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Representing K-parts Integer Partitions

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Abstract

In this paper, representation of K-parts unordered integer partitions is done and numbers of k-parts partitions are derived from such representation.

*In this paper unordered integer partitions for k=2, 3-parts are represented.

INTRODUCTION

2-parts partitions can be represented on a line as x + y=N, 3-parts on a plane (x +y +z=N) etc.

Example-

Unordered Integer partitions of N=5 are:

5,4+1,3+2,3+1+1,2+2+1,2+1+1+1,1+1+1+1+1

In these 2-parts partitions are: 4+1,3+2 which are two in number,3-parts partitions are: 3+1+1,2+2+1 which are two etc. Total numbers of "unordered partitions" of 5 are seven including 5(one-part partition).

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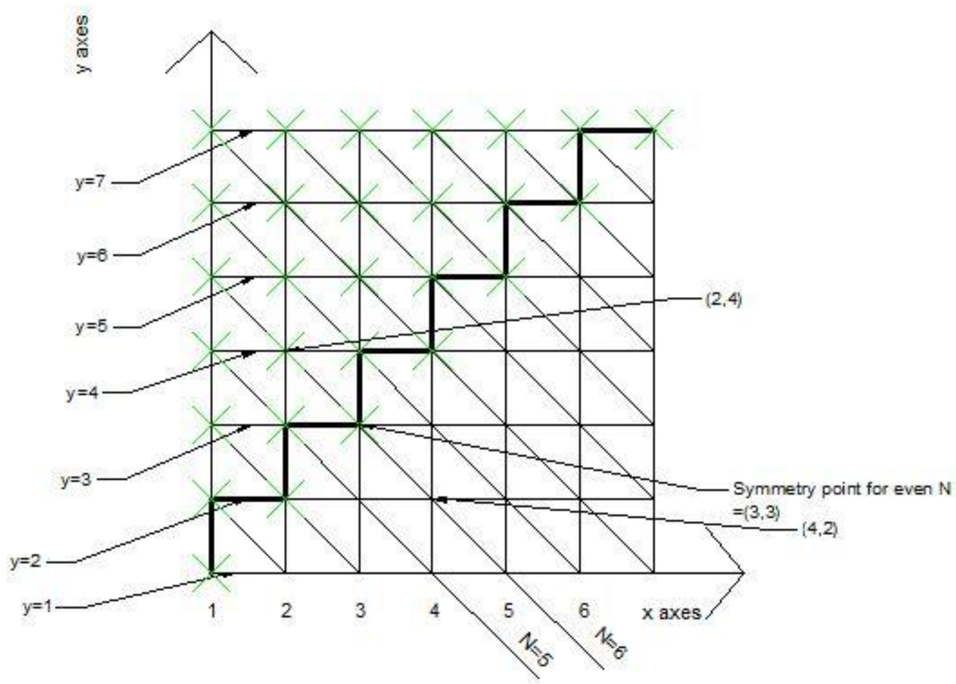
THEORY

1.) <u>2-parts partitions</u>- Of positive integer N can be represented by equation x + y=N, where x is one part and y another, 1≤x≤y. x + y=N is equation of line on Cartesian coordinate system, N-1 is x and y-intercept on X and Y-axis as shown in figure, below. Line x + y=N contains ordered partitions in positive quadrant. We have taken the origin as (1, 1).



1.) <u>For even N-</u>

Line x + y=N contains symmetry point for even N;(N/2,N/2) which separates two copies of unordered partitions. When we look in one direction and second time in other about symmetry point by equal steps we will find that x and y are interchanged for points in opposite direction(up-left & bottom right).For example for N=6 which is even, symmetry point is (3, 3), when we look one step back we see points (2, 4) in up-left direction and (4, 2) in bottom-left. So if there are n steps,1=<n=<((N-2)/2), point in one direction is (x, y)& in opposite direction (y, x) .So, number of ordered partitions =2*p+1 where p are number of unordered partitions -1.Since 2*p+1=N-1 as x goes from 1 to N-1 for x + y=N line .On solving this equation we get p=(N-2)/2 and number of unordered partitions=((N-2)/2)+1=N/2(1 is added to include symmetry point).

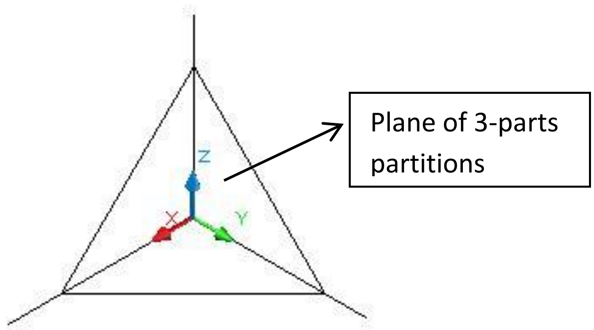
2.)<u>For odd N-</u>

There is no symmetry point for odd N. Number of ordered partitions=N-1 which is even ,when x=(N-1)/2,y=(N+1)/2 next point in the bottom right direction along constant N line=(N+1)/2, (N-1)/2 so x and y are interchanged . This is because ((N-1)/2)-n=((N+1)/2)-(n+1) & ((N+1)/2)+n=((N-1)/2)+(n+1),where n are number of steps in up-left/bottom right direction along line N. Point/partition at n steps from ((N-1)/2 , (N+1)/2) in up-left direction: (((N-1)/2)-n,((N+1)/2)+n) and in bottom-right direction=(((N-1)/2)+(n+1),((N+1)/2)-(n+1)).So, number of unordered partitions =(N-1)/2.

RESULT

P(N,2)=N/2 if 2|N,else P(N,2)=(N-1)/2,P(N, k) is number of partitions of N of k parts.

2.)<u>3-parts partitions</u>- Let the three parts of N be x, y, z ,x>=y>=z>=1 then x + y + z=N which is equation of plane in 3-D.Origin is taken (1,1,1) .So the plane is now an equilateral triangle of length N-2 lying on plane x + y + z=N and it contains 3-parts ordered partitions of N. Ordered partitions/points lie at intersection of constant x-y , y-z , z-x lines with 1=<x, y, z=<N-2. Constant y-lines have slope=√3/2 or 0.866 & constant x-lines have slope -√3/2 or -0.866 on the triangular plane with respect to constant z-lines as shown in the figures below.



