl\ A$. Using the above two inclusions we get that $q_k - int\ A \subseteq A \subseteq q_k - cl\ A$.(iv) The result is obvious .(v)we have $A \cap B \subseteq A$ and $A \cap B \subseteq B$ using these results we get $q_k - int\ (A \cap B) \subseteq q_k - int\ A$ and $q_k - int\ (A \cap B) \subseteq q_k - int\ B$ which implies us that

$q_k - int\ (A \cap B) \subseteq q_k - int\ A \cap q_k - int\ B$(vi) Also since we have $A \cap B \subseteq A$ and $A \cap B \subseteq B$ using these results we get $q_k - cl(A \cap B) \subseteq q_k - cl\ A$ and $q_k - cl(A \cap B) \subseteq q_k - cl\ B$ which implies us that $q_k - cl\ (A \cap B) \subseteq q_k - cl\ A \cap q_k - cl\ B$.(vii) We know that $A \cup B \supseteq A$ and $A \cup B \supseteq B$ it follows that $q_k - int(A \cup B) \supseteq q_k - int\ A$ and $q_k - int(A \cup B) \supseteq q_k - int\ B$ which implies that $q_k - int(A \cup B) \supseteq q_k - int\ A \cup q_k - int\ B$.(viii) We know that $A \cup B \supseteq A$ and $A \cup B \supseteq B$ it follows that $q_k - cl(A \cup B) \supseteq q_k - cl\ A$ and $q_k - cl(A \cup B) \supseteq q_k - clB$ which implies that $q_k - cl(A \cup B) \supseteq q_k - cl\ A \cup q_k - cl\ B$. Since we have $q_k - int\ A \subseteq A$ $A \subseteq q_k - cl\ A$ with the help of this inclusions we can easily establish the proof of (ix)(x) (xi) and (xii). Hence, the proof of the proposition.

Remark :

However, the reverse inclusions of (v)(vi)(vii)(viii)(ix)(x)(xi) and (xii) of proposition 3.2 are not true in general.

The following results can be established easily.

The inclusions may be strict in some cases they are explained with the help of the following example.

Lets us see $for\ the\ following\ inclusion$ A $\subseteq$ B $\Rightarrow q_k - int\ A \subseteq q_k - int\ B$

Let X be the topological spaces. Let $\tau = \{\varphi, \{a,b\}, \{c,d\}, \{a,b,c,d\}, X\}$. Suppose A={a} and B={a,b} then $q_k - int\ A = \varphi$ and $q_k - int\ B = \{a\}$ it follows as A $\subseteq$ B $\Rightarrow q_k - int\ A \subset q_k - int\ B$. If suppose let A = {b} and B = {b,d} then $q_k - int\ A = \varphi$ and $q_k - int\ B = \varphi$ which gives us A $\subseteq$ B $\Rightarrow q_k - int\ A = q_k - int\ B$

Next we check for the inclusion $q_k - cl(A \cup B) \supseteq q_k - cl\ A \cup q_k - cl\ B$ For that Let X be the topological spaces. Let $\tau = \{\varphi, \{a,b\}, \{c,d\}, \{a,b,c,d\}, X\}$. Suppose A ={ a,b} and B = {c,d} then A∪B= { a,b,c,d} here $q_k - cl(A \cup B)$= { a,b.c,d} and $q_k - cl\ A \cup q_k - cl\ B$ = {a,b,c,d,e} which follows that$q_k - cl(A \cup B) \supset q_k - cl\ A \cup q_k - cl\ B$. Also if A={a} and B = {a,b} , $A \cup B = \{a,b\}$$q_k - cl(A \cup B) = \{a,b,e\}$$q_k - cl\ A \cup q_k - cl\ B$ = {a,b,e} which implies us that $q_k - cl(A \cup B) = q_k - cl\ A \cup q_k - cl\ B$.

Next we go for the explanation of strict inclusions of q_k–$int(A \cup B) \supseteq q_k$–$int\ A \cup q_k$ – $int\ \ B$.

For this we consider Let X be the topological spaces. Let $\tau = \{\varphi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$

Here , Suppose A= { a, d} and B ={ a,b,c} A ∪ B ={a,b,c,d} q_k–$int(A \cup B)$= {a,b,c,d}q_k–$int\ A \cup q_k$–$int\ \ B$= {a,b} which implies thatq_k–$int(A \cup B) \supset q_k$–$int\ A \cup q_k$ –$int\ \ B$

Also if suppose A = { c,d,e } B = { b,c,d,e} A ∪ B ={b,c,d,e} then q_k–$int(A \cup B)$= {c,d,e} andq_k–$int\ A \cup\ q_k$ –$int\ B$ = {c,d,e} hence we get that q_k–$int(A \cup B)$=q_k–$int\ A \cup q_k$–$int\ \ B$

Next we see the examples for the strict inclusion of q_k–int (q_k–$int\ A$)⊆q_k–$int\ A$. Let X be the topological spaces. Let $\tau = \{\varphi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$ be the topology on X .Here , Suppose A ={ a,b} then q_k–$int\ A$ = { a,b} and q_k–int (q_k–$int\ A$) = {a,b} which implies us q_k–int (q_k–$int\ A$)=q_k–$int\ A$. Also if suppose A= { b,d,e} q_k–$int\ A$ = { e} and q_k–int (q_k–$int\ A$)= φ which follows that q_k–int (q_k–$int\ A$)⊂q_k–$int\ A$.

