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A new numerical approach for the solution of the modified Burgers' equation using Haar wavelet collocation method

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Abstract

In this study, a new numerical idea is developed to solve the modified Burgers' equation (MBE) via Haar wavelet collocation method. The suggested method has been demonstrated by considering one test example. The L_2 and L_∞ error norms and ROC (Rate of convergence) are studied and compared with various numerical methods which are exist in the literature. The obtained outcomes clearly displays that the suggested method is an outstanding numerical scheme for the solution of the modified Burgers' equation (MBE).

Keywords: Collocation method, Haar wavelets, Modified Burgers' equation.

1. INTRODUCTION

The modified Burgers' equation (MBE) is a notably nonlinear expansion of the Burgers' equation (BE). The word "modified" indicates a difference in the order of nonlinearity. The MBE, which is surveyed here, has the following form,

$$\frac{\partial y}{\partial t}+y^2\frac{\partial y}{\partial x}=v\frac{\partial^2 y}{\partial x^2},\ 0\le x<1, t\ge t_0, \tag{1.1}$$

subject to the initial condition

$$y(x,t_0)=f(x),\ 0\le x<1, \tag{1.2}$$

and the boundary conditions

$$y(0,t)=0,\ y(1,t)=0,\ t\ge t_0. \tag{1.3}$$

Where $'v'$ represents the viscosity parameter. The parameters $'x'$ and $'t'$ indicates the space and time respectively. The equation (1.1) contains the strong non-linear characteristic and also has been extensively used in many model problems which are arises from the different fields like fluid dynamics [6], magnetogasdynamics [7], electric field in a nonlinear isotropic dielectric [8], etc. Many authors are studied both analytical and numerical methods to find the solution of the MBE. The analytical solution of the MBE can be found among the references [9, 10, 11, 12]. Some difficulties are arises in the analytical solution while solving MBE like steady shock wave [13] and also the BE can be completely linearized by the Cole-Hopf transformation but J.J.C. Nimmo and D.G.Crighton [14] shows that this type of transformation (Backlund transformation) is not possible for the MBE. In order to overcome from these difficulties only option is asymptotic studies or direct numerical methods. P.L. Sachdev et al. [10,11] worked on finding the asymptotic solution of the MBE. Also many authors are applied numerical methods to find the solution of the MBE. M. A. Ramadan et al. [1,2] proposed the quintic splines(QSM) and septic B-splines (SBSM) with collocation method for solving the both BE and MBE. A. Bashan et al. [3] obtained the solution of the MBE by Quintic B-spline Differential Quadrature method (QBDQM). Dursun Irk et al. [4] introduced two approaches i.e. Sextic B-spline collocation method1(SBCM1) and Sextic B-spline collocation method 2 (SBCM2) for solving the both BE and MBE. S. B. G. Karakoç et al. [5] have established the quartic B-spline subdomain finite element method (SFEM) for evaluating the numerical solution of the BE and MBE.

Wavelets are a remarkably new mathematical theory founded at the end of the 1980 and also it is a current mathematical tool to divide, analyse and integration of a function. In recent years wavelet found a great number of applications in science and engineering field like in signal analysis [20], numerical analysis[15] and image compression(FBI finger print)[21]. There are different types of wavelets like Coiflet, Haar, Mathieu, Daubechies, etc. are separated according to their properties like compact support, orthogonality and number of vanishing moments and others. A plenty of work has been devoted by many authors like U. Lepik [15], G. Hariharan et al. [18], N. M. Bujurke et al.[19] and A.P.Reddy et al. [16,17] in order to find the approximate solution to the different types of linear and nonlinear differential and integral equations which plays a very vital act in diverse physical phenomena using different types of wavelets. In this paper an iterative method is applied to solve MBE based on Haar wavelet collocation method (HWCM). To check the capability of the proposed method the L_2 and L_∞ error norms and ROC (Rate of convergence) are evaluated and compared with other existing methods which are available in the literature [1, 2, 3, 4, 5].

The outline of the paper contains the following things: In portion 2, we discussed about the Haar wavelet family and the evaluation of their integrals. In portion 3, we have given a procedure for solving the MBE using HWCM. In portion 4, we applied our proposed technique to one test example and compared our results with other methods. At the end of the paper a conclusion is drawn in portion 5.

