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Adomian Polynomial and Elzaki Transform Method of Solving Third Order Korteweg-De Vries Equations

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Abstract

We used Elzaki transform coupled with Adomian polynomial to obtain the exact as well as approximate analytical solutions of nonlinear third order Korteweg-de Vries (KdV) equations. Three third order KdV equations were considered to show the performance and effectiveness of this method. By comparing this method with some other known method, all the problems considered showed that the Elzaki transform method and Adomian polynomial are very powerful and effective integral transform methods in solving some nonlinear Equations.

Keywords: Elzaki transform method, Adomian polynomial, Third order KdV equations

1. INTRODUCTION

In 1985, D.J. Korteweg and G. de Vrie derived Korteweg-de Vries (KdV) equation, they proposed that KdV equation give description of the propagation of shallow water wave. Several attempts have been made by scientists to obtain the solution of KdV equation which is called soliton after its discovery. The word soliton which is a solution to a non-linear partial differential equation was used for the first time by Zabusky and Kruskal[1]. KdV equation was categorized as a typical non-linear partial differential equation that resulting to soliton solutions.

Third order Korteweg-de Vries (KdV) equation is of the form [2], [3]:

$$\phi_t + a\phi\phi_x + b\phi_{xxx} = 0 \tag{1}$$

with the initial conditions

$$\phi(x,0) = f(x) \tag{2}$$

where a and b are constants and ϕ_x and ϕ_t are partial derivatives of function ϕ with respect to space x and time t respectively, and the nonlinear term $\phi\phi_x$ tends to localize the wave, whereas the wave was spread out by dispersion. The formulation of solitons that have a single humped waves was define by delicate balance between $\phi\phi_x$ and ϕ_{xxx}. The displacement which describes how waves evolve under the competing but comparable effects of weak nonlinearity and weak dispersion is denoted by $\phi(x,t)$.

So many methods and approaches have been made to find the approximate analytic solutions and numerical solutions of KdV equations and some nonlinear differential equations such as Homotopy Perturbation Method using Elzaki Transform[4], Homotopy Perturbation method[5], exact solutions, graphical representation of Korteweg-de Vries equation[2], Numerical solutions to a linearized KdV equation on unbounded domain[6], the numerical solutions of Kdv equation using radial basis functions[7], numerical solution of separated solitary waves for KdV equation through finite element technique [8]. Moreover, the focus of some numerous research group have been on study of KdV equations and solitons[9], [10], [11], [12],[13], [14], [15].

The solutions of nonlinear KdV equations by Elzaki transform method and Adomian Polynomial was obtained in this paper. This method gives the solutions as an approximate analytical solutions in series form, most time it yield exact solutions with few iterations.

The structure of this paper is organized as follows. Section 2 contains the basic definitions and the properties of the proposed method. Section 3 shows the theoretical approach of the proposed method on KdV equation. In section 4, we apply the Elzaki transform method and Adomian polynomial to solve three problems in order to show its efficiency.

2. PROPERTIES OF ELZAKI TRANSFORM

Elzaki transform[16], [17], [18], [19], [20], [21], [22] is defined for function of exponential order. Consider the functions in the set A define below

$$A = \left\{ f(t) : \exists M, c_1, c_2 > 0, |f(t)| < Me^{\frac{|t|}{c_j}}, \text{ if } t \in (-1)^j \times [0,\infty) \right\}$$

For any given function in the set A defined above, the constant c_1, c_2 may be either finite or infinite, but M must be infinite.

According to Tarig[18], Elzaki transform is defined as:

$$E[f(t)] = u^2 \int_0^\infty f(ut)e^{-t}dt = T(u), \qquad t \geq 0,\ u \in (c_1, c_2)$$

or

$$E[f(t)] = u \int_0^\infty f(t)e^{-\frac{t}{u}}dt = T(u), \qquad t \geq 0,\ u \in (c_1, c_2) \tag{3}$$

where u in the above definition is used to factor t in the analysis of function f.

