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Mathematical Modelling of Extremism with Sensitization effects in Kenya

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Abstract

Radicalization is a process by which an individual, or group comes to adopt increasingly belief on the importance to have changes on political, social, or religious ideals and aspirations that do not respect contemporary ideas and expressions of the nation or the world. A radicalization model was formulated to explain the spread of extremism with the effects of sensitizing or educating the public. The model included a sensitized or educated compartment. The compartment represented the population of all those who have been sensitized on the dangers associated with extremism of all kind; political, social or religious. The model was analyzed using both qualitative and quantitative approach. The qualitative analysis included the extremism reproductive rate, extremism free equilibrium point, extremism endemic equilibrium and both the local and global stability of the equilibrium point. The quantitative analysis was done through the numerical simulations of the various populations. Through qualitative analysis, the system was determined to be locally asymptotically stable whenever the extremism reproductive number is less than one. Finally, some recommendations have been made, such as improving the parameters and including other compartments by considering social status, age and sex structured model in addition to involving top leadership of al-shabaab in the Somali and as well as Kenya government.

Key words: Radicalization, extremism, extremism mathematical model, equilibrium points, stability analysis, extremism reproductive number.

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1 INTRODUCTION

The outcomes of radicalization are based on the ideas of the society at large. Radicalization can arise as a result of social agreement to resist a particular change or demand some changes in the society, nation or the world at large. Radicalization can be both violent and nonviolent, although most academic literature research work geared towards radicalization which results into violent extremism.

There are multiple causes or reasons of occurrence of the process of radicalization and extremism acts, which can be independent but in most cases mutually reinforcing. Some of these reasons are,grievance, risk and status, isolation and martyrdom among others.

In January 2017, United Nation (U.N) through the secretary general reported that from 2010 to 2016 youths totaling to about 4213 was recruited to join al-shabaab. Similarly, and interview done by [1] interviewing former al-shabaab and displaced individuals in the camps during 2014, did find that 97 percent of those who were recruited to al-shabaab, joined at the age of 10-39 years with youth aged between 15-19 years taking the lead with 40 percent. This was an indication that al-shabaab mainly uses youths as their fighters. In this regard, the mathematical model developed here, treated youths as susceptible population. Given that most of al-shabaab recruits joined the group because of either unemployment, fear of being victimized, propaganda or mental manipulation, this implies that most of them can be sensitized to drop the extremism ideology. .

[2] constructed a mathematical model where an individual behaviour was considered to be erratic, but resultant behaviour of the population is often quite predictable. The model approach was epidemiological which dealt with populations instead of individuals. The foundations of such approach was based on the mimetic human behavior, human herding, and social contagion. The mathematical model was concerned with violent extremism due to the likely or perceived rigged election result which is also a problem in Kenya. But the concerns of this project considered violent extremism due to terrorists' attack. violent extremism due to the likely or perceived rigged election may be short term and as a result of spontaneous response and may not take long before it dies off. While violent extremism due to terrorists' attack may have been planned for a long time and can exist equally for a long period of time.

[10] considered radicalization as a process, that is, before a person enters into a radical class, he or she must pass through the susceptible group from non-core group into indoctrinated class. This model assumed all radicals follow the same systematic path. In this model the radicals were people who had accepted their duties as jihads or holy warriors. It is important to note that some radicals end up being recruiters without necessarily carrying the act of violent attacks. Also the model did not illustrate clearly the population compartment as a result of government intervention.

The mathematical model [18] considered two sets of rival populations. In this model it was rightly assumed to my view, that attacks perpetrated by radicals from one set of population increase under hostilities perpetrated by radicals of the other set of the opposing population. Such mathematical models are best suited to be applied to any political or nationalistic ideology that causes 'violent' rivalries, but not for religious extremism, as competing religious violent radicalism may not be the case in Kenya.

[9, 17] in their Bare-Bones mathematical model of radicalization introduced a simple compartmental model (similar to epidemiology models) to describe the radicalization process. In their second approach the model was similar to the one used in the study of multi-strain diseases. The Bare-Bones model was based on competitive exclusion principle. The Bare-Bones mathematical model assumed that extremists subscribing to one ideology may switch to support the other ideology, but only in one direction. Again Bare-Bones mathematical model may not address the Kenyans situation.

In all the mathematical models discussed above none of them had a component population of those who dropped radical ideology. In this model the major concern, was to include the population who have dropped radical ideology through sensitization.

2 MODEL FORMULATION AND DESCRIPTION

people who recruit others into extremism and $S_E(t)$ represents, the population of all those who have stopped extremism either through sensitization campaign or education through media.