Figure

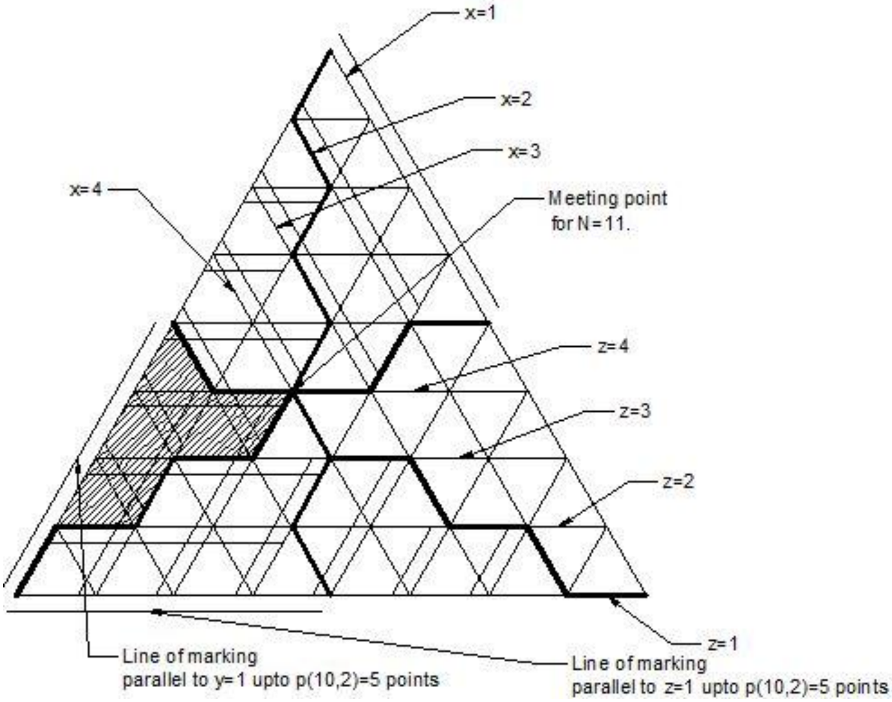
<u>Marking partitions to count number of unordered partitions & to eliminate repeating partitions-</u>

In order to obtain unordered partitions from ordered partitions we have to eliminate repeating partitions using the method of marking which is discussed below.

For a const. x, y or z line that contains 2-parts ordered partitions of N-x, N-y, N-z respectively we can use formula above in <u>Result</u> for 2-parts unordered partitions in which number of 2-parts partitions of integer N=N/2 if N is even& (N-1)/2 if N is odd. Let these unordered partitions of N be represented by p (N, 2).For a constant z line we will mark the line by a line parallel to the line from left to right up to p (N-z,

2) points along the z line& this we will do for every const. z line on the triangle for N as shown in the figure. Similarly for constant y-line we will mark the line by a line parallel to the line from bottom-left to up-right up to p(N-y,2) points along the line & this we will do for every const. y line lying on the triangle of N, for const. x-line we will mark the line by a line up to p(N-x,2) points from up-left to bottom right along the line & this we will do for all const. x lines.

We take this convention of marking, but any convention can be taken with directions of marking along const. lines different from the above convention.



Figure

Different points on the triangle- Points on the triangle for integer N can be categorized depending on their state of marking by the above convention discussed. Example-If a point (x, y, z) is marked along the const. x line,& not marked along const. y, z line then it is given 1 in x,0 in y& z & it is indicated by(1,0,0) or it is given indicator (1,0,0).There are $8(2^3)$ possibilities of such indicators out of which only 6 are feasible. Feasible indicators of points : (0,0,0),(0,1,0),(0,1,1),(1,0,0), (1,0,1), (1,1,1) non-feasible indicators: (0,0,1), (1,1,0).

1).p_0 type points- p_o type points include those points with indicator of marking (0,1,1) which must not be counted in unordered partitions as they are repeating partitions as they are marked on const. y, z lines but not on const. x lines.

2).<u>p_1 type points</u>- p_1 type points include those points with indicator of marking (1,0,1) which must not be counted in unordered partitions as they are repeating partitions as they are marked on const. x, z lines but not on const. y lines.

3).<u>pa_1 type points</u>- pa_1 type points include those points with indicator of marking (1,0,0) which must not be counted in unordered partitions as they are repeating partitions as they are marked only on const. x lines.

4).<u>a_1 type points</u>- a_1 type points include those points with indicator of marking (0,1,0) which must not be counted in unordered partitions as they are repeating partitions as they are marked only on const. y lines.

5).<u>a_2 type points</u>- a_2 type points include those points with indicator of marking (0,0,0) which must not be counted in unordered partitions as they are repeating partitions as they are not marked on const. y, z ,x lines.

6).<u>f_3 type points</u>- f_3 type points include those points with indicator of marking (1,1,1) which must be counted in unordered partitions as (1,1,1) is an indicator showing that it is not repeating in x, y &z.

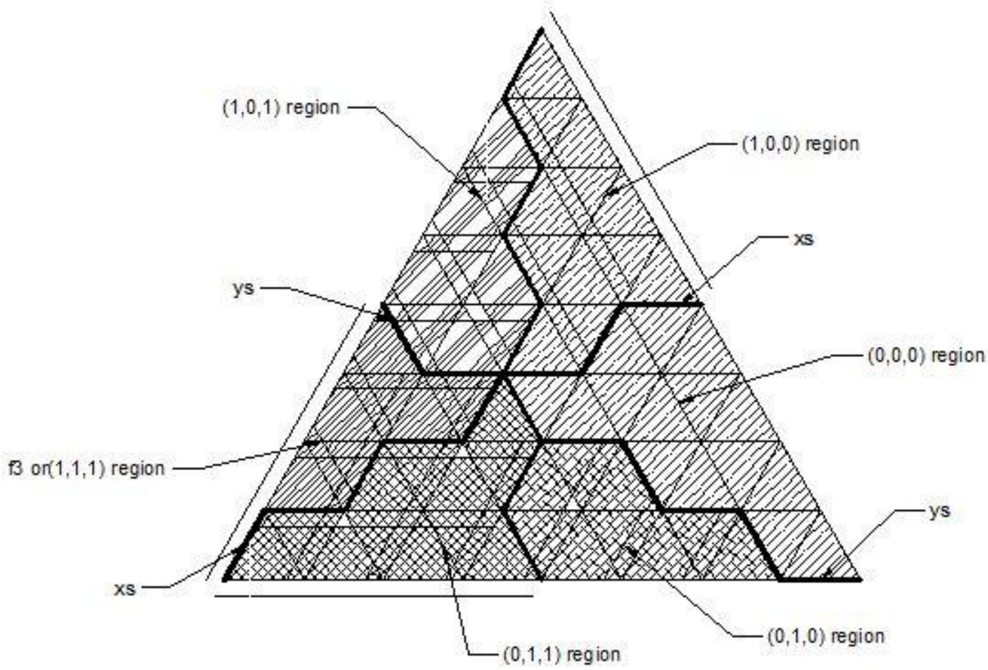
Number of 3-parts unordered partitions of N= Number of 3-parts ordered partitions of N/Total number of points on the triangle-Number of (p_0, p_1, pa_1, a_1, a_2) points =$\sum$(N-i) - Number of (p_0, p_1, pa_1, a_1, a_2) points, where $2 \le i \le N-1$=((N-2)(N-1)/2)- Number of (p_0, p_1, pa_1, a_1, a_2) points.

<u>Different regions as a result of marking-</u>

ys is a zigzag line which joins top (1, 1, 1) marked points of const. y lines as shown in the figure below, xs is a zigzag line which joins bottom (1, 1, 1) marked points of const. x lines as shown in the figure & zs is a zigzag line which joins rightmost (1, 1, 1) marked points of const. z lines as shown in the figure.