Let $T(u)$ be the Elzaki transform of $f(t)$ i.e, $E[f(t)] = T(u)$, then:

(i) $E[f'(t)] = \dfrac{T(u)}{u} - uf(0)$

(ii) $E[f''(t)] = \dfrac{T(u)}{u^2} - f(0) - uf'(0)$

(iii) $E[f^{(n)}(t)] = \dfrac{T(u)}{u^n} - \displaystyle\sum_{k=0}^{n-1} u^{2-n+k} f^{(k)}(0)$

$E[f(t)] = T(u)$ means that $T(u)$ is the Elzaki transform of $f(t)$, and $f(t)$ is the inverse Elzaki transform of $T(u)$. i.e,

$$f(t) = E^{-1}[T(u)].$$

In order to obtain the Elzaki transform of partial derivative, integration by part is used on the definition of Elzaki transform and the resulting expressions is[23]

$$E\left[\frac{\partial f(x,t)}{\partial t}\right] = \frac{T(x,v)}{v} - vf(x,0),$$
$$E\left[\frac{\partial^2 f(x,t)}{\partial t^2}\right] = \frac{T(x,v)}{v^2} - f(x,0) - v\frac{\partial f(x,0)}{\partial t},$$
$$E\left[\frac{\partial f(x,t)}{\partial x}\right] = \frac{d}{dx}[T(x,v)],$$
$$E\left[\frac{\partial^2 f(x,t)}{\partial x^2}\right] = \frac{d^2}{dx^2}[T(x,v)],$$
$$E\left[\frac{\partial^3 f(x,t)}{\partial x^3}\right] = \frac{d^3}{dx^3}[T(x,v)].$$

3. THEORETICAL APPROACH: ELZAKI TRANSFORM METHOD (ETM) ON THIRD ORDER KDV EQUATION

The main focus of this work is to solve the nonlinear partial differential equations which is third order KdV equations, we consider how to use Elzaki transform method to solve the general nonlinear partial differential equation.

According to [24], consider;

$$\frac{\partial^w \phi(x,t)}{\partial t^w} + R\phi(x,t) + N\phi(x,t) = f(x,t), \tag{4}$$

where $w = 1, 2, 3$.

And the initial conditions is given as

$$\frac{\partial^{w-1} \phi(x,t)}{\partial t^{w-1}}\Big|_{t=0} = g_{w-1}(x),$$

the partial derivative of function $\phi(x,t)$ of wth order is the one given as $\frac{\partial^w \phi(x,t)}{\partial t^w}$, R represents the linear differential operator, N indicates the nonlinear terms of differential equations, and $f(x,t)$ is the non-homogeneous/source term.

By applying the Elzaki transform on equation (4) we have;

$$E\left[\frac{\partial^w \phi(x,t)}{\partial t^w}\right] + E\left[R\phi(x,t)\right] + E\left[N\phi(x,t)\right] = E\left[f(x,t)\right]. \tag{5}$$

Note that;

$$E\left[\frac{\partial^w \phi(x,t)}{\partial t^w}\right] = \frac{E[\phi(x,t)]}{v^w} - \sum_{k=0}^{w-1} v^{2-w+k} \frac{\partial^k \phi(x,0)}{\partial t^k}. \tag{6}$$

Substituting equation (6) into equation (5) gives

$$\frac{E[\phi(x,t)]}{v^w} - \sum_{k=0}^{w-1} v^{2-w+k} \frac{\partial^k \phi(x,0)}{\partial t^k} + E\left[R\phi(x,t)\right] + E\left[N\phi(x,t)\right] = E\left[f(x,t)\right].$$

This is the same as

$$\frac{E[\phi(x,t)]}{v^w} = E\left[f(x,t)\right] + \sum_{k=0}^{w-1} v^{2-w+k} \frac{\partial^k \phi(x,0)}{\partial t^k} - \left\{E\left[R\phi(x,t)\right] + E\left[N\phi(x,t)\right]\right\}, \tag{7}$$

simplifying equation (7) gives;

$$E[\phi(x,t)] = v^w E\left[f(x,t)\right] + \sum_{k=0}^{w-1} v^{2+k} \frac{\partial^k \phi(x,0)}{\partial t^k} - v^w \left\{E\left[R\phi(x,t)\right] + E\left[N\phi(x,t)\right]\right\}. \tag{8}$$

Applying the inverse Elzaki transform to equation (8), we have

$$\phi(x,t) = E^{-1}\left[v^w E\left[f(x,t)\right] + \sum_{k=0}^{w-1} v^{2+k}\frac{\partial^k \phi(x,0)}{\partial t^k}\right] - E^{-1}\left[v^w\left\{E\left[R\phi(x,t)\right] + E\left[N\phi(x,t)\right]\right\}\right],$$

this can be written as;

$$\phi(x,t) = F(x,t) - E^{-1}\left[v^w\left\{E\left[R\phi(x,t)\right] + E\left[N\phi(x,t)\right]\right\}\right], \tag{9}$$

where $F(x,t)$ denotes the expression that arises from the given initial conditions and the source terms after simplification.

