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Optimal Two Parameter Logarithmic Estimators for Estimating the Population Variance

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Abstract

Some classes of log-type estimators using information on two auxiliary variables have been proposed for estimating the population variance of the study variable. It has been shown that these classes of log-type estimators have lesser mean squared error under the optimum values of the characterizing scalars as compared to some of the commonly used estimators available in the literature. Further, an extension of the proposed classes using multiple auxiliary information have also initiated in this paper. A numerical study is included as an illustration using two auxiliary variables.

1. INTRODUCTION

In sampling theory, it is a popular trend to use auxiliary information to obtain more efficient estimators for the population parameters to increase the precision of the estimator. Estimators obtained using auxiliary information are supposed to be more efficient than the estimators obtained without using auxiliary information. The ratio, regression, product and difference estimator take advantage of the auxiliary information at the estimation stage. Many authors like, Pandey and Dubey (1988), Upadhyaya and Singh (1999) , Kadilar and Cingi (2003), Singh and Taylor (2003), Singh (2003), Sisodia and Dwivedi (1981), Koyuncu and Kadilar along with many others have proposed various estimators using auxiliary information on various population parameters like coefficient of skewness, kurtosis, standard deviation, correlation coefficient etc. Sometimes, it is more economical to obtain information on more than one auxiliary information; this would probably help in improvising the efficiency of the estimator, used to estimate the parameter under consideration. The literature deals with a wide range of ratio, product, difference and exponential estimators proposed by various renowned authors using multiple auxiliary information (Olkin (1958), Raj (1965), Singh (1967),Shukla(1966), etc.). Recently, Bhushan and Kumari (2018) had made the use of logarithmic relationship between the study variable and auxiliary variable, we have made the use of multiple auxiliary variables x0s for estimating the population variance. The proposed estimators would work in case when the study variable is logarithmically related to the auxiliary variable.

Consider a finite population $U = U_1, U_2, ..., U_N$ of size N from which a sample of size n is drawn according to simple random sampling without replacement (SRSWOR). Let y_i , x_{i_1} and x_{i_2} denotes the value of the study and two auxiliary variable for the ith unit $i = 1, 2, ..., N$ of the population. Further, let $\bar{y}$, $\bar{x}_1$ and $\bar{x}_2$ be the sample means of study variable and two auxiliary variables. Also, $s_y^2 = N^{-1}\sum_{i=1}^{N}(y_i - \bar{y})^2$, $s_{x_1}^2 = n^{-1}\sum_{i=1}^{n}(x_i - \bar{x}_1)^2$ and $s_{x_2}^2 = n^{-1}\sum_{i=1}^{n}(x_i - \bar{x}_2)^2$ be the sample variance of the study and two auxiliary variables respectively.

2. THE SUGGESTED GENERALIZED CLASS OF LOG-TYPE ESTIMATORS

We propose the following new classes of log type estimators for the population variance S_y^2 as

$$T_1 = s_y^2\left[1+\log\left(\frac{S_{x_1}^2}{s_{x_1}^2}\right)\right]^{a_1}\left[1+\log\left(\frac{S_{x_2}^2}{s_{x_2}^2}\right)\right]^{a_2} \tag{2.1}$$

$$T_2 = s_y^2\left[1+b_1\log\left(\frac{S_{x_1}^2}{s_{x_1}^2}\right)\right]\left[1+b_2\log\left(\frac{S_{x_2}^2}{s_{x_2}^2}\right)\right] \tag{2.2}$$

$$T_3 = s_y^2\left[1+\log\left(\frac{S_{x_1}^{2*}}{s_{x_1}^{2*}}\right)\right]^{c_1}\left[1+\log\left(\frac{S_{x_2}^{2*}}{s_{x_2}^{2*}}\right)\right]^{c_2} \tag{2.3}$$

$$T_4 = s_y^2\left[1+d_1\log\left(\frac{S_{x_1}^{2*}}{s_{x_1}^{2*}}\right)\right]\left[1+d_2\log\left(\frac{S_{x_2}^{2*}}{s_{x_2}^{2*}}\right)\right] \tag{2.4}$$

where $S_{x_i}^{2*} = a_i\, S_{x_i}^2 + b_i$ and $s_{x_i}^{2*} = a_i s_{x_i}^2 + b_i$ for i = 1, 2

such that a_i , b_i , c_i and d_i are optimizing scalars or functions of the known parameters of the auxiliary variable x_i's such as the standard deviations S_{x_i} , coefficient of variation C_{x_i} , coefficient of kurtosis b_{2x_i} , coefficient of skewness b_{1x_i} and correlation coefficient $r_{x_i x_j}$ of the population $(i \neq j = 0)$.