2. HAAR WAVELETS AND THEIR INTEGRALS

As mentioned by the wavelet research, any debate of the wavelet begins with the Haar wavelet. Because Haar wavelet is the mathematically simple orthonormal wavelets with compact support [15]. The compact support of the Haar wavelet allows the Haar decomposition to have favorable time localization. There are two types of functions which plays an important role in Haar wavelet which are father wavelet [$h_1(x)$] and the mother wavelet [$h_2(x)$]. These two functions develops a group of functions those can be employed to divide and rebuild a function. Which are defined in the interval $x \in [0,1)$ as follows [15]:

$$h_1(x) = \begin{cases} 1, & \text{for } x \in [0,1), \\ 0, & \text{otherwise.} \end{cases}$$

$$h_2(x) = \begin{cases} 1, & \text{for } x \in \left[0, \frac{1}{2}\right), \\ -1, & \text{for } x \in \left[\frac{1}{2}, 1\right), \\ 0, & \text{otherwise.} \end{cases}$$

For $a > 2$, the Haar wavelet family defined as follows,

$$h_a(x) = \begin{cases} 1, & \text{for } x \in [\alpha(a), \beta(a)), \\ -1, & \text{for } x \in [\beta(a), \gamma(a)), \\ 0, & \text{otherwise.} \end{cases} \qquad (2.1)$$

Where $\alpha(a) = \frac{r}{m}$, $\beta(a) = \frac{r+0.5}{m}$ and $\gamma(a) = \frac{r+1}{m}$, $m = 2^i$ and $i = 0,1,2,...,J$. 'J' symbolized as the maximal level of resolution. The parameter $r = 0,1,2,...,m-1$ is referred as the translation parameter. Where 'a' is indicates the number of daughter Haar wavelets which are evaluated using the formula $a = m + r + 1$. The value of 'a' lies between $2 < a \le 2^{i+1}$. Let us divide the interval [0,1] into 2L subintervals of uniform length $dx = \frac{1}{2L}$, where $L = 2^J$. The collocation points $x_b = \frac{b-0.5}{2L}$, where $b = 1,2,....,2L$.

Let us assume the following notations are as follows:

$$P_{1,a}(x)=\int_0^x h_a(x)\,dx, \qquad (2.2)$$

$$P_{2,a}(x)=\int_0^x P_{1,a}(x)\,dx,$$

Integrating the Haar functions for 'λ' times. We get

$$P_{\lambda,a}(x)=\int_0^x\int_0^x\cdots\int_0^x h_a(s)\,ds^{\lambda},$$

$$P_{\lambda,a}(x)=\frac{1}{(\lambda-1)!}\int_0^x (x-s)^{\lambda-1}h_a(s)\,ds. \qquad (2.3)$$

All these integrals are evaluated directly as explained by U. Lepik [15]. When $a=1$ the equation (2.3) gives

$$P_{\lambda,1}(x)=\frac{1}{(\lambda)!}(x)^{\lambda}. \qquad (2.4)$$

For $a\geq 2$ from the equation (2.3), we get

$$P_{\lambda,a}(x)=\frac{1}{\lambda!}\begin{cases}0, & \text{if } x\in[0,\ \alpha(a)),\\ [x-\alpha(a)]^{\lambda}, & \text{if } x\in[\alpha(a),\beta(a)),\\ \left[[x-\alpha(a)]^{\lambda}-2[x-\beta(a)]^{\lambda}\right], & \text{if } x\in[\beta(a),\gamma(a)),\\ \left[[x-\alpha(a)]^{\lambda}-2[x-\beta(a)]^{\lambda}+[x-\gamma(a)]^{\lambda}\right], & \text{if } x\in[\gamma(a),1).\end{cases} \qquad (2.5)$$

3. EXPLANATION OF THE PROPOSED METHOD:

Consider the MBE (1.1) along with the initial and boundary conditions.