The solution will be in the form of infinite series as indicated below

$$\phi(x,t) = \sum_{n=0}^{\infty} \phi_n(x,t). \tag{10}$$

Nonlinear term can also be decomposed as:

$$N\phi(x,t) = \sum_{n=0}^{\infty} A_n, \tag{11}$$

where A_n is define as the Adomian polynomials which can be calculated by using the formula

$$A_n = \frac{1}{n!}\frac{\partial^n}{\partial \lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i \phi_i\right)\right]_{\lambda=0}, \quad n = 0, 1, \cdots$$

Substituting equation (10) and equation (11) into equation (9) gives

$$\sum_{n=0}^{\infty} \phi_n(x,t) = F(x,t) - E^{-1}\left[v^w\left\{E\left[R\sum_{n=0}^{\infty}\phi_n(x,t)\right] + E\left[\sum_{n=0}^{\infty} A_n\right]\right\}\right]. \tag{12}$$

From equation (12), let

$$\phi_0(x,t) = F(x,t) \tag{13}$$

And the recursive relation will be given as:

$$\phi_{n+1} = -E^{-1}\left[v^w\left\{E\left[R\phi_n(x,t)\right] + E\left[A_n\right]\right\}\right].$$

Here $w = 1, 2, 3$ and $n \geq 0$.

The analytical solution $\phi(x,t)$ can be approximated by truncated series

$$\phi(x,t) = \lim_{N\to\infty}\sum_{n=0}^{N} \phi_n(x,t).$$

4. APPLICATIONS

The effectiveness of the Elzaki transform and Adomian polynomial are demonstarted by solving the following third order Korteweg-De Vries (KdV) Equations.

Example 4.1: Consider the homogeneous KdV equation[5]

$$\phi_t + 6\phi\phi_x + \phi_{xxx} = 0, \tag{14}$$

with initial condition

$$\phi(x,0) = x.$$

Applying the Elzaki transform to equation (14) gives,

$$E[\phi_t] = -E\left[6\phi\phi_x + \phi_{xxx}\right]. \tag{15}$$

Applying the Elzaki transform property and the initial condition given, equation (15) becomes

$$\Phi(x,v) = v^2 x - vE\left[6\phi\phi_x + \phi_{xxx}\right]. \tag{16}$$

By applying the inverse Elzaki transform to equation (16), we have;

$$\phi(x,t) = x - E^{-1}\left\{vE\left[6\phi\phi_x + \phi_{xxx}\right]\right\}. \tag{17}$$

From equation (17), let

$$\phi_0 = x.$$

The recursive relation is given as:

$$\phi_{n+1} = -E^{-1}\left\{vE\left[6A_n + \frac{\partial^3\phi_n}{\partial x^3}\right]\right\}. \tag{18}$$

where A_n is the Adomian polynomial to decompose the nonlinear terms by using the relation:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n} f\left[\sum_{i=0}^{\infty}\lambda^i\phi_i\right]_{\lambda=0}. \tag{19}$$

Let the nonlinear term be represented by

$$f(\phi) = \phi\frac{\partial\phi}{\partial x}. \tag{20}$$

By using equation (20) in equation (19), we obtain;

$$A_0 = \phi_0 \frac{\partial \phi_0}{\partial x},$$
$$A_1 = \phi_1 \frac{\partial \phi_0}{\partial x} + \phi_0 \frac{\partial \phi_1}{\partial x},$$
$$A_2 = \phi_2 \frac{\partial \phi_0}{\partial x} + \phi_1 \frac{\partial \phi_1}{\partial x} + \phi_0 \frac{\partial \phi_2}{\partial x}, \quad \cdots$$

From equation (18), when n=0,

$$\phi_1 = -E^{-1}\left\{vE\left[6A_0 + \frac{\partial^3 \phi_0}{\partial x^3}\right]\right\}.$$