Next we see the examples of strict inclusions for q_k–int (q_k–$cl\ A$) ⊇q_k–$int\ A$. Let X be the topological spaces. Let $\tau = \{\varphi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$ be the topology on X . Suppose A = {a,c} then q_k–int (q_k–$cl\ A$) = {a,b,c,d} and q_k–$int\ A = \varphi$ which implies us thatq_k–int (q_k–$cl\ A$) ⊃ q_k–$int\ A$**.** Also if suppose A = { e} then q_k–int (q_k–$cl\ A$) = {e} and q_k–$int\ A$ = {e} which follows that q_k–int (q_k–$cl\ A$) = q_k–$int\ A$.

Proposition 3.3:

If A is a q_k– set then q_k–$int\ A = A = q_k$–$cl\ A$.

Proposition 3.4:

If A is a q_k– set then q_k–cl (q_k–$int\ A$) $= A = q_k$– int (q_k– $cl\ A$) .

Proposition 3.5 :

If A is pre *- open and A is q_k- set then A is semi*-open .

Proof :

Suppose A is pre *- open and A is q_k- set which implies that $A \subseteq int^*cl\ A$ and if $cl^*(int\ A) \supseteq int^*(cl\ A)$ it follows that $A \subseteq cl^*int\ A \Rightarrow$ A is semi *- open .

Proposition 3.6 :

If A is semi *- open and A is p_k- set then A is pre*-open .

Result 3.7 :

Every α^{s*}-open is pre*-open

Proposition 3.7 :

If A is α^{s*}- open and A is q_k- set then A is semi*-open .

Proof : The proof of the proposition follows from the above result and proposition.

Proposition 3.8:

For any subset $Y \subseteq X$, q_k–$int\ Y = X - q_k$–$cl\ (X - Y)$.

Proposition 3.9:

Suppose $A \subseteq B$ with $cl\ A = cl\ B$. If A is a q_k – set then B is a q_k– set .

Proof:

Suppose $A \subseteq B$ and A is a q_k – set which implies us that $int^{*}cl\ B = int^{*}cl\ A \subseteq cl^{*}int\ A \subseteq cl^{*}int\ B$. Hence B is a q_k – set .

Proposition 3.10:

Suppose $A \subseteq B$ with $cl\ A = cl\ B$. If A and B are p_k – set then A is a p_k– set .

4. Algorithm of p_k & q_k –sets

The objectives of the algorithms are to generate $p_k\ or\ q_k$ –sets if any one of the sets is clopen

Theorem 4.1:

In a topological space, (X, τ), where $\tau = \{\varphi, U, V, U', U' \cup V, X\}$ where U' is a subset of U and if A is clopen Then A is both p_k & q_k – sets .

Proof:

Case (i) If A is clopen then A is both p_k & q_k – sets.

Case (ii)

Sub case (i)

If A is exact open then A = U' or $U' \cup V$.

Sub case (ii)

If A = U' then int A = U' which implies us that U= $cl\ (U')$. Hence , U' is not g- closed . Thus we get $cl\ (U') = cl^{*}(U') = U$ Hence, $cl^{*}(int\ A) = cl^{*}(U') = U$. Also , $int^{*}(cl\ A) = int^{*}(U) = U$.

Case (iii)

If A = $U' \cup V$, $cl\ (A) = X\ , int\ (A) = A$ which implies $cl^{*}(A) = X\ , int^{*}\ (\ A) = A$. Hence A is both p_k & q_k – sets. Thus we get $cl^{*}int\ (A) = X$ and $int\ ^{*}cl\ (A) = X$.

Case (iv)

If A is exactly closed ,then A = $X - U'$ is closed . Hence , U' is open which implies us U' is both p_k & q_k – sets . Thus , we get $X - U' = A$ is both p_k & q_k – sets.

Case (v)

If A is neither open nor closed

Sub case (i) :

Suppose $cl(A)$ is closed and $int(A) = \varphi$ is open ,Hence $int^{*}cl\ (A) = int^{*}(A)$ which implies that $cl^{*}int\ (A) = cl^{*}(intA) = \varphi$.Thus we get $cl^{*}int\ (A) \subseteq int^{*}(cl\ A)$ Hence, The set is a p_k – set .