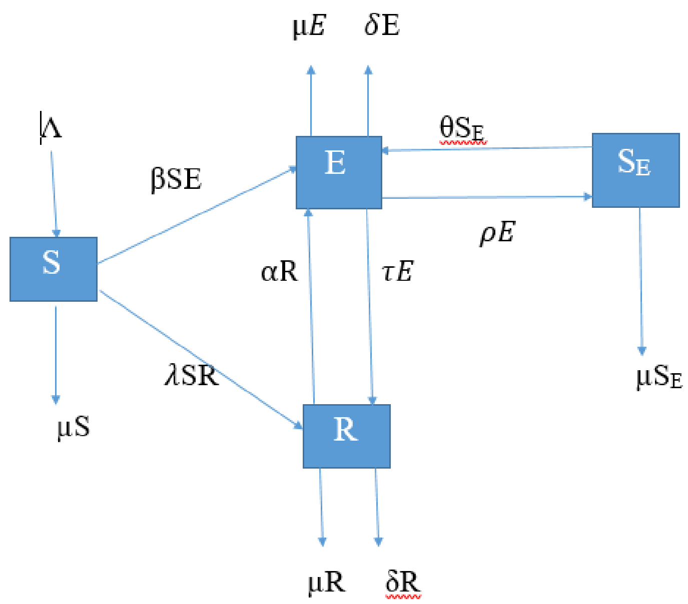


Figure 1: Radicalization model.

The total population of the compartments $N(t)$, considered in this model, is obtained using the formula below;

$N(t) = S(t) + E(t) + R(t) + S_E(t)$

Variable	Definition	Description
$S(t)$	Susceptible	Population at risk of radicalization
$E(t)$	Extremists	Population of extremists
$R(t)$	Recruiters	Population of recruiters
$S_E(t)$	Sensitized	Population sensitized and dropped extremism

Table 1: The model variables

Parameter	Description
Λ	Entry rate into susceptible
β	Contact rate between extremism and susceptible
λ	Rate of susceptible into recruiter
α	Rate of recruiters into extremism
δ	Recruiters/Extremist death rate due to their involvement in radicalization
μ	Natural death rate of all population
θ	Rate of treated into extremism
τ	Recruitment rate of extremists into recruiters
ρ	Rate of extremism into sensitized population

Table 2: The model parameters

3 MODEL EQUATIONS

$N(0) = S(0)$

$N(t) = S(t) + E(t) + R(t) + S_E(t)$

The following are the set of differential equations that are obtained from the model:

$$\left.\begin{aligned}\frac{dS}{dt} &= \Lambda - \beta SE - \lambda SR - \mu S\\ \frac{dE}{dt} &= \beta SE + \theta S_E + \alpha R - (\tau + \mu + \delta + \rho)E\\ \frac{dR}{dt} &= \lambda SR + \tau E - (\alpha + \mu + \delta)R\\ \frac{dS_E}{dt} &= \rho E - (\theta + \mu)S_E\end{aligned}\right\}$$

4 THE POSITIVITY AND SOLUTION BOUNDEDNESS

The region of invariant refers to the region in which solutions of radicalization model is uniformly bounded in the subset $\Psi \subset R^4_+$. By considering the total human population of the model at any time total human population is given by;

$N(t) = S(t) + E(t) + R(t) + S_E(t)$

$\frac{dN(t)}{dt} = \frac{dS(t)}{dt} + \frac{dE(t)}{dt} + \frac{dR(t)}{dt} + \frac{S_E(t)}{dt}$

The total population given by the sum of all the susceptible, recruiters, extremist and sensitized humans;

$$\left.\begin{array}{l} \frac{dN}{dt} = \Lambda - \beta SE - \lambda SR - \mu S + \beta SE + \theta S_E + \alpha R + \lambda SR + \\ \tau E - (\tau + \mu + \delta + \rho)E - (\alpha + \mu + \delta)R + \rho E - (\theta + \mu)S_E \end{array}\right\}$$

The result after the simplification of the above equation is;

$\frac{dN}{dt} = \Lambda - \mu S - \mu E - \delta E - \mu R - \delta R - \mu S_E$

In the absence of radicalization and radicalization into extremism, there will be no sensitized group and the above equation reduces to;

$\frac{dN}{dt} = \Lambda - \mu S$

$\frac{dN}{dt} = \Lambda - \mu N$

$\frac{dN}{\Lambda - \mu N} \leq dt$

$\int \frac{dN}{\Lambda - \mu N} \leq \int dt$

$\frac{\ln(\Lambda - \mu N)}{-\mu} \leq t + A$

$(\Lambda - \mu N) \leq -\mu t - \mu A$

$\Lambda - \mu N \leq e^{-\mu t - \mu A}$

$\Lambda - \mu N \leq e^{-\mu t} e^{-\mu A}$

$e^{-\mu A} \leq C$ where C is a constant and therefore;

$\Lambda - \mu N \leq Ce^{-\mu t}$

Using initial conditions $N = N(0)$ we have;

$\Lambda - \mu N(0) = C$

$\Rightarrow \Lambda - \mu N = (\Lambda - \mu N(0))e^{-\mu t}$

By making N the subject, and simplifying, the above equation becomes;

$N \leq \frac{\Lambda}{\mu} - \frac{(\Lambda - \mu N(0))e^{-\mu t}}{\mu}$

As $t \to 0$, $e^{-\mu t} = 1$

Therefore,

$N \leq \frac{\Lambda}{\mu} - \frac{(\Lambda - \mu N(0))}{\mu}$

$N \leq \frac{\Lambda}{\mu} - \frac{\Lambda}{\mu} + \frac{\mu N(0))}{\mu}$

$N \leq N(0)$

As $t \to \infty$, $e^{-\mu t} \to 0$

Therefore,

$N \leq \frac{\Lambda}{\mu}$

Hence

$\Psi = \left\{ (S, E, R, S_E) \in \mathbb{R}_+^4 : S, E, R, S_E \leq \frac{\Lambda}{\mu} \right\}$ [11, 6, 4].