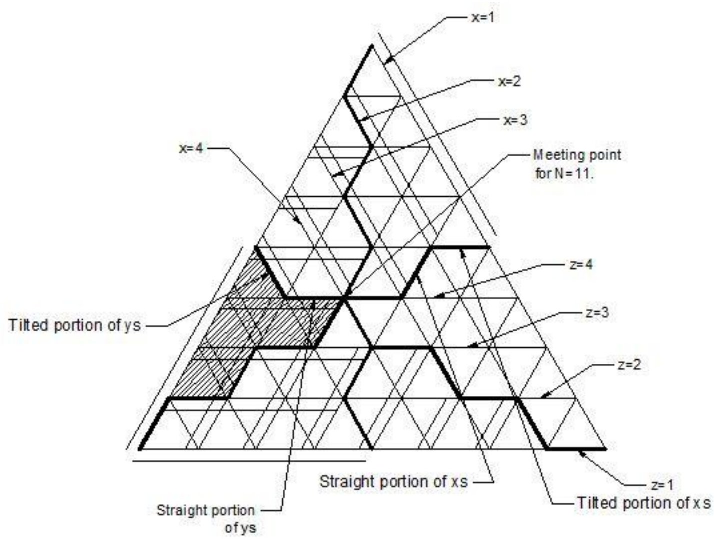
1).<u>f3 region</u>-It is region containing points of f3 type :(1,1,1).It is in the region between ys& xs line up to the meeting point from left side or region common to down-left side of ys line, up-left side of xs line& left side of zs line. This region also includes point lying on the ys, xs lines only up to the meeting point. Points lying in this region are counted in unordered 3-parts partitions.

Other regions are similarly constructed and are shown in the figure below:



Figure

<u>Analysis of the triangle of integer N</u>:-

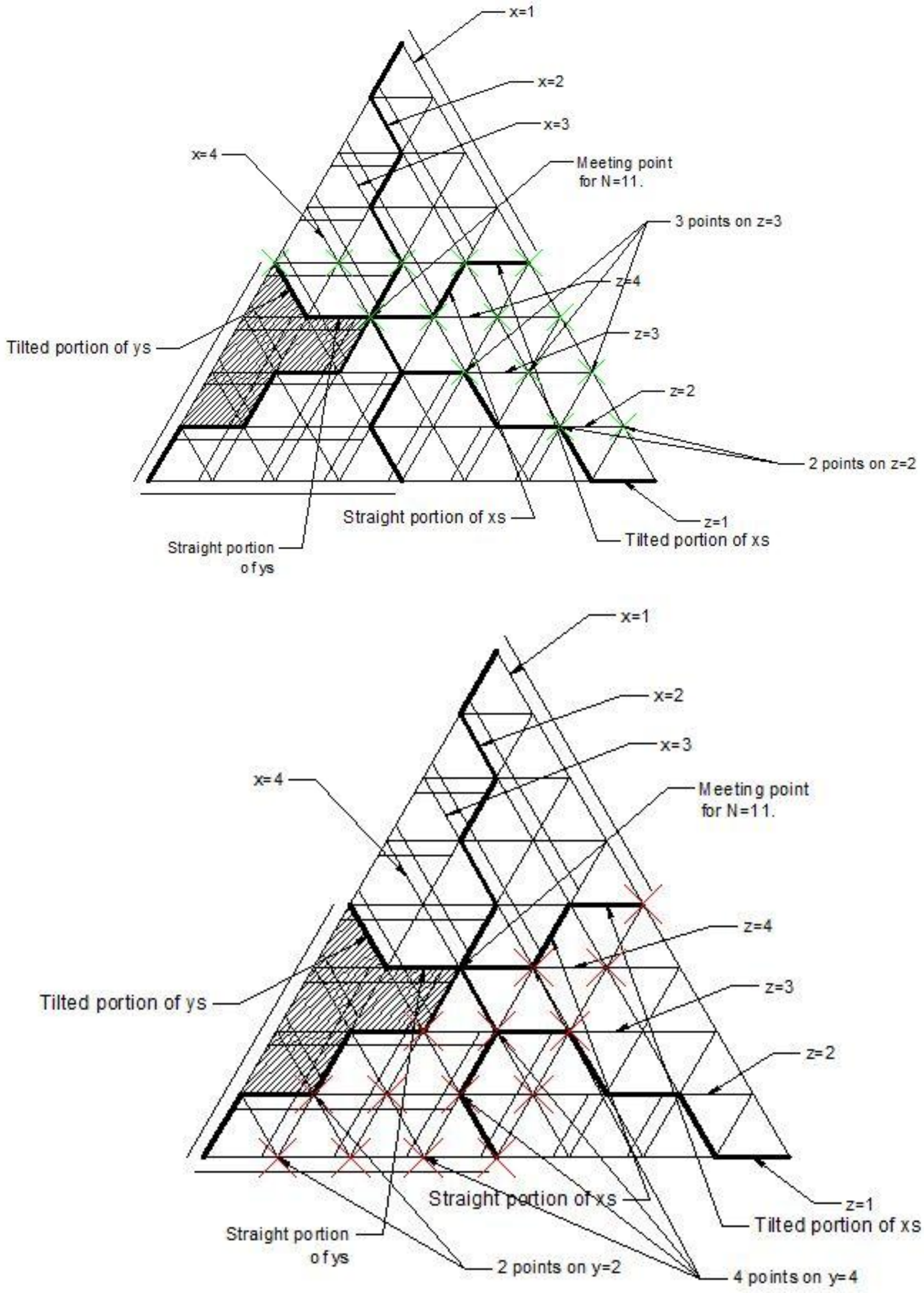


Figure

In the figure above, ys line in y= [1, 2] has slope -√3/2 with respect to const. z is called "tilted" ys &y= [2, 3] has slope o is straight portion of ys. Also in the figure xs line in x= [1, 2] has slope o with respect to const. z lines is called "tilted" xs with respect to const. y& in x= [2, 3] has slope √3/2 is straight portion of xs with respect to const. y.

Now we determine whether xs, ys lines are tilted or straight.