3. PROPERTIES OF THE SUGGESTED CLASS OF ESTIMATORS

In order to obtain the bias and mean square error (MSE), let us consider

$$\varepsilon_0 = \frac{\left(s_y^{\,2} - S_y^{\,2}\right)}{S_y^{\,2}},\ \varepsilon_1 = \frac{\left(s_{x_1}^{\,2} - S_{x_1}^{\,2}\right)}{S_{x_1}^{\,2}} \text{ and } \varepsilon_2 = \frac{\left(s_{x_2}^{\,2} - S_{x_2}^{\,2}\right)}{S_{x_2}^{\,2}}$$

$E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_2) = 0$, $E\left(\varepsilon_0^{\,2}\right) = Ib_{2y}^{*}$, $E\left(\varepsilon_1^{\,2}\right) = Ib_{2x_1}^{*}$, $E\left(\varepsilon_2^{\,2}\right) = Ib_{2x_2}^{*}$, $E(\varepsilon_0\varepsilon_1) = I\, I_{22yx_1}^{*}$, $E(\varepsilon_0\varepsilon_2) = I\, I_{22yx_2}^{*}$ and $E(\varepsilon_1\varepsilon_2) = I\, I_{22x_1x_2}^{*}$ where $b_{2y}^{*} = b_{2y} - 1$, $b_{2x_1}^{*} = b_{2x_1} - 1$, $b_{2x_2}^{*} = b_{2x_2} - 1$ and $I_{22yx_1}^{*} = I_{22yx_1} - 1$, $I_{22yx_2}^{*} = I_{22yx_2} - 1$, $I_{22x_1x_2}^{*} = I_{22x_1x_2} - 1$; $I_{pq} = m_{pq} \big/ m_{20}^{p/2}\, m_{02}^{q/2}$, $m_{pq} = \sum_{i=1}^{N} \left(Y_i - \bar{Y}\right)^p \left(X_i - \bar{X}\right)^q \Big/ N$, $I = 1/N$, $b_{2y} = m_{40} \big/ m_{20}^2$, $b_{2x} = m_{04} \big/ m_{02}^2$ are the coefficient of kurtosis of y and x respectively.

Theorem 1 *The bias of the proposed estimators are given as*

$$Bias(T_1) = S_y^2\, I\left[\frac{a_1^2}{2} b_{2x_1}^{*} + \frac{a_2^2}{2} b_{2x_2}^{*} - a_1\, r_{yx_1}\sqrt{b_{2y}^{*}\, b_{2x_1}^{*}} - a_2\, r_{yx_2}\sqrt{b_{2y}^{*}\, b_{2x_2}^{*}} + a_1\, a_2 r_{x_1x_2}\sqrt{b_{2x_1}^{*} b_{2x_2}^{*}}\right]$$

$$Bias(T_2) = S_y^2\, I\left[\frac{b_1}{2} b_{2x_1}^{*} + \frac{b_2}{2} b_{2x_2}^{*} - b_1\, r_{yx_1}\sqrt{b_{2y}^{*}\, b_{2x_1}^{*}} - b_2\, r_{yx_2}\sqrt{b_{2y}^{*}\, b_{2x_2}^{*}} + b_1\, b_2 r_{x_1x_2}\sqrt{b_{2x_1}^{*} b_{2x_2}^{*}}\right]$$