Step 1: Let us approximate the highest derivative in the equation (1.1) in terms of Haar wavelets as follows

$$\dot{y}''(x,t)=\sum_{a=1}^{2L}c_a h_a(x). \qquad (3.1)$$

Where '.' ≡ differentiation with respect to 't', $t\in[t_n,t_{n+1}]$,

' " ' ≡ differentiation with respect to 'x', $x\in[0,1)$,

c_a = Haar wavelet coefficient,

$h_a(x)$ = Haar wavelet family.

Step 2: Let us determine the unknowns $y''(x,t)$, $y'(x,t)$, $y(x,t)$ and $\dot{y}(x,t)$ by integrating the equation (3.1) once w.r.t. $'t'$ and twice w.r.t. $'x'$ in terms of integrated Haar functions. We obtain the following equations as follows,

$$y''(x,t) = y''(x,t_n) + (t-t_n)\sum_{a=1}^{2L} c_a(t_n)h_a(x), \tag{3.2}$$

$$y'(x,t) = y'(0,t) + y'(x,t_n) - y'(0,t_n) + (t-t_n)\sum_{j=1}^{2L} a_j(t_n)P_{1,j}(x), \tag{3.3}$$

$$y(x,t) = (t-t_n)\sum_{a=1}^{2L} c_a(t_n)P_{2,a}(x) + y(x,t_n) - y(0,t_n) + x\left[y'(0,t) - y'(0,t_n)\right] + y(0,t), \tag{3.4}$$

$$\dot{y}(x,t) = \sum_{a=1}^{2L} c_a(t_n)P_{2,a}(x) + x\,\dot{y}'(0,t) + \dot{y}(0,t). \tag{3.5}$$

Step 3: By substituting the boundary conditions in the equations (3.2) to (3.5), we obtain the following equations which are,

$$\dot{y}'(0,t) = -\sum_{a=1}^{2L} c_a(t_n)P_{2,a}(1), \tag{3.6}$$

$$\left[y'(0,t) - y'(0,t_n)\right] = -(t-t_n)\sum_{a=1}^{2L} c_a(t_n)P_{2,a}(1). \tag{3.7}$$

Step 4: Substitute the equations (3.6) and (3.7) in (3.2) to (3.5) along with that discritizing the equations $x \to x_b$ and $t \to t_{n+1}$,we obtain the following equations.

$$y''\left(x_b,t_{n+1}\right)=y''\left(x_b,t_n\right)+\left(t_{n+1}-t_n\right)\sum_{a=1}^{2L}c_a\left(t_n\right)h_a(x_b), \tag{3.8}$$

$$y'\left(x_b,t_{n+1}\right)=y'\left(x_b,t_n\right)+\left(t_{n+1}-t_n\right)\sum_{a=1}^{2L}c_a\left(t_n\right)\left[P_{1,a}(x_b)-P_{2,a}(1)\right], \tag{3.9}$$

$$y\left(x_b,t_{n+1}\right)=\left(t_{n+1}-t_n\right)\sum_{a=1}^{2L}c_a\left(t_n\right)\left[P_{2,a}(x_b)-x_bP_{2,a}(1)\right]+y\left(x_b,t_n\right), \tag{3.10}$$

$$\dot{y}\left(x_b,t_{n+1}\right)=\sum_{a=1}^{2L}c_a\left(t_n\right)\left[P_{2,a}(x_b)-x_bP_{2,a}(1)\right]. \tag{3.11}$$

Step 5: Where c_a's (Haar wavelet coefficients) are unknowns in the equation (3.10). In order to find these unknowns, let us consider the equation (1.1) as follows,

$$\frac{\partial}{\partial t}y\left(x_b,t_{n+1}\right)=-\left[y\left(x_b,t_n\right)\right]^2\frac{\partial}{\partial x}y\left(x_b,t_n\right)+v\frac{\partial^2}{\partial x^2}y\left(x_b,t_n\right). \tag{3.12}$$

Let us break the time interval $\left[t_0,T\right]$ into 'E' subinterval of uniform length $dt=\frac{\left[T-t_0\right]}{E}$. At the beginning we begin an iteration scheme by replacing $t_n=t_0$ and $t_{n+1}=t_n+dt$ for $n=1,2,...,E.$

Step 6: Lastly we obtain the Haar wavelet coefficients after successful iteration. Then substituting these coefficients into the equation (3.10), we will have the Haar wavelet based set of solutions at different $x\in[0,1)$ and time 't'.