A_0 is computed as:

$$A_0 = x$$

$$\phi_1 = -E^{-1}\left\{vE\left[6x + \frac{\partial^3}{\partial x^3}[x]\right]\right\}. \tag{21}$$

By simplifying equation (21) we obtain;

$$\phi_1 = -6xt. \tag{22}$$

When $n = 1$, we have;

$$\phi_2 = -E^{-1}\left\{vE\left[6A_1 + \frac{\partial^3 \phi_1}{\partial x^3}\right]\right\}.$$

A_1 is computed as:

$$A_1 = -12xt$$

$$\phi_2 = -E^{-1}\left\{vE\left[-72xt + \frac{\partial^3}{\partial x^3}[-6xt]\right]\right\}. \tag{23}$$

Simplifying equation (23) gives;

$$\phi_2 = 36xt^2.$$

When $n = 2$,

$$\phi_3 = -E^{-1}\left\{vE\left[6A_2 + \frac{\partial^3 \phi_2}{\partial x^3}\right]\right\}.$$

A_2 is computed as:

$$A_2 = 108xt^2$$

$$\phi_3 = -E^{-1}\left\{vE\left[648xt^2 + \frac{\partial^3}{\partial x^3}\left[36xt^2\right]\right]\right\}. \tag{24}$$

By simplifying equation (24) we obtain;

$$\phi_3 = -216xt^3.$$

The approximate series solution is

$$\phi(x,t) = \phi_0 + \phi_1 + \phi_2 + \cdots$$

$$\phi(x,t) = x - 6xt + 36xt^2 - 216xt^3 + \cdots$$

This can be written as

$$\phi(x,t) = x\left[1 - 6t + (6t)^2 - (6t)^3 + \cdots\right]$$

Using Taylor's series, the closed form solution is

$$\phi(x,t) = \frac{x}{1+6t}$$

This closed form solution for equation (14) agree with the one obtained by Homotopy perturbation transform method[5].

Figure 1 shows the 3D graph of the solution of equation (14).

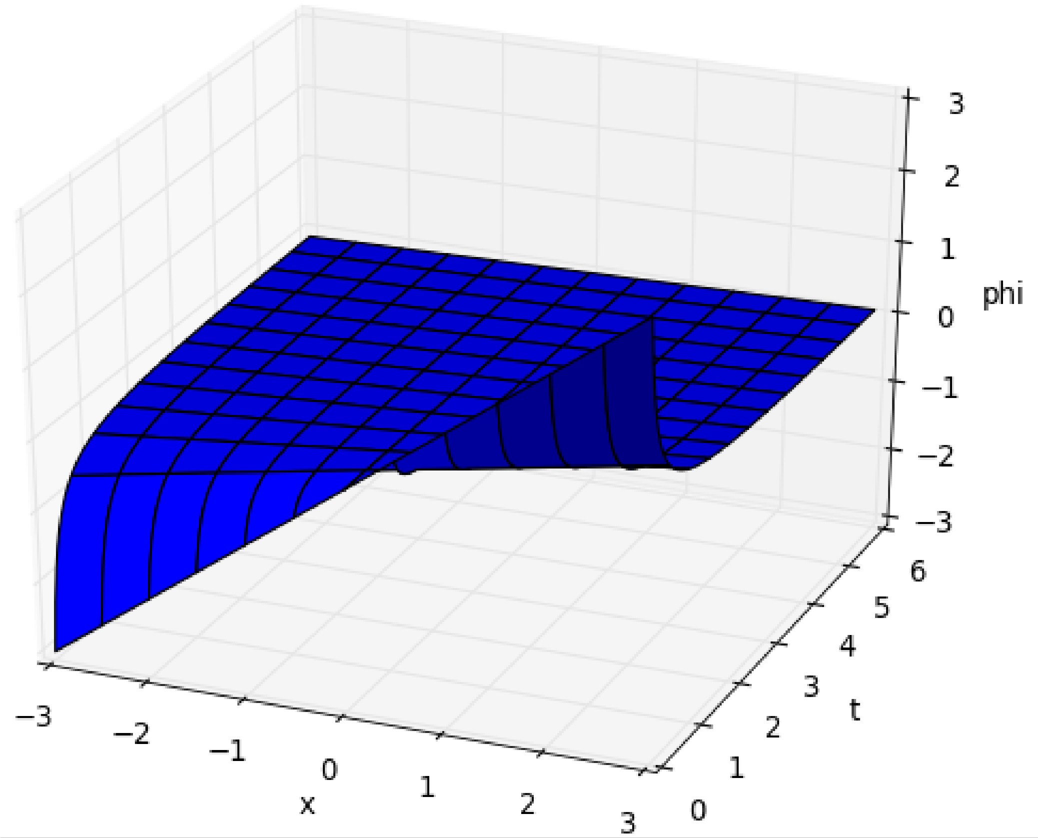

Figure 1: The solution of the first third order KdV equation by ETM in Equation (14)