5 EXTREMISM FREE EQUILIBRIUM POINTS

At this points the system of differential equations are set to zero.

At extremism free equilibrium there are no people being influenced to join recruiters or extremist group and hence there are no people being sensitized to leave recruiters or extremist group. This implies that $E = 0$, $R = 0$ and $S_E = 0$; and since $E = 0$, $R = 0$ and $S_E = 0$, the above equations reduces to;

$\frac{dS}{dt} = \Lambda - \beta SE - \lambda SR - \mu S = 0$

$\Lambda - \beta SE - \lambda SR - \mu S = 0$

but $R = 0$ and $E = 0$

$\Lambda - \mu S = 0$

$S = \frac{\Lambda}{\mu}$

Therefore the susceptible population S^*, at extremism free equilibrium is given by;

$S^* = \frac{\Lambda}{\mu}$

$(EFE) = (S^*, 0, 0, 0)$

$(EFE) = (\frac{\Lambda}{\mu}, 0, 0, 0)$

6 EXTREMISM ENDEMIC EQUILIBRIUM POINTS.

At equilibrium points the rate of changes is zero. Therefore the system of differential equations are set to zero.

$\frac{dS}{dt} = \Lambda - \beta SE - \lambda SR - \mu S = 0$

$\Lambda - \beta SE - \lambda SR - \mu S = 0$

$\Lambda - (\beta E + \lambda R + \mu)S = 0$

$$S = \frac{\Lambda}{\beta E + \lambda R + \mu}$$
$$S^* = \frac{\Lambda}{\beta E + \lambda R + \mu}$$
$$\frac{dE}{dt} = \beta SE + \theta S_E + \alpha R - (\tau + \mu + \delta + \rho)E = 0$$
$$\beta SE + \theta S_E + \alpha R - (\tau + \mu + \delta + \rho)E = 0$$
$$(\tau + \mu + \delta + \rho)E = \beta SE + \theta S_E + \alpha R$$
$$E^* = \frac{\theta S_E + \alpha R}{\tau + \mu + \delta + \rho - \beta S}$$
$$\frac{dR}{dt} = \lambda SR + \tau E - (\alpha + \mu + \delta)R = 0$$
$$\lambda SR + \tau E - (\alpha + \mu + \delta)R = 0$$
$$(\alpha + \mu + \delta)R - \lambda SR = \tau E$$
$$(\alpha + \mu + \delta - \lambda S)R = \tau E$$
$$R^* = \frac{\tau E}{\alpha + \mu + \delta - \lambda S}$$
$$\frac{dS_E}{dt} = \rho E - (\theta + \mu)S_E = 0$$
$$\frac{dS_E}{dt} = \rho E - (\theta + \mu)S_E \rho E - (\theta + \mu)S_E = 0$$
$$S_E^* = \frac{\rho E}{\theta + \mu}$$
$$EE = (S^*, E^*, R^*, S_E^*)$$
$$EE = (\frac{\Lambda}{\beta E + \lambda R + \mu}, \frac{\theta S_E + \alpha R}{\beta E + \lambda R + \mu}, \frac{\tau E}{\alpha + \mu + \delta - \lambda S}, \frac{\rho E}{\theta + \mu})$$

7 STABILITY OF THE EXTREMISM ENDEMIC EQUILIBRIUM

Theorem 3. If $R_0 > 1$, then the extremism endemic equilibrium is globally asymptotically stable.

Proof:

Consider a Lyapunov function defined as;

$$\left.\begin{array}{l} L(S^*E^*, R^*, S_E^*) = (S - S^* - S^* \ln \frac{S^*}{S}) + (E - E^* \\ -E^* \ln \frac{E^*}{E}\Big) + (R - R^* - R^* \ln \frac{R^*}{R}) + (S_E - S_E^* - S_E^* \ln \frac{S_E^*}{S_E}) \end{array}\right\}$$

Determining the derivative of the L along the solutions of the system of equations directly;

$$\frac{dL}{dt} = \left(\frac{S - S^*}{S}\right)\frac{dS}{dt} + \left(\frac{E - E^*}{E}\right)\frac{dE}{dt} + \left(\frac{R - R^*}{R}\right)\frac{dR}{dt} + \left(\frac{S_E - S_E^*}{S}\right)\frac{dS_E}{dt}$$

By replacing $\frac{dS}{dt},\frac{dE}{dt},\frac{dR}{dt}$ and $\frac{dS_E}{dt}$

$$\left.\begin{array}{l}\frac{dL}{dt}=(\frac{S-S^*}{S})(\Lambda-\beta SE-\lambda SR-\mu S)+(\frac{E-E^*}{E})(\beta SE+\theta S_E+\alpha R-(\tau+\mu\\ +\delta+\rho)E)+(\frac{R-R^*}{R})(\lambda SR+\tau E-(\alpha+\mu+\delta)R)+(\frac{S_E-S_E^*}{S})(\rho E-(\theta+\mu)S_E)\end{array}\right\}$$