4. NUMERICAL STUDIES AND DISCUSSION:

In this portion, we will discuss the one test problem of the MBE whose analytical solution is known [1, 2] by applying the proposed technique. In order to show the robustness and numerical accuracy of our method, we compared the proposed method results with the other methods [1,2,3,4,5] in terms of L_2 and L_∞ error norms and ROC (Rate of convergence) which are defined as follows.

$$L_\infty = \max_{1\le p\le N} \left|y(p)-y_{ex}(p)\right|,\ \ L_2 = \sqrt{h\sum_{p=1}^{N} \left|y(p)-y_{ex}(p)\right|^2},$$

$$\mathrm{ROC} = \frac{\ln\left(\dfrac{F(N_1)}{F(N_2)}\right)}{\ln\left(\dfrac{N_2}{N_1}\right)}.$$

Where $N =$ number of nodes, $h = \frac{1}{N}$, $y(p) =$ numerical solution at $x = x_p$, $y_{ex}(p) =$ analytical solution at $x = x_p$, $F(N_P) =$ L_2 or L_∞ error norm with N_P node points at $x = x_p$ for $p = 1,2,..,N$ respectively.

Example 1. Consider the MBE (1.1) with the following conditions [1,2],

$$\frac{\partial y}{\partial t} + y^2\frac{\partial y}{\partial x} = v\frac{\partial^2 y}{\partial x^2},\ 0\le x<1, t\ge 1, \tag{4.1}$$

subject to the initial condition

$$y(x,1) = \frac{x}{1+\left(\dfrac{1}{c_0}\right)e^{\left(\frac{x^2}{4v}\right)}}, 0\le x<1,$$

and the boundary conditions

$$y(0,t)=0,\ y(1,t)=0,\ t\ge 1.$$

Where $c_0 = 0.5$ is a constant.

The analytical solution of the MBE [1,2] is given by

$$y(x,t) = \frac{\left(\dfrac{x}{t}\right)}{1+\left(\dfrac{\sqrt{t}}{c_0}\right)e^{\left(\frac{x^2}{4tv}\right)}}, 0\le x<1, t\ge 1.$$

The obtained numerical results are compared and discussed in terms of Tables and graphs with different viscosity parameters(v), different times(t) and with different resolution(J) for chosen length(dt) and step size(h).

Table 1: Comparison of L_2 and L_∞ error norms for v=0.01 with J=4 and dt=0.01.

h	**Methods**	L_∞ **×10^{-4}** **t=2**	L_2 **×10^{-4}** **t=2**	L_∞ **×10^{-4}** **t=4**	L_2 **×10^{-4}** **t=4**	L_∞ **×10^{-4}** **t=6**	L_2 **×10^{-4}** **t=6**	L_∞ **×10^{-}** **t=10**	L_2 **×10^{-4}** **t=10**
1/32	HWCM	8.403971	3.83983	6.25117	3.31656	4.81811	3.33852	12.21814	5.46523
1/200	QSM[1]	12.169876	5.230818	9.313639	5.16252	7.22492	4.902369	12.812487	6.40075
1/200	SBSM[2]	17.03092	7.90431	9.96452	5.57679	7.61053	5.16723	18.02394	8.00256
1/50	QBDQM[3]	14.06115	6.88316	10.47041	5.51891	8.14737	5.36003	17.42584	8.00131
1/200	SBCM1[4]	8.29340	3.84890	-	-	-	-	12.8127	5.48260
1/200	SBCM2[4]	8.2734	3.9078	-	-	-	-	12.8127	5.4612
1/50	SFEM[5]	13.795979	7.95586	9.76415	5.25053	6.374950	3.45221	14.747885	6.04293