Example 4.2: Consider the homogeneous KdV equation[5]

$$\phi_t - 6\phi\phi_x + \phi_{xxx} = 0, \tag{25}$$

with initial condition

$$\phi(x, 0) = 6x.$$

Applying Elzaki transform to both sides of equation (25) gives;

$$E[\phi_t] = E\left[6\phi\phi_x - \phi_{xxx}\right]. \tag{26}$$

Recall that

$$E\left[\phi_t\right] = \frac{\Phi(x, v)}{v} - v\phi(x, 0),$$

Equation (26) becomes;

$$\frac{\Phi(x,v)}{v} - v\phi(x,0) = E\left[6\phi\phi_x - \phi_{xxx}\right]. \tag{27}$$

Applying the given initial conditions on equation (27) and simplifying, we obtain;

$$\Phi(x,v) = 6v^2x + vE\left[6\phi\phi_x - \phi_{xxx}\right]. \tag{28}$$

Applying the inverse Elzaki transform to equation (28), gives;

$$\phi(x,t) = 6x + E^{-1}\left\{vE\left[6\phi\phi_x - \phi_{xxx}\right]\right\}. \tag{29}$$

From equation (29), let

$$\phi_0 = 6x.$$

The recursive relation is given as:

$$\phi_{n+1} = E^{-1}\left\{vE\left[6A_n - \frac{\partial^3\phi_n}{\partial x^3}\right]\right\}, \tag{30}$$

where A_n is the Adomian polynomial to decompose the nonlinear terms by using the relation:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n} f\left[\sum_{i=0}^{\infty}\lambda^i\phi_i\right]_{\lambda=0}. \tag{31}$$

Let the nonlinear term be represented by

$$f(\phi) = \phi\frac{\partial\phi}{\partial x}. \tag{32}$$

By using equation (32) in equation (31), we obtain;

$$\begin{aligned}
A_0 &= \phi_0\frac{\partial\phi_0}{\partial x}, \\
A_1 &= \phi_1\frac{\partial\phi_0}{\partial x} + \phi_0\frac{\partial\phi_1}{\partial x}, \\
A_2 &= \phi_2\frac{\partial\phi_0}{\partial x} + \phi_1\frac{\partial\phi_1}{\partial x} + \phi_0\frac{\partial\phi_2}{\partial x}, \quad \cdots
\end{aligned}$$

From equation (30), when n=0,

$$\phi_1 = E^{-1}\left\{vE\left[6A_0 - \frac{\partial^3\phi_0}{\partial x^3}\right]\right\}.$$

A_0 is computed as:

$$A_0 = 36x.$$

$$\phi_1 = E^{-1}\left\{vE\left[216x - \frac{\partial^3}{\partial x^3}[6x]\right]\right\}. \tag{33}$$

Simplifying equation (33) gives;

$$\phi_1 = 216xt. \tag{34}$$

When $n = 1$, we have;

$$\phi_2 = E^{-1}\left\{vE\left[6A_1 - \frac{\partial^3 \phi_1}{\partial x^3}\right]\right\}.$$

A_1 is computed as:

$$A_1 = 2592xt.$$

$$\phi_2 = E^{-1}\left\{vE\left[15552xt - \frac{\partial^3}{\partial x^3}[216xt]\right]\right\}. \tag{35}$$

By simplifying equation (35) we obtain;

$$\phi_2 = 7776xt^2.$$

When $n = 2$,

$$\phi_3 = E^{-1}\left\{vE\left[6A_2 - \frac{\partial^3 \phi_2}{\partial x^3}\right]\right\}.$$

A_2 is computed as:

$$A_2 = 139968xt^2.$$

$$\phi_3 = E^{-1}\left\{vE\left[839808xt^2 - \frac{\partial^3}{\partial x^3}\left[7776xt^2\right]\right]\right\}. \tag{36}$$