1).<u>Determination of Tilted Condition</u>-

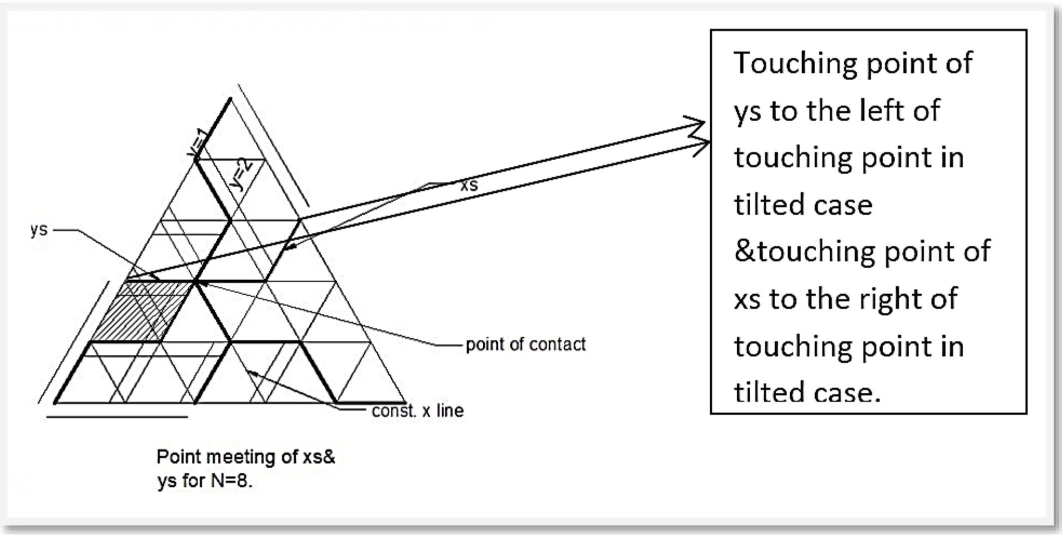


Figure

In the above figures, in first figure for tilted ys , in x= [1 ,2] there are two points on z=2, in x=[1,3] there are three points on z=3, in x= [1, 4] there are four points on z=4& so on up to the left touching point of ys with y=1 side of Triangle.N-z-1 is number of points on const. z line & is also the x-coordinate of touching point of ys with y=1.So, at touching point N-z-1=z ,which implies z=(N-1)/2 or N=2z+1 is odd. So, for odd N, ys is tilted.

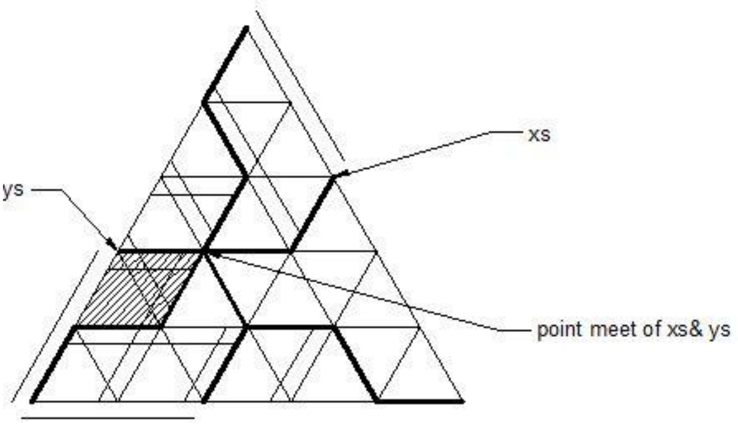
Similar to ys- in z= [1, 2] there are two points on y=2,in z= [1, 3]there are three points on y=3,in z= [1, 4]there are four points on z=4& so on up to the right touching point of xs with x=1 side of triangle.N-y-1 is number of points on const. y line & is also the x-coordinate of touching point of xs with x=1.So, at touching point N-y-1=N-z-1=z=(N-1)/2,which implies y=(N-1)/2 or N=2y+1 is odd. So, for odd N, xs is tilted. So, when N is odd xs& ys are tilted.

2).Determination of Straight Condition-

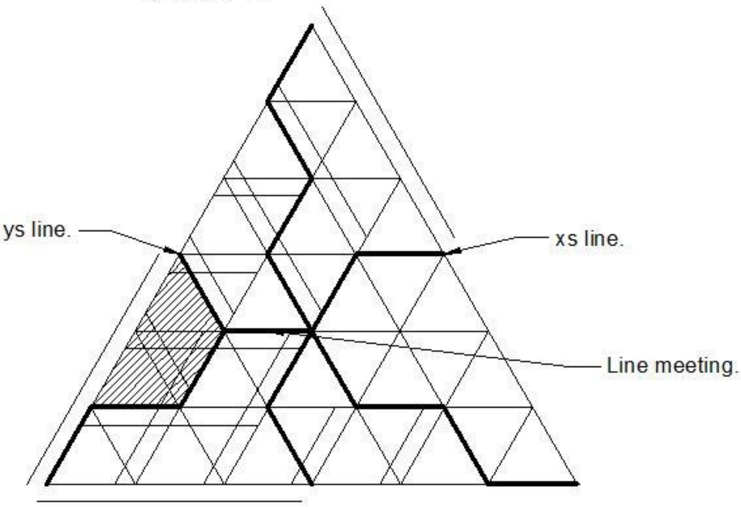


In straight ys touching point is one step to the left of touching point in tilted case, so: N-z-1=z+1, implies z= (N-2)/2 or N=2(z+1) which is even. In straight xs touching point is one step to the up-right of touching point in tilted case, so: N-y-1=y+1, implies y=(N-2)/2 or N=2(y+1) which is even. So, when N is even xs& ys are straight.

Meeting at a point, line for xs,ys&zs-



Point meeting of xs&
ys for N=8.



Line meeting of xs&
ys for N=9.

Figure

We will draw table showing whether there is point contact or line contact (two points contact) between xs-ys, ys-zs, xs-zs for different integers N. L denotes line contact, and P denotes point contact or meeting.

N	xs-ys	ys-zs	xs-zs
6	L	P	P
7	P	P	L
8	P	L	P
9	L	P	P
10	P	P	L

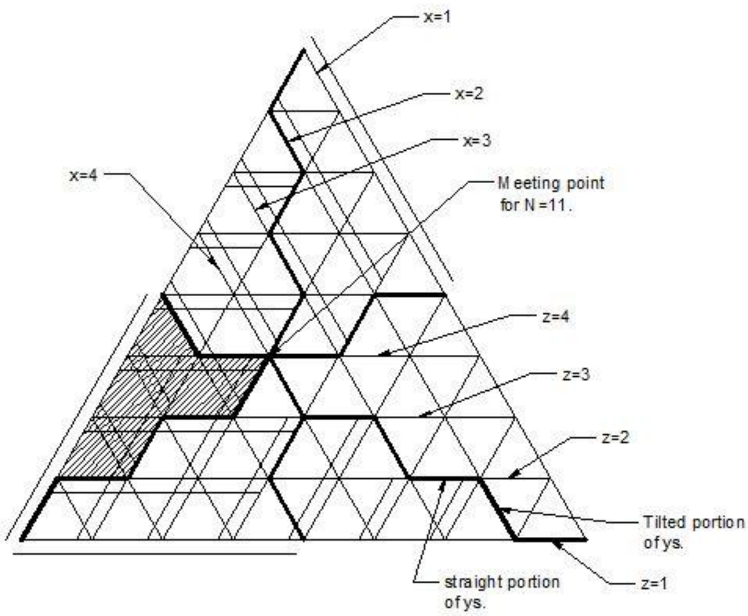
11	P	L	P
12	L	P	P
13	P	P	L
14	P	L	P

Looking at this table we see that the sequence LPP, PPL, PLP repeats after every three steps. So, by this assumption we can say that for N=15, Sequence will be L, P, P for xs-ys. We will prove this later. Line, point meet of xs, ys is important and we will study this while line, point meet of xs-zs, ys-zs is unimportant because f3 region is made by xs and ys.