Table 2: Comparison of L_2 and L_∞ error norms for v=0.001 with J=6 and dt=0.01

h	**Methods**	L_∞ **×10^{-4}** **t=2**	L_2 **×10^{-4}** **t=2**	L_∞ **×10^{-4}** **t=4**	L_2 **×10^{-4}** **t=4**	L_∞ **×10^{-4}** **t=6**	L_2 **×10^{-4}** **t=6**	L_∞ **×10^{-4}** **t=10**	L_2 **×10^{-4}** **t=10**
1/128	HWCM	2.633796	0.678996	1.95958	0.57915	1.489729	0.505499	1.027189	0.414035
1/200	QSM [1]	2.796704	0.670396	2.18566	0.66695	1.71765	0.604622	1.212998	0.501085
1/200	SBSM [2]	8.185211	1.83549	3.56354	1.14411	2.13485	0.814175	1.39431	0.551152
1/166	QBDQM [3]	4.57137	1.27227	3.33200	0.98569	2.54679	0.84073	1.75928	0.68599
1/200	SBCM1[4]	2.62330	0.68430	-	-	-	-	1.0295	0.40800
1/200	SFEM [5]	4.45389	1.370701	3.25839	1.01976	2.484289	0.84949	1.711034	0.680383

In **Table 1** and **2** the acquired numerical results are studied in terms of L_2 and L_∞ error norms with the parameters v = 0.01, dt = 0.01, J=4 and v = 0.001, dt = 0.01, J=6 at different times t=2, 4, 6, 10 respectively and also compared with the other methods[1,2,3,4,5]. Moreover it is evidently observed from the **Tables 1** and **2** that as the time increases the L_2 and L_∞ error norms decreases. This displays that our proposed scheme is quite stable. Another thing is also observed from the **Tables 1** and **2** that as the value of the viscosity parameter (v) reduces the L_2 and L_∞ error norms will also reduces. Finally we can say that we may get better results if the value of the viscosity parameter is small.

Table 3: ROC calculations for v=0.005, dt=0.01 at t=6.

N	L_∞	**ROC**	L_2	**ROC**
2	0.031753750342047	-	0.022588508459772	-
4	0.022684481183504	0.485221330210295	0.014092899208365	0.680620576221818
8	0.002301032788224	3.301352217915852	0.001135187995427	3.633965291903219
16	0.0005706188345132239	2.011682269077052	0.0002738674173324916	2.051381697687605
32	0.0003441466698703203	0.729503817181940	0.0001817786691794366	0.591294718948715
64	0.0003308384855810918	0.056896480569521	0.0001674886996267114	0.118119155405148

In **Table 3** we observed that when the value of the resolution level(J) increases, the L_2 and L_∞ error norms decreases. The results of ROC (L_2) and ROC (L_∞) are varies in the interval of [0.057, 3.3] and [0.12, 3.6] respectively.

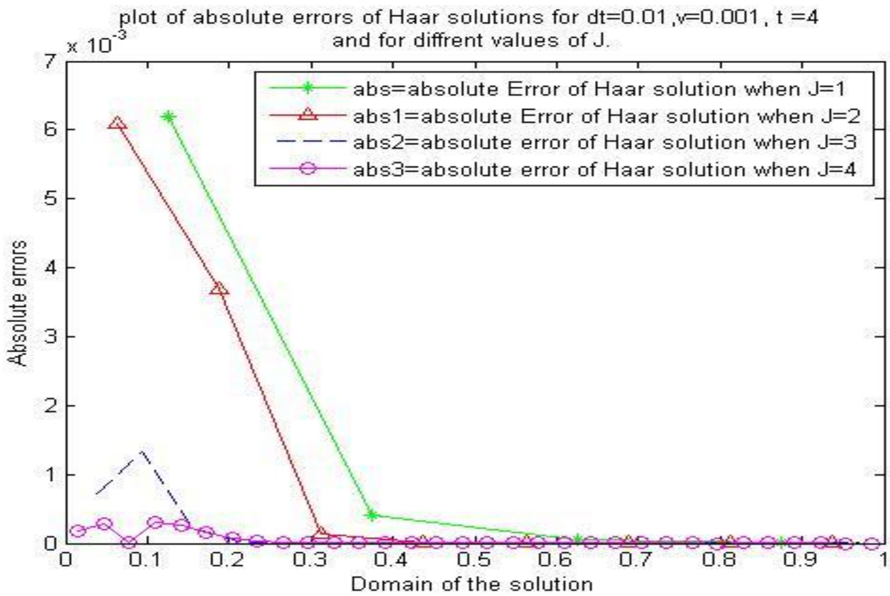

Figure 1: Solution performance of the MBE with v=0.001, dt=0.01 with different J at t=4.