By expansion and rearranging;

Let;

$$\frac{dL}{dt}=P-Q$$

where P are the positive terms and Q the negative terms.

$$\left.\begin{array}{l}P=\Lambda+\beta S^*E+\lambda S^*R+\mu S^*+\beta SE+\theta S_E+\alpha R+(\tau+\mu+\delta+\rho)E^*\\ \qquad +\lambda SR+\tau E+(\alpha+\mu+\delta)R^*+\rho E+(\theta+\mu)S_E^*\end{array}\right\}$$

and

$$\left.\begin{array}{l}Q=\beta SE+\lambda SR+\mu S+\frac{\Lambda S^*}{S}+(\tau+\mu+\delta+\rho)E)+\beta SE^*+\frac{\theta S_E E^*}{E}\\ +\frac{\alpha RE^*}{E}+(\alpha+\mu+\delta)R+\lambda SR^*+\frac{\tau ER^*}{R}+(\theta+\mu)S_E+\frac{\rho ES_E^*}{S_E}\end{array}\right\}$$

If $P<Q$ then the derivative of the Lyapunov function is less than or equal to zero

If $P<Q$ then $\frac{dL}{dt}\leq 0$

Moreover , $\frac{dL}{dt}=0$ if and only if $S=S^*$,$E=E^*$,$R=R^*$ and $S_E=S_E^*$

Hence the largest compact invariant set in $\{S,E,R,S_E\}\in\Psi$:$\frac{dL}{dt}=0$in singleton ξ^* where$\xi^*$is the endemic equilibrium

Therefore, the extremism endemic equilibrium is globally asymptotically stable in the invariant Ψ if $P<Q$ [7, 14, 15]

8 THE EXTREMISM REPRODUCTIVE NUMBER (R_0)

By using the Jacobian matrix approach in [16, 12, 8];

$$J(S,E,R,S_E)=\begin{pmatrix}\frac{\partial S}{\partial S} & \frac{\partial S}{\partial E} & \frac{\partial S}{\partial R} & \frac{\partial S}{\partial S_E}\\ \frac{\partial E}{\partial S} & \frac{\partial E}{\partial E} & \frac{\partial E}{\partial R} & \frac{\partial E}{\partial S_E}\\ \frac{\partial R}{\partial S} & \frac{\partial R}{\partial E} & \frac{\partial R}{\partial R} & \frac{\partial R}{\partial S_E}\\ \frac{\partial S_E}{\partial S} & \frac{\partial S_E}{\partial E} & \frac{\partial S_E}{\partial R} & \frac{\partial S_E}{\partial S_E}\end{pmatrix}$$

$$\begin{pmatrix}-\beta E-\lambda R-\mu & -\beta S & -\lambda S & 0\\ \beta E & \beta S-(\tau+\mu+\delta+\rho) & \alpha & \theta\\ \lambda R & \tau & \lambda S-(\alpha+\mu+\delta) & 0\\ 0 & \rho & 0 & -(\theta+\mu)\end{pmatrix}$$

Now computing the Jacobian matrix at Extremism free equilibrium point $J(S^*,0,0,0)$

$$\begin{pmatrix} -\mu & \frac{-\beta\Lambda}{\mu} & \frac{-\lambda\Lambda}{\mu} & 0 \\ 0 & \frac{\beta\Lambda}{\mu}-(\tau+\mu+\delta+\rho) & \alpha & \theta \\ 0 & \tau & \frac{\lambda\Lambda}{\mu}-(\alpha+\mu+\delta) & 0 \\ 0 & \rho & 0 & -(\mu+\theta) \end{pmatrix}$$

$$\begin{vmatrix} -\mu & \frac{-\beta\Lambda}{\mu} & \frac{-\lambda\Lambda}{\mu} & 0 \\ 0 & (\frac{\beta\Lambda}{\mu}-(\tau+\mu+\delta+\rho)) & \alpha & \theta \\ 0 & \tau & (\frac{\lambda\Lambda}{\mu}-(\alpha+\mu+\delta)) & 0 \\ 0 & \rho & 0 & -(\mu+\theta) \end{vmatrix} = 0$$

Hence; determining the eigenvalues of the Jacobian matrix and selecting the dominant eigenvalue, the extremism reproductive number R_0 is given by the relation;

$$R_0 = \frac{\beta\Lambda}{\mu}-(\tau+\mu+\delta+\rho)+\frac{\lambda\Lambda}{\mu}-(\mu+\delta+\alpha)$$

$$R_0 = \frac{(\beta+\lambda)\Lambda}{\mu}-(\tau+2\mu+2\delta+\rho)$$

$$R_0 = \frac{(\beta+\lambda)\Lambda-\mu(\tau+2\mu+2\delta+\rho)}{\mu}$$

$R_0 = 0.03281$ (by substituting the values of the parameters)

Since $R_0 < 1$ extremism free equilibrium point is locally asymptotically stable [13, 11].