Notations- p (N-x, 2) is number of unordered 2-parts partitions of N-x on the const. line x. (x_m, y_m, z_m) is meeting point.

1.)Point meet of xs and ys-y_m=p(N-x_m,2) as y increases from left to right on const. x line& x is marked from top left to right bottom .x_m =(N-y_m-1)-p(N-y_m,2)+1 as N-y_m-1 are total number of points on line y=p(N-x_m,2)&x increases from right to left on const. y line.1 is added to include the meeting point which gets removed when subtracting p(N-y_m,2) from (N-y_m-1). We can find z_m.-As we move from base of triangle or z=1, z increases by one and so does the marked point on the y line in counting. So, the number of points up to the meeting point =z_m= p (N-y_m, 2)

A).For tilted xs and ys-



Figure

N is odd for tilted case, z_m=N-y_m- p (N-y_m, 2) only when y_m is odd, not when y_m is even because p(N-y_m, 2)= (N-y_m)/2 when y_m is odd.

So, for tilted xs and ys, meeting point=N-y_m-p (N-y_m, 2), p (N-x_m, 2), p (N-y_m, 2)

1). <u>When x_m is even</u>-

Since, y_m=p (N-x_m, 2) = (N-x_m-1)/2 & x_m= N-y_m-p(N-y_m,2)-(i)

When y_m is even: x_m= (N-y_m)- (N-y_m-1)/2= (N-y_m+1)/2,2x_m=N-y_m+1,y_m=N+1-2x_m=(N-x_m-1)/2(from -(i)),2N+2-4x_m=N-x_m-1,(N+3)/3=x_m, x_m=1+N/3

Implies y_m=(N/3)-1(from -(i))& z_m= p(N-y_m,2)= (N-y_m-1)/2=(N-1-(N/3)+1)/2=N/3.

When y_m is odd: x_m= (N-y_m)- (N-y_m)/2= (N-y_m)/2,2x_m=N-y_m, y_m=N-2x_m=(N-x_m-1)/2(from (i)),2N-4x_m =N- x_m -1, (N+1)/3= x_m

Implies y_m=(N-2)/3(from -(i))& z_m= p(N- y_m,2)= (N- y_m)/2=(N-((N-2)/3))/2=(3N-N+2)/6=(N+1)/3.

2). <u>When x_m is odd</u>-

Since, y_m =p(N- x_m,2)=(N- x_m)/2 & x_m = N- y_m–p(N- y_m,2) -(ii)

When y_m is even: x_m= (N- y_m)- (N- y_m-1)/2= (N- y_m +1)/2, 2x_m =N- y_m +1, y_m =N+1-2x_m =(N- x_m)/2(from - (ii)), 2N+2-4x_m =N-x_m, (N+2)/3= x_m

Implies y_m=(N-1)/3(from -(ii))& z_m= p(N- y_m,2)= (N- y_m-1)/2=(3N-(N-1)-3)/6 =(2N-2)/6=(N-1)/3.

When y_m is odd: x_m= (N- y_m) - (N- y_m)/2= (N- y_m)/2, y_m =N-2x_m = (N- x_m)/2(from (ii)), 2N-4x_m =N- x_m, N/3= x_m

Implies y_m=N/3(from - (ii)) & z_m= p (N- y_m, 2) = (N- y_m)/2= (N-N/3)/2=N/3.

So,

(x_m, y_m, z_m)=

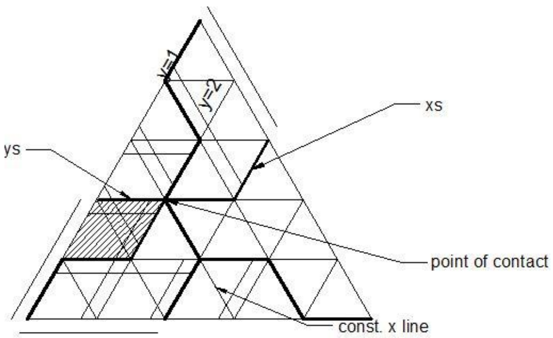
1). for even x_m& even y_m: ((N/3) +1, (N/3) -1, N/3)

2). for even x_m& odd y_m: ((N+1)/3, (N-2)/3, (N+1)/3)

3). for odd x_m& even y_m: ((N+2)/3, (N-1)/3, (N-1)/3)

4). for odd x_m& odd y_m: (N/3, N/3, N/3)-In this case x_m= z_m, which is to be noticed as it is useful in line meet.

B).<u>For straight xs and ys</u>-



Point meeting of xs& ys for N=8.

Figure

N is even for straight case, $z_m=x_m-1$ only when = $N-y_m-p(N-y_m,2)-1=z_m=p(N-y_m,2)$,implies $p(N-y_m,2)=(N-y_m-1)/2$ which further implies that y_m is odd. So, $z_m=x_m-1$ when y_m is odd. For straight xs and ys meeting point=$N-y_m-p(N-y_m, 2)$, $p(N-x_m,2)$, $p(N-y_m,2)$.

1). When x_m is even-

Since, $y_m=p(N-x_m,2)=(N-x_m)/2$ & $x_m=N-y_m-p(N-y_m,2)$-(i)

When y_m is even: $x_m=(N-y_m)-(N-y_m)/2=(N-y_m)/2, 2x_m=N-y_m$, $y_m=N-2x_m=(N-x_m)/2$(from -(i)),$2N-4x_m=N-x_m$, $N/3=x_m$,

Implies $y_m=(N-N/3)/2=N/3$(from -(i)) & $z_m=p(N-y_m,2)=(N-y_m)/2=(N-N/3)/2=N/3$.

When y_m is odd: $x_m=(N-y_m)-(N-y_m-1)/2=(N-y_m+1)/2, 2x_m=N-y_m+1, y_m=N-2x_m+1=(N-x_m)/2$(from (i)),$2N-4x_m+2=N-x_m,(N+2)/3=x_m$

Implies $y_m=(N-1)/3$(from -(i)) & $z_m=p(N-y_m,2)=(N-y_m-1)/2=(N-1-((N-1)/3))/2=(3N-N-3+1)/6=(N-1)/3$.

2). When x_m is odd-

Since, $y_m=p(N-x_m,2)=(N-x_m-1)/2$ & $x_m=N-y_m-p(N-y_m,2)$ -(ii)

When y_m is even: $x_m=(N-y_m)-(N-y_m)/2=(N-y_m)/2$, $2x_m=N-y_m$, $y_m=N-2x_m=(N-x_m-1)/2$(from -(ii)),$2N-4x_m=N-x_m-1,(N+1)/3=x_m$

Implies $y_m=(N-2)/3$(from -(ii))& $z_m=p(N-y_m,2)=(N-y_m)/2=(3N-(N-2))/6=(2N+2)/6=(N+1)/3$.