$$Bias(T_3) = S_y^2\, I\left[\frac{\eta_1^2 c_1^2}{2} b_{2x_1}^{*} + \frac{\eta_2^2 c_2^2}{2} b_{2x_2}^{*} - \eta_1 c_1\, r_{yx_1}\sqrt{b_{2y}^{*}\, b_{2x_1}^{*}} - \eta_1 c_2\, r_{yx_2}\sqrt{b_{2y}^{*}\, b_{2x_2}^{*}} + \eta_1\eta_2 c_1\, c_2 r_{x_1x_2}\sqrt{b_{2x_1}^{*} b_{2x_2}^{*}}\right]$$

$$Bias(T_4) = S_y^2\, I\left[\frac{\eta_1^2 d_1}{2} b_{2x_1}^{*} + \frac{\eta_1^2 d_2}{2} b_{2x_2}^{*} - \eta_1 d_1\, r_{yx_1}\sqrt{b_{2y}^{*}\, b_{2x_1}^{*}} - \eta_1 d_2\, r_{yx_2}\sqrt{b_{2y}^{*}\, b_{2x_2}^{*}} + \eta_1\eta_2 d_1\, d_2 r_{x_1x_2}\sqrt{b_{2x_1}^{*} b_{2x_2}^{*}}\right]$$

where $r_{yx_1} = \dfrac{I_{22yx_1}^{*}}{\sqrt{b_{2y}^{*}\, b_{2x_1}^{*}}}$, $r_{yx_2} = \dfrac{I_{22yx_2}^{*}}{\sqrt{b_{2y}^{*}\, b_{2x_2}^{*}}}$ and $r_{x_1x_2} = \dfrac{I_{22\,x_1x_2}^{*}}{\sqrt{b_{2x_1}^{*} b_{2x_2}^{*}}}$; $\eta_1 = \dfrac{a_1 S_{x_1}^2}{a_1 S_{x_1}^2 + a_2}$, $\eta_2 = \dfrac{a_1 S_{x_2}^2}{a_1 S_{x_2}^2 + a_2}$

Proof. Consider the estimator

$$T_1 = s_y^2 \left[1+\log\left(\frac{S_{x_1}^2}{s_{x_1}^2}\right)\right]^{a_1} \left[1+\log\left(\frac{S_{x_2}^2}{s_{x_2}^2}\right)\right]^{a_2}$$

$$T_1 - S_y^2 = S_y^2 \left[\frac{a_1^2\, \varepsilon_1^2}{2} + \frac{a_2^2\, \varepsilon_2^2}{2} + a_1 a_2 \varepsilon_1 \varepsilon_2 - a_1 \varepsilon_0 \varepsilon_1 - a_2 \varepsilon_0 \varepsilon_2\right] \quad (3.1)$$

Taking expectation on both the sides, we get

$$Bias(T_1) = S_y^2\, I \left[\frac{a_1^2}{2} b_{2x_1}^* + \frac{a_2^2}{2} b_{2x_2}^* - a_1\, r_{yx_1} \sqrt{b_{2y}^*\, b_{2x_1}^*} - a_2\, r_{yx_2} \sqrt{b_{2y}^*\, b_{2x_2}^*} + a_1\, a_2 r_{x_1x_2} \sqrt{b_{2x_1}^* b_{2x_2}^*}\right] \quad (3.2)$$

Proceeding in a similar way, we obtain the bias of the remaining proposed estimators.

Theorem 2 *The mean squared error of the proposed estimator considered up to the terms of order n^{-1} are given by*

$$MSE(T_1) = S_y^4\, I \left[b_{2y}^* + a_1^2 b_{2x_1}^* + a_2^2 b_{2x_2}^* - 2a_1 r_{yx_1} \sqrt{b_{2y}^* b_{2x_1}^*} - 2a_2 r_{yx_2} \sqrt{b_{2y}^* b_{2x_2}^*} + 2a_1 a_2 r_{x_1x_2} \sqrt{b_{2x_1}^* b_{2x_2}^*}\right]$$

$$MSE(T_2) = S_y^4\, I \left[b_{2y}^* + b_1^2 b_{2x_1}^* + b_2^2 b_{2x_2}^* - 2b_1 r_{yx_1} \sqrt{b_{2y}^* b_{2x_1}^*} - 2b_2 r_{yx_2} \sqrt{b_{2y}^* b_{2x_2}^*} + 2b_1 b_2 r_{x_1x_2} \sqrt{b_{2x_1}^* b_{2x_2}^*}\right]$$