9. PARAMETER CONTRIBUTION (SENSITIVITY ANALYSIS)

The sensitivity analysis of a model parameter is normally evaluated by relating each parameter to the reproduction number, (R_0) [8, 5, 3]. The sensitivity $S_m^{R_0}$ of a variable m is given by the relation;

$$S_m^{R_0} = \frac{\partial R_0}{\partial_m} * \frac{m}{R_0} \tag{1}$$

Sensitivity index Λ

$$S_\Lambda^{R_0} = \frac{\partial R_0}{\partial \Lambda} \times \frac{\Lambda}{R_0}$$

$$S_\Lambda^{R_0} = \frac{(\beta+\lambda)\Lambda}{(\beta+\lambda)\Lambda-\mu(\tau+2\mu+2\delta+\rho)}$$

Sensitivity index β

$$S_{\beta}^{R_0} = \frac{\partial R_0}{\partial \beta} \times \frac{\beta}{R_0}$$

$$S_{\beta}^{R_0} = \frac{\beta \Lambda}{(\beta+\lambda)\Lambda - \mu(\tau+2\mu+2\delta+\rho)}$$

Sensitivity index λ

$$S_{\lambda}^{R_0} = \frac{\partial R_0}{\partial \lambda} \times \frac{\lambda}{R_0}$$

$$S_{\lambda}^{R_0} = \frac{(\beta+\lambda)\Lambda}{(\beta+\lambda)\Lambda - \mu(\tau+2\mu+2\delta+\rho)}$$

Sensitivity index α

$$S_{\alpha}^{R_0} = \frac{\partial R_0}{\partial \alpha} \times \frac{\alpha}{R_0}$$

$$S_{\alpha}^{R_0} = \frac{-\alpha\mu}{(\beta+\lambda)\Lambda - \mu(\tau+2\mu+2\delta+\rho)}$$

Sensitivity index δ

$$S_{\delta}^{R_0} = \frac{\partial R_0}{\partial \delta} \times \frac{\delta}{R_0}$$

$$S_{\delta}^{R_0} = \frac{-2\delta\mu}{(\beta+\lambda)\Lambda - \mu(\tau+2\mu+2\delta+\rho)}$$

Sensitivity index μ

$$S_{\mu}^{R_0} = \frac{\partial R_0}{\partial \mu} \times \frac{\mu}{R_0}$$

$$S_{\mu}^{R_0} = \frac{-((\beta+\lambda)\Lambda + \mu^2)}{(\beta+\lambda)\Lambda - \mu(\tau+2\mu+2\delta+\rho)}$$

Sensitivity index θ

$$S_{\theta}^{R_0} = \frac{\partial R_0}{\partial \theta} \times \frac{\theta}{R_0}$$

Sensitivity index τ

$$S_{\tau}^{R_0} = \frac{\partial R_0}{\partial \tau} \times \frac{\tau}{R_0}$$

$$S_{\tau}^{R_0} = \frac{-\tau\mu}{(\beta+\lambda)\Lambda - \mu(\tau+2\mu+2\delta+\rho)}$$

Sensitivity index ρ

$$S_{\rho}^{R_0} = \frac{\partial R_0}{\partial \rho} \times \frac{\rho}{R_0}$$

$$S_{\rho}^{R_0} = \frac{-\rho\mu}{(\beta+\lambda)\Lambda - \mu(\tau+2\mu+2\delta+\rho)}$$

Parameter	Sensitivity level	Sensitivity Index +/-
Λ	14.156	+
β	10.6168	+
λ	3.53895	+
α	-0.731556	-
δ	-3.59682	-
μ	-14.8752	-
θ	0	
τ	-0.4877	-
ρ	-7.62038	-

Table 3: Sensitivity level

When the $S_m^{R_0}$ is positive or negative, the parameter m causes an increase or decrease in reproduction number respectively. Therefore from the above sensitivity analysis, the increase in the parameters such as recruitment rate into susceptible, contact rate between extremist and susceptible and interaction rate between recruiters and susceptible cause an increase in reproductive number. The increase in the rest of the parameters causes a reduction of reproductive number while the rate of sensitized into extremism may have no effect.

10. NUMERICAL SIMULATIONS

Numerical simulations was performed on the radicalization parameters in-order to observe dynamics of the population of susceptible, extremists, recruiters and sensitized people, so that to determine how each population compartment changes with time. This may give insight to the concerned groups fighting al-shabaab and terrorism in general.The simulation was done using the *Fehlberg fourth and fifth order Runge-Kutta* method using a software.

In the numerical simulations additional assumptions were made as follows;

(i) Most Kenyans radicals are people of Somali origin and therefore the number of radicals from non-Somali origin is negligible. Hence, this means the population compartments of the Kenyan situation is proportional to the Somali situation.

(ii) Number of individuals Λ entering into susceptible population is constant throughout the years and is a percentage of initial population of susceptible.

(iii) Susceptible population is taken as individuals whose age lies between 10 years and 39 years and that this population is approximately three fifth of the total population.