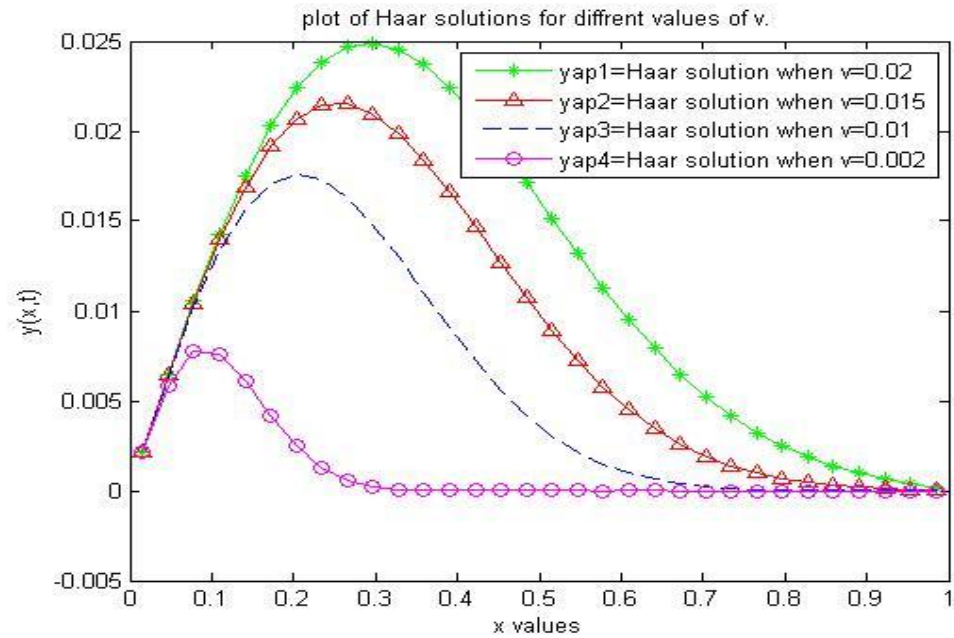

Figure 2: Solution performance of the MBE with J=4, dt=0.01 with different values of v at t=4.

The **Figure 1 and 2** demonstrate the performance of the numerical solution for v=0.001,dt=0.01 with different resolution J=1,2,3,4 and for J=4, dt = 0.01 with different viscosity parameters v = 0.02, 0.005, 0.001, 0.002 respectively. From the **Figure 1** it is noted that as the resolution level J increases the absolute error of the numerical solution decays very fastly. The numerical solutions for different values 'v' are captured in **Figure 2** in order to study how the numerical solution of the proposed method changes with the different viscosity parameters (v).

5. CONCLUSION

In this article, an iterative scheme based on HWCM has been successfully implemented for solving the MBE type equations. The final conclusion of the paper is outlined as follows.

a) Our proposed method is capable of finding the solution for MBE type equations without using any linearization and transformation process.

b) The obtained L_2 and L_∞ error norm results are compatible with some previously published results [1, 2, 3, 4, 5]. Hence we may suggest to use our method to solve the MBE type equations instead of using other methods which are mentioned in our article.

c) Our method is capable of finding the numerical solution of the MBE for small value of viscosity parameter (v) also.

d) Numerically ROC of numerical calculations was also acquired.

e) Our method will give accurate solution with less number of node points and also by increasing the resolution level (J) we will get more accurate results which is observed in **Figure 2**.

f) Finally we conclude that our method is simple, fast, reliable and minimum cost to obtain the numerical solution of the MBE type equations which plays a very important role in many physical phenomena.

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REFERENCES

[1] M. A. Ramadan and T. S. El-Danaf, "Numerical treatment for the modified Burgers equation," *Mathematics and Computers in Simulation,* vol.70, no.2, pp.90–98, 2005.