$$MSE(T_3) = S_y^4\, I \left[b_{2y}^* + \eta_1^2 c_1^2 b_{2x_1}^* + \eta_2^2 c_2^2 b_{2x_2}^* - 2\eta_1 c_1 r_{yx_1} \sqrt{b_{2y}^* b_{2x_1}^*} - 2\eta_2 c_2 r_{yx_2} \sqrt{b_{2y}^* b_{2x_2}^*} \right.$$
$$\left. + 2\eta_1 \eta_2 c_1 c_2 r_{x_1x_2} \sqrt{b_{2x_1}^* b_{2x_2}^*}\right]$$

$$MSE(T_4) = S_y^4\, I \left[b_{2y}^* + \eta_1^2 d_1^2 b_{2x_1}^* + \eta_2^2 d_2^2 b_{2x_2}^* - 2\eta_1 d_1 r_{yx_1} \sqrt{b_{2y}^* b_{2x_1}^*} - 2\eta_2 d_2 r_{yx_2} \sqrt{b_{2y}^* b_{2x_2}^*} \right.$$
$$\left. + 2\eta_1 \eta_2 d_1 d_2 r_{x_1x_2} \sqrt{b_{2x_1}^* b_{2x_2}^*}\right]$$

Proof. Consider the estimator,

$$T_1 = s_y^2 \left[1+\log\left(\frac{S_{x_1}^2}{s_{x_1}^2}\right)\right]^{a_1} \left[1+\log\left(\frac{S_{x_2}^2}{s_{x_2}^2}\right)\right]^{a_2}$$

$$T_1 - S_y^2 = S_y^2 \left[\varepsilon_0 - a_1 \varepsilon_1 - a_2 \varepsilon_2\right]$$

Squaring on both the sides and taking expectation, we get

$$MSE(T_1) = S_y^4\, I \left[b_{2y}^* + a_1^2 b_{2x_1}^* + a_2^2 b_{2x_2}^* - 2a_1 r_{yx_1} \sqrt{b_{2y}^* b_{2x_1}^*} - 2a_2 r_{yx_2} \sqrt{b_{2y}^* b_{2x_2}^*} \right.$$
$$\left. + 2a_1 a_2 r_{x_1x_2} \sqrt{b_{2x_1}^* b_{2x_2}^*}\right] \quad (3.3)$$

Proceeding in a similar way, we obtain the required expressions for MSE of the proposed estimators.

Corollary 1. *The optimum values of constant are obtained as*

$$a_{1opt}=\left[\frac{r_{yx_1}-r_{yx_2}\,r_{x_1x_2}}{1-r^2_{x_1x_2}}\right]\frac{\sqrt{b^*_{2y}}}{\sqrt{b^*_{2x_1}}}$$

$$a_{2opt}=\left[\frac{r_{yx_2}-r_{yx_1}\,r_{x_1x_2}}{1-r^2_{x_1x_2}}\right]\frac{\sqrt{b^*_{2y}}}{\sqrt{b^*_{2x_2}}}$$

$$c_{1opt}=\left[\frac{r_{yx_1}-r_{yx_2}\,r_{x_1x_2}}{1-r^2_{x_1x_2}}\right]\frac{\sqrt{b^*_{2y}}}{\eta_1\sqrt{b^*_{2x_1}}}$$

$$c_{2opt}=\left[\frac{r_{yx_2}-r_{yx_1}\,r_{x_1x_2}}{1-r^2_{x_1x_2}}\right]\frac{\sqrt{b^*_{2y}}}{\eta_2\sqrt{b^*_{2x_2}}}$$

The optimum mean squared error is given by

$$M\left(T_1\right)_{opt}=I\,S_y^4\,\left(1-R^{2*}_{y.x_1x_2}\right) \tag{3.4}$$

where $R^{2*}_{y.x_1x_2}$ *is the transformed multiple correlation coefficient between* y *and* x_1, x_2