Data for the Numerical Simulation

The following information in the tables below were used to estimate some of the parameters and initial conditions.

Detail	Quantity	Year/Period of Time
Population of Somali P	10 million	2013
Population of Somali growth	2.90%	2015
Al-shabaab Population taken as E	7000-9000	2014
Population of new recruited soldiers taken as βSE	4213	1/4/2010-31/7/2016
Natural death rate in Somali μ	11.8 in a 1000	2015

Table 4: Somali and Al-shabaab population

Age (Years)	< 10	10-14	15-19	20-24	25-29	30-34	35-39	> 40	**Source**[1]
Percentage	1	4	40	25	21	5	2	2	

Table 5: Age at which one joined al-shabaab

Roles	Percentage
Casual workers	17
Money collectors	6
Fighters	60
Intelligence	2
Recruiters	5
Religious scholars	3
Security	5
Trainers	2
Source[1]	

Table 6: Core duties in al-shabaab

Estimation of initial condition taken as the year near 2014 and assumed to be the time sensitization was introduced together with the parameters Λ, β, λ, μ and δ were then estimated as shown below;

Therefore the initial condition and the values of some parameters were estimated as follows;

$S(0) \approx \frac{3}{5} \times 10000000 \approx 6000000$

$E(0) \approx 8000$

$R(0) \approx \frac{1}{8}E(0) \approx 1000$

Estimation of Λ considering initial susceptible population

$\Lambda \approx \frac{2.9}{100} \times 6000000$

$\Lambda \approx 174000$

Estimation of β and λ

λ is assumed to be approximately equal to $\frac{1}{3}\beta$

$\beta SE = 4213$ for a period of about 6 years.

$6000000 \times 8000 \times \beta = \frac{4213}{6}$

$\beta \approx 0.000000015$

$\lambda \approx 0.000000005$

Estimation of δ, death rate due to being radicalized as a percentage of the already radicalized population.

δ is also assumed to be five times the natural death μ

$\delta \approx 5\mu$

$\delta \approx 5 \times 0.0118 \approx 0.059$

The table below shows estimated and assumed parameters and how they were used to obtain various graphs.

Parameter	**Estimated/Assumed**	**Value**
Λ	Estimated	174000
β	Estimated	0.000000015
λ	Estimated	0.000000005
α	Assumed	0.024
δ	Estimated	0.059
μ	Estimated	0.0118
θ	Assumed	0.012
τ	Assumed	0.016
ρ	Assumed	0.25

Table 7: Values of estimated and assumed parameters

Susceptible population

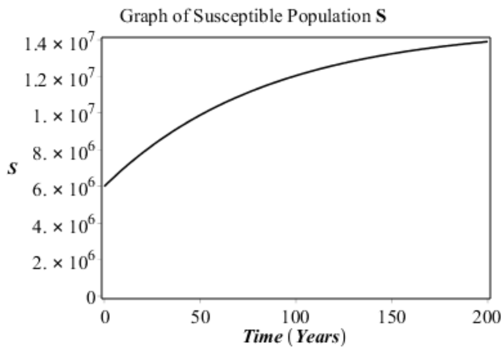


Figure 2: A graph of Susceptible Population **S**

The susceptible population continues to increase in a way similar to most of the general populations of the world where a carry capacity exists. This could be due the high number of new individuals entering susceptible compared to the ones leaving through various means, that is, rate of radicalization into extremism and recruiters together with natural death of susceptible is less than the rate at which people enter the susceptible compartment

Extremists population

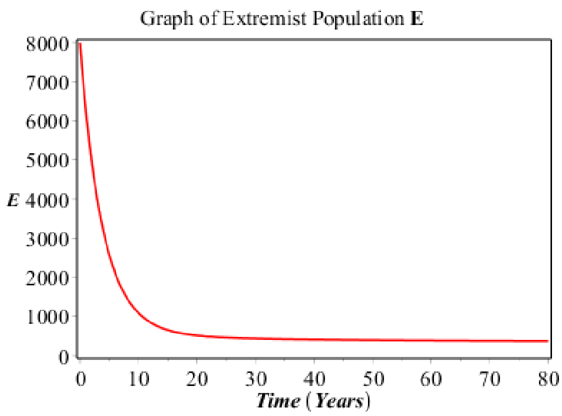


Figure 3: A graph of Extremists' population **E**

The number of extremists reduces with time to about 5 percent of the initial value and almost stabilizes at low value. The reduction may be mainly because of the sensitization after which the sensitized population increases. The high sensitized population also increases the number of sensitized population crossing back to extremism if θ remains constant. This increased population crossing from sensitized group to extremists tends to make the extremists population to stabilize.

Generally, from the graph representing the extremists population, it is very clear that, at the rate of sensitization of 25 percent the extremists population per year and other parameters remaining constant as assumed, the extremists will continue existing, and they are the most dangerous group to the general public.