When y_m is odd: $x_m=(N-y_m)-(N-y_m-1)/2=(N-y_m+1)/2$, $y_m=N-2x_m+1=(N-x_m-1)/2$(from (ii)),$2N-4x_m+2=N-x_m-1$, $(N/3)+1=x_m$

Implies $y_m=(N/3)-1$(from - (ii)) & $z_m=p(N-y_m,2)=(N-y_m-1)/2=(N-N/3)/2=N/3$.

So,

(x_m, y_m, z_m)=

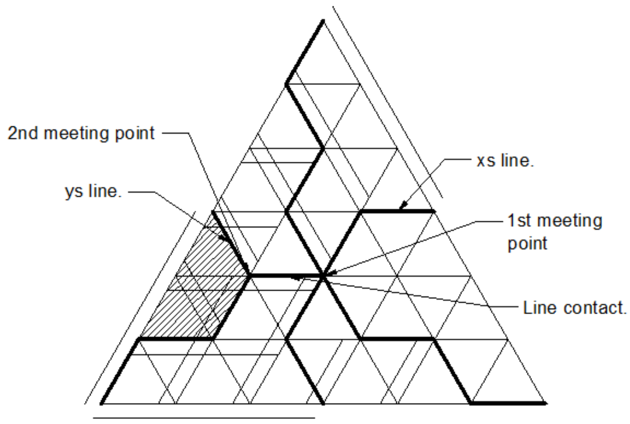
1). for even x_m& even y_m: (N/3, N/3, N/3) - In this case x_m= z_m, which is to be noticed as it is useful in line meet.

2). for even x_m& odd y_m: ((N+2)/3, (N-1)/3, (N-1)/3)

3). for odd x_m& even y_m: ((N+1)/3, (N-2)/3, (N+1)/3)

4). for odd x_m& odd y_m: ((N/3) +1, (N/3)-1, N/3)

2).Line contact of xs & ys-



Figure

Notation- (x_{m1}, y_{m1}, z_{m1}) is first meeting point, (x_{m2}, y_{m2}, z_{m2}) is second meeting point.

When xs & ys line meet there are two meeting points common in xs &ys & both are f3 type points. First meeting point is right to second meeting point in the triangle shown. $z_{m1}= z_{m2}=z_m$ because xs &ys meet on line segment parallel to const. z lines, $z_m=x_{m1}$=p (N-y_{m1},2), $x_{m2}= x_{m1}+1$, $y_{m1}= y_{m2}+1$.

1).for tilted xs & ys – Since, $z_{m1}= z_{m2}= z_m$ so only two meeting points out of four in point meet of tilted case are feasible for two meeting points:

i). for even x_m& even y_m: ((N/3) +1, (N/3) -1, N/3)-Second meeting point.

ii). for odd x_m& odd y_m: (N/3, N/3, N/3). Since, $x_{m2}= x_{m1}+1$ &$y_{m1}= y_{m2}+1$ which is kept by the two points in (i) & (ii).So, first meeting point is (N/3, N/3, N/3) and second is ((N/3) +1, (N/3) -1, N/3).

2).for straight xs & ys – Since, $z_{m1}= z_{m2}= z_m$ so only two meeting points out of four in point meet of tilted case are feasible for two meeting points:

i).<u>for even x_m& even y_m</u>: (N/3, N/3, N/3) - First meeting point.

ii).<u>for odd x_m& odd y_m</u>: ((N/3) +1, (N/3)-1, N/3). Since, $x_{m2}= x_{m1}+1$ &$y_{m1}= y_{m2}+1$ which is kept by the two points in (i) & (ii).So, first meeting point is (N/3, N/3, N/3) and second is ((N/3) +1, (N/3) -1, N/3).

So, for line contact in tilted case in which Nis odd, 3|N(3 divides N) or N=3k={3, 9, 15, 21, ...}=3(2q-1),q≥1 and in straight case in which N is even, 3|N (3 divides N) or N=3k`= {6, 12, 18, 24, ...}=3*2q=6q, q≥1.Hence,proved that line contact between xs& ys repeats after every 3 steps for N or for line contact in xs-ys, N={3,6,9,12,15,18,21,24,..}.

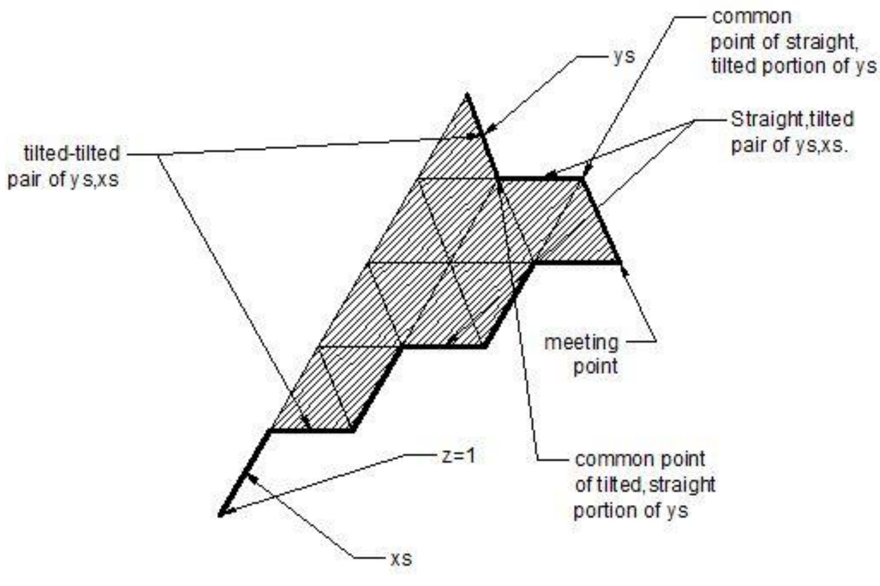
<u>Calculating number of points in f3 region-</u>

In the figure, shown below for tilted – point meet of xs & ys, in y= [1 ,2] there is tilted-tilted pair portion of xs- ys and in y= [2 ,3] there is tilted-straight pair portion of xs- ys respectively and so on up to the meeting point.

<u>Notation:</u> (x, y, z) is touching point of ys with y=1 or left end point of ys.

1).<u>When xs & ys are tilted and they point meet</u>-

A).<u> When y_m is even</u>- There are unequal number of tilted-tilted pair portions and straight-tilted pair portions of ys, xs respectively. Number of tilted-tilted pair portions=1+number of straight-tilted pair portions. Meeting point is common point of tilted-straight portion of ys (first tilted then straight, in order).