4. Multivariate extension of proposed class of estimators

Let there are k auxiliary variables then we can use the variables by taking a linear combination of these k estimators of the form given in section 2, calculated for every auxiliary variable separately, for estimating the population variance. Then the estimators for population variance will be defined as

$$T_1^*=s_y^2\;\pi_{i=1}^{k}\left[1+\log\left(\frac{S^2_{x_i}}{s^2_{x_i}}\right)\right]^{a_i}$$

$$T_2^*=s_y^2\;\pi_{i=1}^{k}\left[1+b_i\log\left(\frac{S^2_{x_i}}{s^2_{x_i}}\right)\right]$$

$$T_3^*=s_y^2\;\pi_{i=1}^{k}\left[1+\log\left(\frac{S^{2*}_{x_i}}{s^{2*}_{x_i}}\right)\right]^{c_i}$$

$$T_4^*=s_y^2\;\pi_{i=1}^{k}\left[1+d_i\log\left(\frac{S^{2*}_{x_i}}{s^{2*}_{x_i}}\right)\right]$$

where a_i, b_i, c_i and d_i are the optimizing scalars i = 1,2,...,k.

5. PROPERTIES OF PROPOSED CLASS OF ESTIMATORS USING MULTIPLE AUXILIARY INFORMATION

Theorem 4.*The bias of the proposed estimators are given by*

$$Bias\left(T_1^*\right)=S_y^2\, I\left[\sum_{i=1}^{k}\frac{a_i^2}{2}b_{2x_i}^* -\sum_{i=1}^{k}a_i\, r_{yx_i}\sqrt{b_{2y}^*\, b_{2x_i}^*}+\sum_{i\neq j=1}^{k}a_i\, a_j r_{x_i x_j}\sqrt{b_{2x_i}^* b_{2x_j}^*}\right]$$

$$Bias\left(T_2^*\right)=S_y^2\, I\left[-\sum_{i=1}^{k}\frac{b_i^2}{2}b_{2x_i}^* +\sum_{i=1}^{k}b_i\, r_{yx_i}\sqrt{b_{2y}^*\, b_{2x_i}^*}+\sum_{i\neq j=1}^{k}b_i\, b_j r_{x_i x_j}\sqrt{b_{2x_i}^* b_{2x_j}^*}\right]$$

$$Bias\left(T_3^*\right)=S_y^2\, I\left[\sum_{i=1}^{k}\frac{\eta_i^2 c_i^2}{2}b_{2x_i}^* -\sum_{i=1}^{k}\eta_i c_i\, r_{yx_i}\sqrt{b_{2y}^*\, b_{2x_i}^*}+\sum_{i\neq j=1}^{k}\eta_i\eta_j c_i\, c_j r_{x_i x_j}\sqrt{b_{2x_i}^* b_{2x_j}^*}\right]$$

$$Bias\left(T_4^*\right)=S_y^2\, I\left[-\sum_{i=1}^{k}\frac{\eta_i^2 d_i^2}{2}b_{2x_i}^* +\sum_{i=1}^{k}\eta_i d_i\, r_{yx_i}\sqrt{b_{2y}^*\, b_{2x_i}^*}+\sum_{i\neq j=1}^{k}\eta_i\eta_j d_i\, d_j r_{x_i x_j}\sqrt{b_{2x_i}^* b_{2x_j}^*}\right]$$

Theorem 5. *The mean squared error of the suggested class of estimators are given by*

$$MSE\left(T_1^*\right)=S_y^4\, I\left[b_{2y}^*+\sum_{i=1}^{k}a_i^2 b_{2x_i}^* -2\sum_{i=1}^{k}a_i r_{yx_i}\sqrt{b_{2y}^* b_{2x_i}^*}+2\sum_{i\neq j=1}^{k}a_i a_j r_{x_i x_j}\sqrt{b_{2x_i}^* b_{2x_j}^*}\right]$$

$$MSE\left(T_2^*\right)=S_y^4\, I\left[b_{2y}^*+\sum_{i=1}^{k}b_i^2 b_{2x_i}^* -2\sum_{i=1}^{k}b_i r_{yx_i}\sqrt{b_{2y}^* b_{2x_i}^*}+2\sum_{i\neq j=1}^{k}b_i b_j r_{x_i x_j}\sqrt{b_{2x_i}^* b_{2x_j}^*}\right]$$