Population recruiters

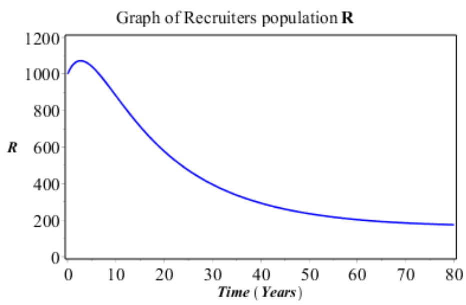


Figure 4: A graph of Recruiters population **R**

Recruiters population increases initially and then reduces with time and finally appears to stabilize for a period indicated on the graph. This could be attributed to initial high number extremists becoming recruiters. As more and more extremists get sensitized, the number of extremists crossing to recruiters reduces.

Sensitized population

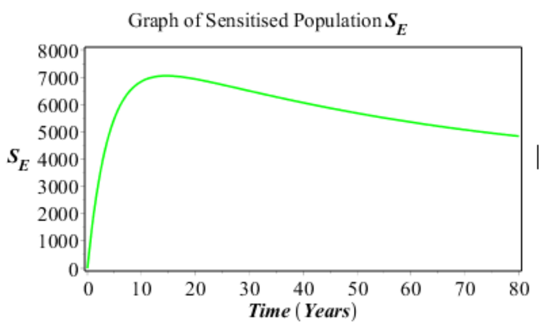


Figure 5: A graph of Sensitized population S_E

The sensitized population increases rapidly from zero and then start reducing gradually. This could be attributed to initial high population of extremists. The number of people being sensitized in a given time is proportional to the number of extremists provided θ remains constant. Therefore, at low population of extremists very few number of extremists are sensitized to drop radical ideology. At low number of extremists, the population being sensitized may be less than the sum of the sensitized population crossing back to extremism and the ones dying naturally hence causing a decrease in the sensitized population.

General population dynamics

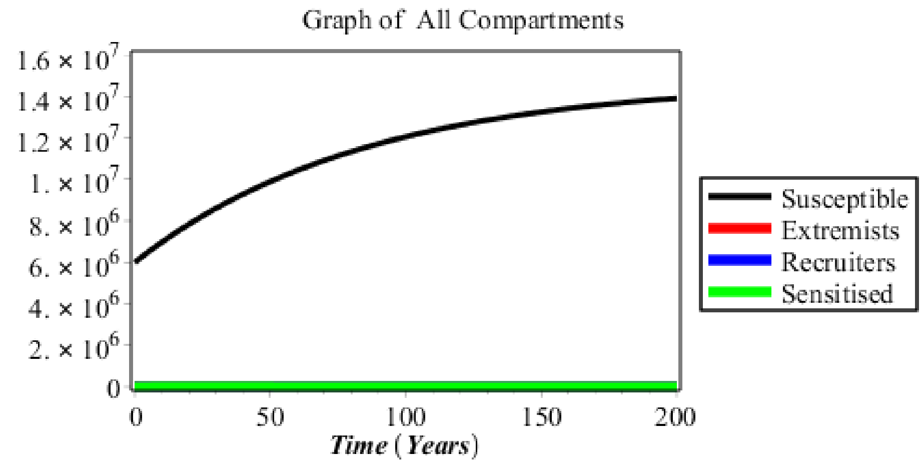
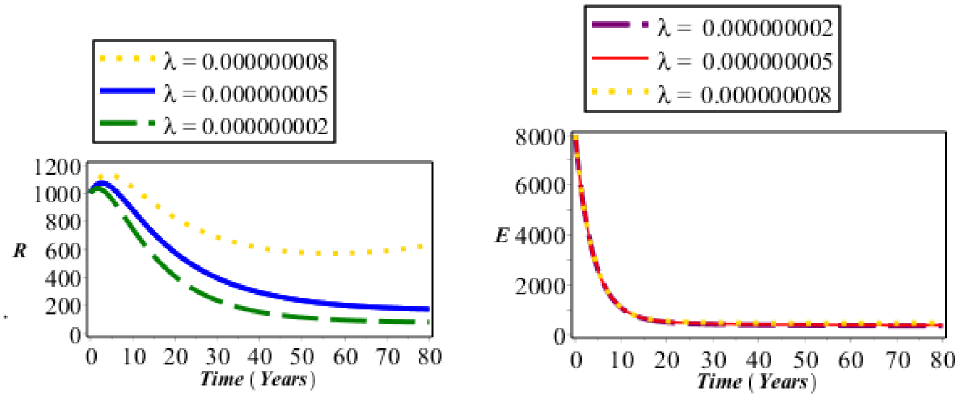


Figure 6: A graph of All compartments

This graph shows that, the rest of the population is very small or almost negligible as compared to susceptible population or the general population of the nation. But this negligible radicalized population in particular can not be ignored as observed practically in their extremism activities. The effort should be aimed at eliminating the radicalized mind in the society. This should be done mainly through sensitization and other prevention measures.

From figure 3 and figure 4, it can be observed that, population of radicals stabilizes at low value, it is most likely that, that is the time when the radicals' group activities are done in latent design until they regain. At this point in time, the government and other stakeholders in the world fighting terrorists activities should not relent on the fight against terrorism.