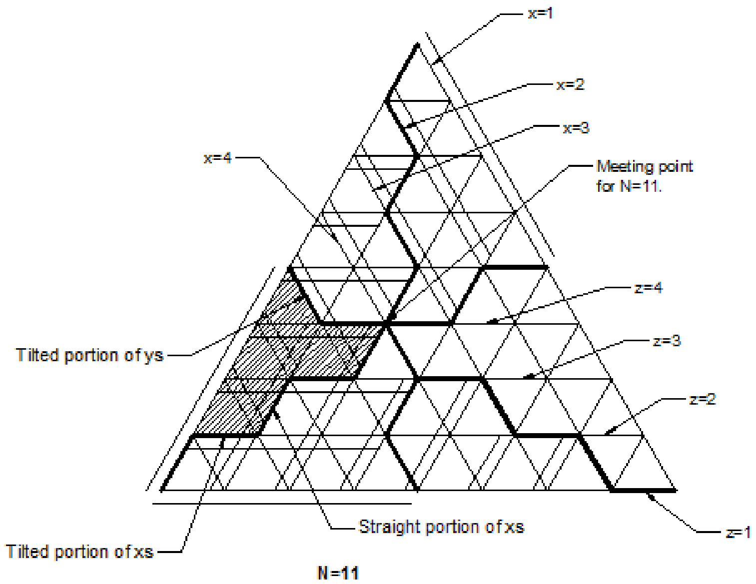
f3 region for N=13

Figure

Number of f3 points = F3=z+(z-2+z-3)+(z-5+z-6)+(z-8+z-9)+…..+(z-(z-1)+z-(z-3))=z+(z-2+z-5+z-8+….+z-(z-1))+(z-3+z-6+z-9+…+z-(z-3)). (z-2+z-5+z-8+….+z-(z-

1)) is summation of number of points in f3 region lying on the const. y line which passes through the common point of tilted-straight portion of ys (first tilted then straight ,in order)and (z-3+z-6+z-9+…+z-(z-3)) is summation of number of points in f3 region lying on the const. y line which passes through the common point of straight-tilted portion of ys (first straight then tilted ,in order) , are A.P.s with forward difference in terms=-3.So, z-1 (in z-(z-1) in first series)=2+3(n-1),implies n=z/3 and z-3 (in z-(z-3) in second series) =3+3(n-1), implies n=(z/3)-1.So, $F3=z+z^2/3-(2+5+8+\ldots+z-1) + z^2/3 - z - (3+6+9+\ldots+z-3)=2(z^2/3) - (z/6)*(4+3((z-3)/3)) - ((z-3)/6)(6+3((z-6)/3))= 2(z^2/3) - (z/6)(z+1) - ((z-3)/6)z=2(z^2/3) - 2(z/6)(z-1)=(z/3)(2z-z+1)= z(z+1)/3$.Since,z=(N-1)/2 for tilted case ,$F3=(N^2-1)/12$.

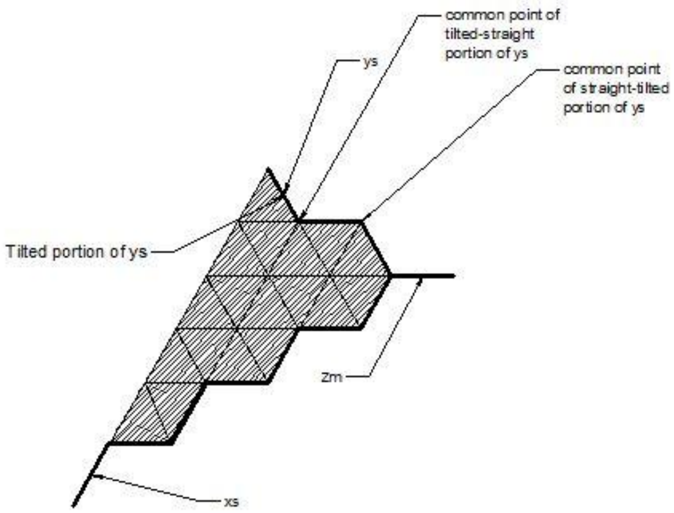
B). <u>When y_m is odd</u>- There are an equal number of tilted-tilted pair portions and straight-tilted pair portions of ys- xs. Meeting point is common point of straight-tilted portion of ys (first straight then tilted, in order).



Figure

Number of f3 points = F3=z+ (z-2+z-3) + (z-5+z-6) + (z-8+z-9) +…..+ (z-(z-3)+z-(z-1))=z+(z-2+z-5+z-8+….+z-(z-3))+(z-3+z-6+z-9+…+z-(z-1)). (z-2+z-5+z-8+….+z-(z-3)) is summation of number of points in f3 region lying on the const. y line which passes through the common point of tilted-straight portion of ys (first tilted then straight ,in order) and (z-3+z-6+z-9+…+z-(z-1)) is summation of number of points in f3 region lying on the const. y line which passes through the common point of straight-tilted portion of ys (first straight then tilted ,in order) , are A.P.s with forward difference in terms=-3.So, z-1 (in z-(z-1) in second series)=3+3(n-1),implies n=(z-1)/3 and z-3 (in z-(z-3) in first series) =2+3(n-1), implies n=(z-2)/3.So, F3=z+(z(z-2)/3) - (2+5+8+…+(z-3)) + (z(z-1)/3) - (3+6+9+…+(z-1))= z + (z(2z-3)/3) - ((z-2)/6)(4+3((z-5)/3)) - ((z-1)/6)(6+3((z-4)/3))= z + (z(2z-3)/3) - ((z-2)/6)(z-1) - ((z-1)/6)(z+2)= z+(z(2z-3)/3)- z(z-1)/3= z + z(z-2)/3=z(z+1)/3. Since, z=(N-1)/2 for tilted case, $F3= (N^2-1)/12$.

2).<u>When xs & ys are tilted and they line meet</u>-



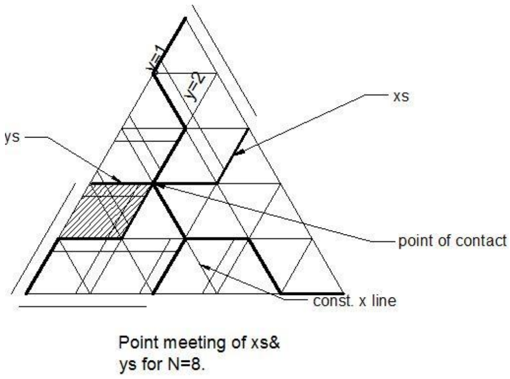
Figure

Since, y_{m2}=(N/3)-1 in line meet is even because N is odd for tilted case is only case possible in this. First meeting point is right to second meeting point which is to be added to F3.