[2] M. A. Ramadan, T. S. El-Danaf and F. E. A. Alaal, "A numerical solution of

the Burgers' equation using septic B-splines," *Chaos, Solitons and Fractals,* vol. 26, no.3, pp. 1249–1258, 2005.

[3] A. Bashan, S. Karako and T. Geyikli, "B-spline differential quadrature method for the modified Burgers' equation," *Cankaya University Journal of Science and Engineering,* vol.12, no.1, pp.001-013, 2015.

[4] Dursun Irk, "Sextic B-spline collocation method for the modified Burgers' equation," *Emerald Group Publishing Limited,* vol.*38*, no.9, pp. 1599-1620, 2009.

[5] S. B. G. Karakoç, A. Bashan and T. Geyikli, "Two different methods for numerical solution of the modified Burgers' equation," *The Scientific World Journal,* vol.2014, no. 5, pp. 1-13, 2014.

[6] J.M. Burger, "A mathematical model illustrating the theory of turbulence," *Advances in Applied Mechanics,* vol.1, pp. 171–99, 1948.

[7] G. Nariboli and W. Lin, "A new type of Burgers' equation," *Journal of Applied Mathematics and Mechanics,* vol.53, no.8, pp. 505–510, 1973.

[8] K.Gorschkov, L. Ostrovsky and E. Pelinovsky, "Some problems of asymptotic theory of nonlinear waves," *Proceedings of the IEEE,* vol. 62, no.11, pp.1511–1517, 1974.

[9] S.E. Harris, "Sonic shocks governed by the modified Burgers' equation," *European Journal of Applied Mathematics,* vol.7, no.2, pp.201-222, 1996.

[10] P.L. Sachdev and Ch. Srinivasa Rao, "N-wave solution of modified Burgers equation," *Applied Mathematics Letters,* vol. 13, no.4, pp.1-6, 2000.

[11] P.L. Sachdev, Ch. Srinivasa Rao and B.O. Enflo, "Large-time asymptotics for periodic solutions of the modified Burgers equation," *Studies in Applied Mathematics,* vol.114, no.3, pp.307-323, 2005.

[12] I. Inan and Y. Ugurlu, "Exp-function method for the exact solutions of fifth order KdV equation and modified Burgers equation," *Applied Mathematics and Computation,* vol.217, no.7, pp.2912, 2009.

[13] J.D.Cole, "On a quasilinear parabolic equation occurring in aerodynamics," *Quarterly of Applied Mathematics*, vol. 9, pp.225–236, 1951.

[14] J.J.C. Nimmo and D.G.Crighton, "Backlund transformations for nonlinear parabolic equations: the general results," *Proceedings of the Royal Society A,* vol.384, no.1787, pp. 381-401, 1982.

[15] U. Lepik and H. Hein, "Haar wavelets with applications," *Springer,*2014.

[16] A.P.Reddy, S.H.Manjula and C. Sateesha, "A numerical approach to solve eighth order boundary value problems by Haar wavelet collocation method," *Journal of Mathematical Modeling*, vol.5,no.1,pp.61-75, 2017.

[17] A.P.Reddy, C. Sateesha and S.H.Manjula, "Investigation of Haar Wavelet Collocation Method to Solve Ninth Order Boundary Value Problems," *Global*

Journal of Pure and Applied Mathematics, vol.13, no.5, pp. 1415-1428, 2017.

[18] G. Hariharan, and K. Kannan, "Haar wavelet method for solving Cahn-Allen equation," *Applied Mathematical Sciences,* Vol. 3, no.51, pp. 2523 – 2533,2009.

[19] N.M. Bujurke, C.S. Salimath and S.C. Shiralashetti, "Numerical solution of stiff systems from nonlinear dynamics using single-term Haar wavelet series," *Nonlinear Dynamics,* vol.51**,** no 4,pp.595–605, 2008.

[20] H. R. Karimi and K. G. Robbersmyr, "Signal analysis and performance evaluation of a vehicle crash test with a fixed safety barrier based on Haar wavelets," *International Journal of Wavelets, Multiresolution and Information Processing*, vol. 9, no.1, pp. 131–149, 2011.

[21] A. Graps, "An introduction to wavelets," *IEEE Computational Science and Engineering,* vol.2, no.2, pp. 50–61, 1995.