Effect of contact rate between recruiters and susceptible population

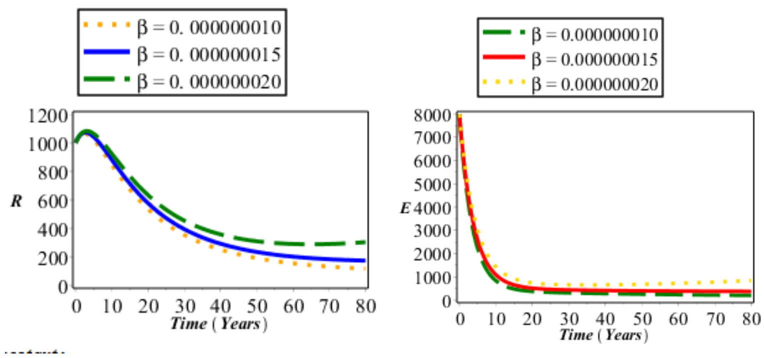


(a) Effect on Recruiters **R** **(b)** Effect on Extremists **E**

Figure 7: Graphs showing effect of contact rate between Recruiters and Susceptible population

The higher the contact rate between the recruiters and the susceptible make the population of the recruiters to be at higher level, although in an increasing then decreasing trend. While on the extremists' population, the change in the contact rate has no significant effect.

Effect of contact rate between extremists and susceptible population

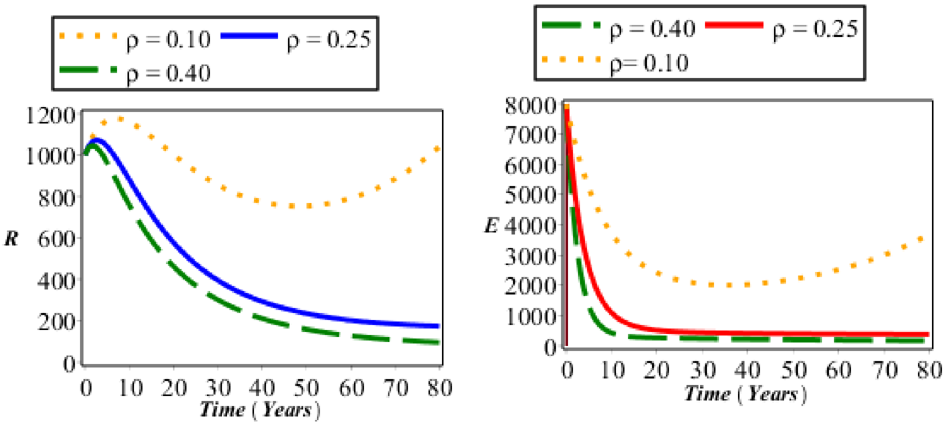


(a) Effect on Recruiters **R** **(b)** Effect on Extremists **E**

Figure 8: Graphs showing effect of contact rate between Extremists and Susceptible population

When the contact rate between the extremists and the susceptible increases, both the extremists' and recruiters' population are at higher level although in decreasing trend.

Effect of sensitization level



(a) Effect on Recruiters **R** **(b)** Effect on Extremists **E**

Figure 9: Graphs showing effect of sensitization level

Although the extremists are the ones being sensitized, the change in the sensitization level also have an impact on the population of recruiters. Increase in the sensitization level, reduces both the extremists' and recruiters' population. At very low level of sensitization, for example10 percent per year, the recruiters' and extremists' population reduces then start increasing again.

11 CONCLUSION REMARKS

In this project, an epidemic model to study radicalization/extremism was proposed. The model was analyzed using both qualitative and numerical approach. Sensitivity analysis was employed to determine the effect of the constants in the model. The Lyapunov function was used to show that the extremism endemic equilibrium is globally asymptotically stable as discussed in section 6 while the derivation and calculation of the reproduction number was used to show that the system is locally asymptotically stable. During the numerical simulation, Somali population and the number of al-shabaab soldiers in the year 2014, were assumed to be proportional to the Kenyans situation. From the numerical simulation, the model revealed that, the population of the susceptible will continue to increase with time. While extremists and radicals will decrease and tend to stabilize at some points. The number of the sensitized population increased until almost the extremists' population stabilized and reduces thereafter. From the simulation, using both the estimated and assumed parameters as they are, it is clear that, the radicalized population will continue existing unless stern measures are taken or rate of sensitization is increased.

When modelling social dynamics, several simplifying assumptions has to be made. The model studied in this paper is not exceptional from such assumptions. These assumptions may end up not giving a complete true picture of the situation. One issue, for instance, some of the parameters which were completely assumed need to be determined through research and more so from the radical group themselves, although this may be a risky undertaking. Also the model can still be further improved by considering a population sensitized before joining extremists' population. In addition, the likelihood of a particular social-status, sex or age joining extremists or recruiters compartment should be taken into consideration. Therefore, a social-status, age and sex structured model may be a better one. The plan is to address some of these issues in the future modelling and equally in cooperate some other ideas which may be offered by readers of this model or any other person or source. Last but not least, since stern measures need to be taken in order to eliminate radicalization, one of the proposed measure should include, bringing the leadership of the radicalized group on re-conciliatory table and where possible technically, involve them in governance of a nation.

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