Sub case (ii) :

Suppose $cl(A)$ = X and $int(A) = \varphi$ is open .Then, $cl^{*}int\ (A) = cl^{*}(A)$ and $int^{*}cl\ (A) = X$. Thus , we get $cl^{*}int\ (A) \subseteq int^{*}(cl\ A)$ Hence, The set is a p_k – set .

Theorem 4.2

If (X, τ) is a topological space and $\tau = \{\ \varphi, U, V, U', U' \cup V, X\ \}$ where U and V are clopen and $U' \subseteq U$ then the collection of q_k- set is equal to τ and the collection of p_k- set is 2^{x}.

Proof of the this theorem is similar to algorithm 4.1

Theorem 4.3:

In a Topological space (X,τ) where τ = {ϕ,X,U,V} where U∩V = ϕ and U∪V = X then the member of 2^{X} are p_k-set.

Proof:

Let A be any subset of X

Case (i): If A = U or V

Then A is clopen . hence A is p_k-set.

Case(ii) : If A ≠U and V

Then A is not clopen. Then cl(A) and int(A) are clopen.Hence int*cl(A) =cl(A) and cl*(int(A)) = int (A).Since int(A) ⊆ cl(A), cl*(int(A)) ⊆ int*cl(A). Hence A is p_k-set.

Proposition 4.4 :

If (X, τ) is a topological spaces and any one of the given set is clopen then the collection of q_k – sets are of the form { A⊆X \ A is either open or closed or clopen }.

Proposition 4.5 :

If (X, τ) is a topological spaces and any one of the given set is clopen then the collection of p_k – sets are of the form { A⊆X \ A is either open nor closed and $cl(A) = X$ or $int\ A = \varphi$ or A and $cl(A)$ are open or A and $int\ (A)$ are closed}

Remark 4.6 :

If (X, τ) is a topological spaces and any one of the given set is clopen and A is neither open nor closed and $cl(A) \neq X$ and $int\ A \neq \varphi$.Then A is neither p_k-set nor q_k- set .

5. Topology Generated by p_k & q_k −sets

Proposition 5.1 :

If A is clopen and B is q_k- set then $A \cap B$ and $A \cup B$ are q_k- set

Proposition 5.2:

If A is clopen and B is p_k- set then $A \cap B$ and $A \cup B$ are p_k- set

Proposition 5.3:

Let (X, τ) be a topological space Suppose A is Clopen and B is a q_k- set then $N = \{\varphi, A \cap B, A, A - (A \cap B), X\}$ is a topology and every member of N is a q_k- set .

Proof:

$N = \{\varphi, A \cap B, A, A - (A \cap B), X\}$ is clearly a topology on X .

Clearly, φ, A, X are all q_k- sets. Since A is Clopen and B is a q_k- set, By using the algorithm $A \cap B$ is a q_k- set. Since the complement of q_k- set is again a q_k- set, $X - (A \cap B)$ is a q_k- set. This proves that $A - (A \cap B) = A \cap (X - (A \cap B)$ is a q_k- set. Hence every member of N is a q_k- set.

Proposition 5.4:

Let (X, τ) be a topological space Suppose A is Clopen and B is a q_k- set then $N = \{\varphi, A, B, A \cap B, A \cup B, X\}$ is a topology and every member of N is a q_k- set .

Proof:

$N = \{\varphi, A \cap B, A, A - (A \cap B), X\}$ is clearly a topology on X .

Clearly, φ, A, X are all q_k- sets. Since A is Clopen and B is a q_k- set, by using algorithm we get , $A \cap B$ and $A \cup B$ are q_k- sets. Hence every member of N is a q_k- set.

Proposition 5.5:

Let (X, τ) be a topological space Suppose A is Clopen and B is a p- set then $N = \{\varphi, A \cap B, A, A - (A \cap B), X\}$ is a topology and every member of N is a p_k- set .

Proof:

$N = \{\varphi, A \cap B, A, A - (A \cap B), X\}$ is clearly a topology onX.

Clearly, φ, A, X are all p_k- sets. Since A is Clopen and B is a p_k- set, by using the algorithms we get, $A \cap B$ is a p_k- set. Since the complement of p_k- set is again a p_k- set, $X - (A \cap B)$ is a p_k- set. This proves that $A - (A \cap B) = A \cap (X - (A \cap B)$ is a p_k- set. Hence every member of N is a p_k- set.

Conclusion

Thus in this paper we have introduced p_k- sets and q_k- sets . Also some of their properties were discussed . Henceforth, we are discussing about some more characteristics of p_k- sets and q_k- sets in the further papers the results will be established.

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