Simplifying equation (36) yields;

$$\phi_3 = 279936xt^3.$$

The approximate series solution is

$$\phi(x,t) = \phi_0 + \phi_1 + \phi_2 + \cdots$$

$$\phi(x,t) = 6x + 216xt + 7776xt^2 + 279936xt^3 + \cdots$$

This can be written as

$$\phi(x,t) = 6x\left[1 + 36t + (36t)^2 + (36t)^3 + \cdots\right]$$

Using Taylor's series, the closed form solution is

$$\phi(x,t) = \frac{6x}{1 - 36t}$$

This closed form solution for equation (25) agree with the one obtained by Homotopy perturbation transform method[5].

Figure 2 shows the 3D graph of the solution of equation (25).

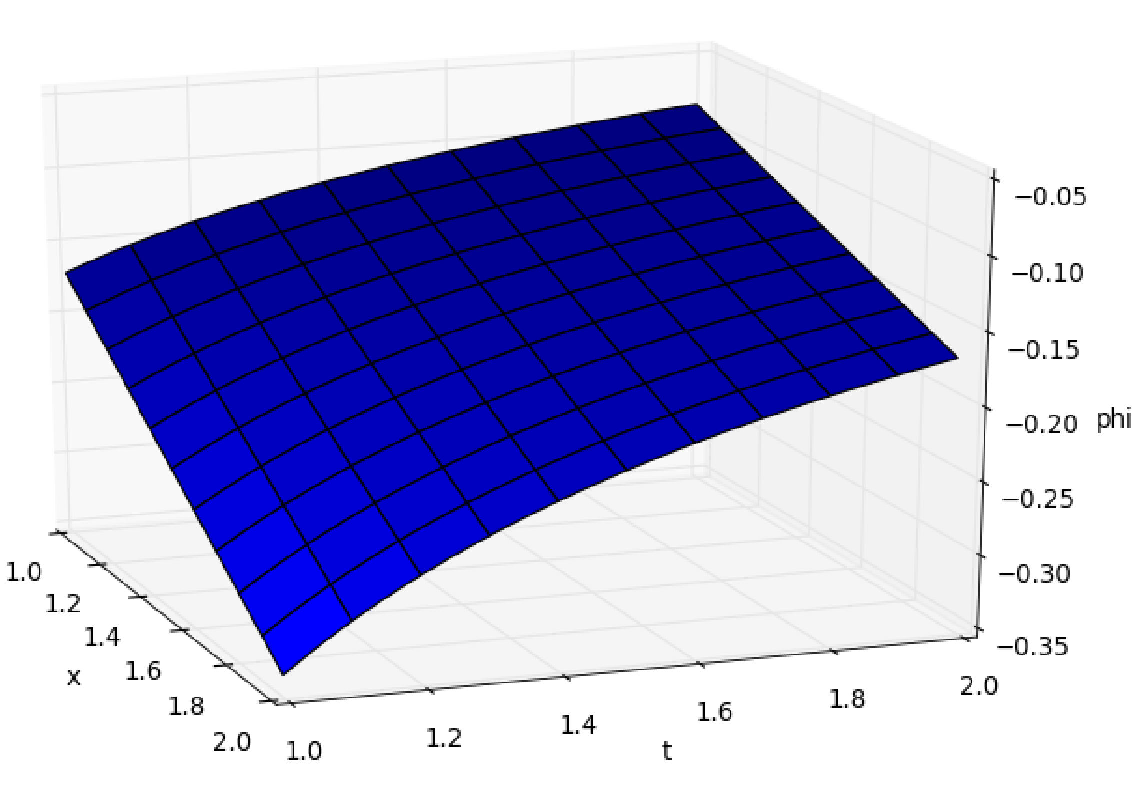

Figure 2: The solution of the second third order KdV equation by ETM in Equation (25)