Number of f3 points =F3=1+z+ (z-2+z-3) + (z-5+z-6) + (z-8+z-9) +…..+ (z-(z-2)+z-(z-4))=1+z+(z-2+z-5+z-8+….+z-(z-2))+(z-3+z-6+z-9+…+z-(z-4))(1 is added for first meeting point). (z-2+z-5+z-8+….+z-(z-2)) is summation of number of points in f3 region lying on the const. y line which passes through the common point of tilted-straight portion of ys (first tilted then straight ,in order) and (z-3+z-6+z-9+…+z-(z-3)) is summation of number of points in f3 region lying on the const. y line which passes through the common point of straight-tilted portion of ys (first straight then tilted ,in order) , are A.P.s with forward difference in terms=-3.So, z-2 (in z-(z-2) in first series)=2+3(n-1),implies n=(z-1)/3 and z-4 (in z-(z-4) in second series) =3+3(n-1), implies n=(z-4)/3.So, F3=1+z+(z(z-1)/3)-(2+5+8+…+(z-2))+ (z(z-4)/3) -(3+6+9+…+(z-4))= 1+z+(z(z-1)/3) + (z(z-1)/3) – z -((z-1)/6)(4+3((z-4)/3)) - ((z-4)/6)(6+3((z-7)/3))= 1+(2z(z-1)/3)-(z(z-1)/6) -((z-4)/6)(z-1)=1+ (2z(z-1)/3)-(z-1)(z-2)/3)= 1+(z-1)(z+2)/3.Since, z=(N-1)/2 , so F3=1+(N^2-9)/12=(N^2+3)/12.

1).<u>When xs & ys are straight and they point meet</u>-

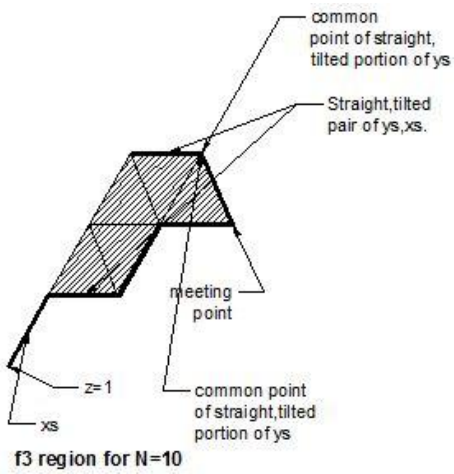
A).<u> When y_m is even</u>- There are unequal number of tilted-tilted pair portions and straight-tilted pair portions of ys- xs. Number of straight (of ys)-tilted (of xs) pair portions=1+number of tilted-tilted pair portions. Meeting point is common point of straight-tilted portion of ys (first straight then tilted, in order).



Figure

Number of f3 points = F3=z+ (z-1+z-3) + (z-4+z-6) + (z-7+z-9) +…..+ (z-(z-2)+z-(z-3))=z+(z-1+z-4+z-7+….+z-(z-2))+(z-3+z-6+z-9+…+z-(z-3)). (z-1+z-4+z-7+….+z-(z-2)) is summation of number of points in f3 region lying on the const. y line which passes through the common point of straight- tilted portion of ys (first straight then tilted ,in order) and (z-3+z-6+z-9+…+z-(z-3)) is summation of number of points in f3 region lying on the const. y line which passes through the common point of tilted-straight portion of ys (first tilted then straight ,in order) , are A.P.s with forward difference in terms=-3.So, z-2 (in z-(z-2) in first series)=1+3(n-1),implies n=z/3 and z-3 (in z-(z-3) in second series) =3+3(n-1), implies n=(z/3)-1.So, $F3=z+z^2/3-(1+4+7+\ldots+z-2)+z^2/3- z - (3+6+9+\ldots+z-3)=2(z^2/3) - (z/6)*(2+3((z-3)/3)) - ((z-3)/6)(6+3((z-6)/3))= 2(z^2/3)- (z/6)(z-1) - (z(z-3)/6)=2(z^2/3)-2(z/6)(z-2)=(z/3)(2z-z+2)= z(z+2)/3$.Since, z=(N-2)/2 for straight case ,$F3=(N^2-4)/12$.

B). <u>When y_m is odd</u>- There are an equal number of tilted-tilted pair portions and straight-tilted pair portions of ys- xs. Meeting point is common point of tilted-straight portion of ys (first tilted then straight, in order).



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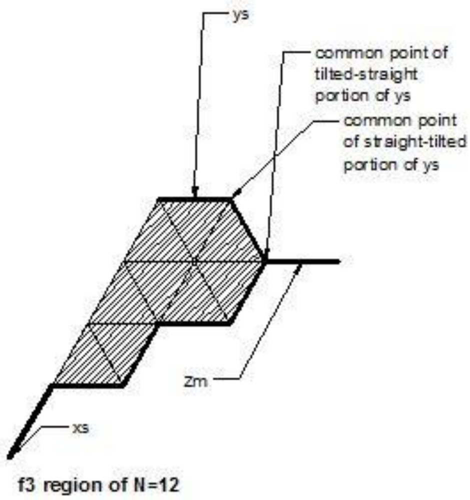
2).<u>When xs & ys are straight and they line meet</u>-



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Number of f3 points =F3=1+z+ (z-1+z-3) + (z-4+z-6) + (z-7+z-9) +…..+ (z-(z-4)+z-(z-2))=1+z+(z-1+z-4+z-7+….+z-(z-4))+(z-3+z-6+z-9+…+z-(z-2))(1 is added for first meeting point). (z-1+z-4+z-7+….+z-(z-4)) is summation of number of points in f3 region lying on the const. y line which passes through the common point of straight-tilted portion of ys (first straight then tilted ,in order) and (z-3+z-6+z-9+…+z-(z-2)) is summation of number of points in f3 region lying on the const. y line which passes through the common point of tilted-straight portion of ys (first tilted then straight ,in order) , are A.P.s with forward difference in terms=-3.So, z-4 (in z-(z-4) in first series)=1+3(n-1),implies n=(z-2)/3 and z-2 (in z-(z-2) in second series) =3+3(n-1), implies n=(z-2)/3.So, F3=1+z+(2z(z-2)/3)-(1+4+7+…+(z-4))-(3+6+9+…+(z-2))= 1+z+(2z(z-2)/3) - ((z-2)/6)(2+3((z-5)/3)) - ((z-2)/6)(6+3((z-5)/3))= 1+z+(2z(z-2)/3)-

((z-2)/6)(z-3) - ((z-2)/6)(z+1)= 1+z+(2z(z-2)/3) - ((z-1)(z-2)/3)= 1+z+(z+1)(z-2)/3.Since, z=(N-2)/2 , so F3=(N/2)((1+(N/6)-1))= N^2/12.

RESULT

1). for odd N&N≠3k where k= (2q-1), q≥1

F3=p(N,3)= (N^2-1)/12.

2). for odd N&N=3k where k= (2q-1), q≥1

F3=p(N,3)= (N^2+3)/12.

3). for even N&N≠3k where k= 2q, q≥1

F3=p(N,3)= (N^2-4)/12.

4). for even N&N=3k where k= 2q, q≥1

F3=p(N,3)= N^2/12.

Verification of above result

The above formulas for p (N, 3) can be verified by calculating p (N, 3) for values of N≥3 and comparing these values of p (N, 3) with computed values of p (N, 3) or values of p (N, 3) in [1] taking N=0 as N=3,N=1 as N=4 and so on in [1] because smallest part in 3-parts partitions in[1] is zero while in this paper is one.

REFERENCES

[1] A001399-OEIS ("The On-Line Encyclopedia of Integer Sequences")-number of partitions of n into at most 3 parts.