$$MSE\left(T_3^*\right)=S_y^4\, I\left[b_{2y}^*+\sum_{i=1}^{k}\eta_i^2 c_i^2 b_{2x_i}^* -2\sum_{i=1}^{k}\eta_i c_i r_{yx_i}\sqrt{b_{2y}^* b_{2x_i}^*}+2\sum_{i\neq j=1}^{k}\eta_i\eta_j c_i c_j r_{x_i x_j}\sqrt{b_{2x_i}^* b_{2x_j}^*}\right]$$

$$MSE\left(T_4^*\right)=S_y^4\, I\left[b_{2y}^*+\sum_{i=1}^{k}\eta_i^2 b_i^2 d_{2x_i}^* -2\sum_{i=1}^{k}\eta_i d_i r_{yx_i}\sqrt{b_{2y}^* b_{2x_i}^*}+2\sum_{i\neq j=1}^{k}\eta_i\eta_j d_i d_j r_{x_i x_j}\sqrt{b_{2x_i}^* b_{2x_j}^*}\right]$$

6. EFFICIENCY COMPARISON

In this section, we compare the proposed classes of estimators with some important estimators. The comparison will be in terms of their MSE up to the order of n^{-1}. The optimum mean squared error of proposed estimator is given by

$$M\left(T_1\right)_{opt}=I\, S_y^4\left(1-R_{y.x_1x_2}^{2*}\right)$$

6.1 General variance estimator

$$\hat{S}_y^2=s_y^2$$

It's mean squared error is given by

$$MSE(\hat{S}_y^2)=S_y^4\, I\, b_{2y}^* > MSE(T_1)_{opt}$$

6.2 The usual ratio type variance estimator

$$\hat{S}_r^2 = s_y^2 \left(\frac{S_{x_1}^2}{s_{x_1}^2}\right)\left(\frac{S_{x_2}^2}{s_{x_2}^2}\right)$$

It's mean squared error is given by

$$MSE(\hat{S}_r^2) = S_y^4\, I\, \left[b_{2y}^* + b_{2x_1}^* + b_{2x2}^* - 2I_{22yx_1}^* - 2I_{22yx_2}^* + 2I_{22x_1x_2}^*\right] > MSE(T_1)_{opt}$$

6.3 The product type variance estimator

$$\hat{S}_p^2 = s_y^2 \left(\frac{s_{x_1}^2}{S_{x_1}^2}\right)\left(\frac{s_{x_2}^2}{S_{x_2}^2}\right)$$

Its mean squared error is given by

$$MSE(\hat{S}_r^2) = S_y^4\, I\, \left[b_{2y}^* + b_{2x_1}^* + b_{2x2}^* + 2I_{22yx_1}^* + 2I_{22yx_2}^* + 2I_{22x_1x_2}^*\right] > MSE(T_1)_{opt}$$

6.4 Isaki (1983) variance estimator

$$\hat{S}_I^2 = w_1 \left(\frac{s_y^2}{s_{x_1}^2}\right) S_{x_1}^2 + w_2 \left(\frac{s_y^2}{s_{x_2}^2}\right) S_{x_2}^2$$

The mean squared error is given by

$$MSE\left(\hat{S}_I^2\right)_{opt} = I\, S_y^4 \left[b_{2y}^* + b_{2x_2}^* - 2I_{22x_2}^* - \frac{\left(b_{2x_2}^* - I_{22x_2}^*\right)^2}{b_{2x_1}^* + b_{2x_2}^* - 2I_{22x_1x_2}^*}\right] > MSE\left(T_1\right)_{opt}$$

6.5 Singh, Chauhan, Sawan and Smarandache (2011) type variance estimator

$$\hat{S}_s^2 = s_y^2 \exp\left(\frac{S_{x_1}^2 - s_{x_1}^2}{S_{x_1}^2 + s_{x_1}^2}\right)\left(\frac{S_{x_2}^2 - s_{x_2}^2}{S_{x_2}^2 + s_{x_2}^2}\right)$$