Example 4.3: Consider the homogeneous KdV equation[5]

$$\phi_t - 6\phi\phi_x + \phi_{xxx} = 0, \tag{37}$$

with initial condition

$$\phi(x,0) = -\frac{2k^2e^{kx}}{(1+e^{kx})^2}.$$

Equation (37) can be written as:

$$\phi_t = [6\phi\phi_x - \phi_{xxx}].$$

Applying Elzaki transform into both sides

$$E[\phi_t] = E[6\phi\phi_x - \phi_{xxx}]. \tag{38}$$

Since

$$E[\phi_t] = \frac{\Phi(x,v)}{v} - v\phi(x,0),$$

equation (38) becomes;

$$\frac{\Phi(x,v)}{v} - v\phi(x,0) = E\left[6\phi\phi_x - \phi_{xxx}\right]. \tag{39}$$

Applying the given initial conditions on equation (39) and simplifying, we obtain;

$$\Phi(x,v) = -v^2\frac{2k^2e^{kx}}{(1+e^{kx})^2} + vE\left[6\phi\phi_x - \phi_{xxx}\right]. \tag{40}$$

Applying the inverse Elzaki transform into equation (40) gives;

$$\phi(x,t) = E^{-1}\left\{-v^2\frac{2k^2e^{kx}}{(1+e^{kx})^2}\right\} + E^{-1}\left\{vE\left[6\phi\phi_x - \phi_{xxx}\right]\right\}.$$

The resulting expression is

$$\phi(x,t) = -\frac{2k^2e^{kx}}{(1+e^{kx})^2} + E^{-1}\left\{vE\left[6\phi\phi_x - \phi_{xxx}\right]\right\}. \tag{41}$$

From equation (41), let

$$\phi_0 = -\frac{2k^2e^{kx}}{(1+e^{kx})^2}.$$

The recursive relation is given as:

$$\phi_{n+1} = E^{-1}\left\{vE\left[6A_n - \frac{\partial^3\phi_n}{\partial x^3}\right]\right\}, \tag{42}$$

where A_n is the Adomian polynomial to decompose the nonlinear terms by using the relation:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}f\left[\sum_{i=0}^{\infty}\lambda^i\phi_i\right]_{\lambda=0}. \tag{43}$$

Let the nonlinear term be represented by

$$f(\phi) = \phi\frac{\partial\phi}{\partial x}. \tag{44}$$

By using equation (44) in equation (43), we obtain;

$$\begin{aligned} A_0 &= \phi_0\frac{\partial\phi_0}{\partial x}, \\ A_1 &= \phi_1\frac{\partial\phi_0}{\partial x} + \phi_0\frac{\partial\phi_1}{\partial x}, \\ A_2 &= \phi_2\frac{\partial\phi_0}{\partial x} + \phi_1\frac{\partial\phi_1}{\partial x} + \phi_0\frac{\partial\phi_2}{\partial x}, \quad \cdots \end{aligned}$$

From equation (42), when n=0,

$$\phi_1 = E^{-1}\left\{vE\left[6A_0 - \frac{\partial^3\phi_0}{\partial x^3}\right]\right\}.$$

A_0 is computed as:

$$A_0 = -\frac{4k^5e^{2kx}(e^{kx}-1)}{(e^{kx}+1)^5}.$$

Then,

$$\phi_1 = E^{-1}\left\{vE\left[-\frac{24k^5e^{2kx}(e^{kx}-1)}{(e^{kx}+1)^5} - \frac{\partial^3}{\partial x^3}\left[-\frac{2k^2e^{kx}}{(1+e^{kx})^2}\right]\right]\right\}. \quad (45)$$

Simplifying equation (45) gives;

$$\phi_1 = -\frac{2k^5e^{kx}(e^{kx}-1)}{(e^{kx}+1)^3}t. \quad (46)$$

When $n = 1$, we have;

$$\phi_2 = E^{-1}\left\{vE\left[6A_1 - \frac{\partial^3\phi_1}{\partial x^3}\right]\right\}.$$

A_1 is computed as:

$$A_1 = -\frac{8k^8e^{2kx}(e^{2kx}-3e^{kx}+1)}{(e^{kx}+1)^6}t.$$

So,

$$\phi_2 = E^{-1}\left\{vE\left[-\frac{48k^8e^{2kx}(e^{2kx}-3e^{kx}+1)}{(e^{kx}+1)^6}t - \frac{\partial^3}{\partial x^3}\left[-\frac{2k^5e^{kx}(e^{kx}-1)}{(e^{kx}+1)^3}t\right]\right]\right\}. \quad (47)$$