It's mean squared error is given by

$$MSE(\hat{S}_s^2) = S_y^4\, I \left[b_{2y}^* + \frac{b_{2x_1}^*}{4} + \frac{b_{2x2}^*}{4} - I_{22yx_1}^* - I_{22yx_2}^* + \frac{I_{22x_1x_2}^*}{4}\right] > MSE(T_1)_{opt}$$

6.6 Olufadi and Kadilar (2014) variance estimator

$$\hat{S}_K^2 = s_y^2 \left(\frac{S_{x_1}^2}{s_{x_1}^2} \right)^{a_1} \left(\frac{S_{x_2}^2}{s_{x_2}^2} \right)^{a_2}$$

It's mean squared error is given by

$$MSE(\hat{S}_K^2) = MSE(T_1)_{opt}$$

6.7 Das and Tripathi (1978) type variance estimator

$$\hat{S}_D^2 = s_y^2 \left(\frac{S_{x_1}^2}{S_{x_1}^2 + a_1 \left(s_{x_1}^2 - S_{x_1}^2 \right)} \right) \left(\frac{S_{x_2}^2}{S_{x_2}^2 + a_2 \left(s_{x_2}^2 - S_{x_2}^2 \right)} \right)$$

It's mean squared error is given by

$$MSE(\hat{S}_D^2) = MSE(T_1)_{opt}$$

7. EMPIRICAL STUDY

The data on which we performed the numerical calculation is taken from some natural populations. The source of the data is given as follows.

Population 1. (Chochran, Pg. no. 155). The data concerns about weekly expenditure on food per family.

y : weekly expenditure on food

x_1 : number of persons

x_2 : the weekly family income

Population 2. (Singh S., Pg. no. 1114). The data concerns Apples commercial crop, season average price (in \$) per pound, by states 1994-1996.

y : season average price (in \$) per pound in 1996

x_1 : season average price (in \$) per pound in 1995

x_2 : season average price (in \$) per pound in 1994.

Population 3. (Singh S., Pg. no. 1123). The data concerns age specific death rates from 1990-2065. y : x1 : x2 : per 100,000 births in 2000

y : per 100,000 births in 2040

x_1 : per100,000 births in 1990

x_2 : per 100,000 births in 2000

Population 4. (Choudhary F. S., Pg. no. 117).

y : area under wheat (in acres) in 1974

x_1 : area under wheat (in acres) in 1971

x_2 : area under wheat (in acres) in 1973

The summary and the percent relative efficiency of the following estimators are as follows:

Table 2: Parameters of the data

Parameter	Population 1	Population 2	Population 3	Population 4
N	33	36	22	34
n	11	11	8	10
b^*_{2y}	4.032	2.632	9.433	2.725
$b^*_{2x_1}$	1.388	2.402	7.105	12.366
$b^*_{2x_1}$	1.143	2.345	7.766	1.912
$I^*_{22yx_1}$	0.305	1.835	8.140	0.224
$I^*_{22yx_2}$	1.155	2.014	8.538	2.104
$I^*_{22x_1x_2}$	0.492	2.182	7.423	0.152

Table 3: PRE of the estimators

Estimator	Pop. 1	Pop. 2	Pop. 3	Pop. 4
$\hat{S}_y^2$	100	100	100	100
$\hat{S}_r^2$	87.167	65.093	162.742	21.544
$\hat{S}_p^2$	38.525	13.593	13.009	12.407
$\hat{S}_s^2$	116.860	248.212	5107.79	67.423
$\hat{S}_I^2$	141.940	277.598	574168.1	637.142
$\hat{S}_D^2$	142.235	293.078	605514.2	666.034
$\hat{S}_K^2$	142.235	293.078	605514.2	666.034
$T_{1_{opt}}$	142.235	293.078	605514.2	666.034

8. CONCLUSION

The present study extends the idea regarding the effective use of auxiliary information if the relationship between the study variable and the auxiliary variable is of logarithmic type. The present study provides some novel estimators that may be used when two auxiliary information is available. This study is also supported through an empirical study and the results of this study are quite encouraging both theoretically and empirically.

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