Simplifying equation (47) gives;

$$\phi_2 = -\frac{k^8e^{kx}(e^{2kx}-4e^{kx}+1)}{(e^{kx}+1)^4}t^2.$$

The approximate series solution is

$$\phi(x,t) = \phi_0 + \phi_1 + \phi_2 + \cdots$$

$$\phi(x,t) = -\frac{2k^2e^{kx}}{(e^{kx}+1)^2} - \frac{2k^5e^{kx}(e^{kx}-1)}{(e^{kx}+1)^3}t - \frac{k^8e^{kx}(e^{2kx}-4e^{kx}+1)}{(e^{kx}+1)^4}t^2 + \cdots$$

Using Taylor's series, the closed form solution is

$$\phi(x,t) = -\frac{2k^2e^{k(x-k^2t)}}{(e^{k(x-k^2t)}+1)^2} \quad (48)$$

This closed form solution for equation (37) agree with the one obtained by Homotopy perturbation transform method[5].

Figure 3 shows the 3D graph of the solution of equation (37).

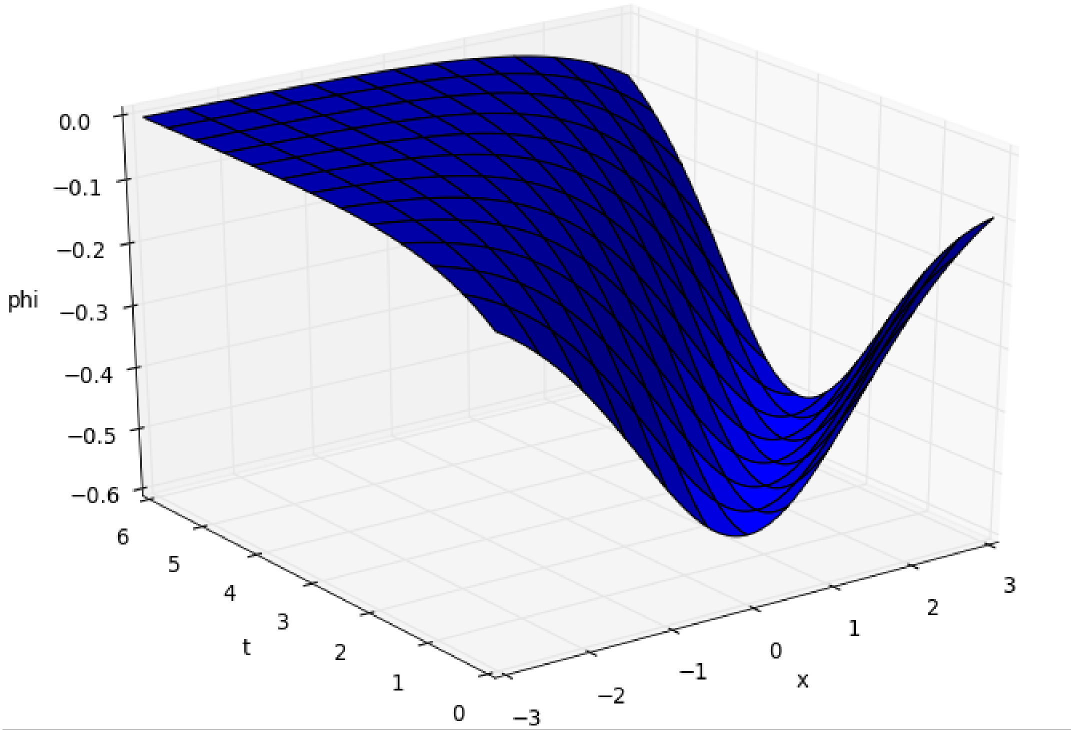

Figure 3: The solution of the third third order KdV equation by ETM in Equation (37)

5. CONCLUSION

We applied combination of Elzaki transform and Adomian polynomial in this paper to solve third order Korteweg-De Vries (KdV) equations approximately. The problems considered showed that this method is a very powerful integral transform method in solving third order KdV equations because it give highly accurate solutions for nonlinear equations compare with some other methods. Using this method make us to realise how potent it is because the solutions of all the three problems obtained in series form converges to exact solutions with few iterations. All these solutions also agree with the solutions obtained when Homotopy perturbation transform method is used as in the reference. In addition, three dimensional graph were plotted to show the behaviour of the solutions. Solving some other nonlinear differential equations (whether partial or ordinary differential equations) is very easy by using this powerful method.

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