Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 15, Number 3 (2019), pp. 241-250

http://www.ripublication.com/gjpam.htm

Common Fixed Point Theorems on Complete Metric Space for Two self Maps Using Generalized Altering Distance Functions in Five Variables and Deficit functions

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Abstract

The concept of existence and uniqueness of fixed points by altering distance between points have been explored by many authors. In this paper, we obtain a common fixed point result for two and four self mappings by altering distances sub compatible functions in five variables and deficit functions with generalization of contractive type condition.

2010 MSC: 47H10, 54E35

Keywords: Altering distance functions, Sub compatible, Generalized altering distance.

1. INTRODUCTION:

Fixed point theory plays an important role in functional analysis. This is a very extensive and wider field. The concept of a metric space was introduced by **M. Ferchet [11]**. Fixed point theory beginning from Banach contraction principle of **Banach [1]** (1922) with complete metric spaces as a background and went back to Brouwer fixed point theorem of **Brouwer [6,7]** (1910) with $\mathbb{R}^n$ as background. It has wider applications in differential and integral equations in mathematical science and engineering. Many authors have extended, generalized and improved Banach's fixed point theorem in different ways.

The study of the existence and uniqueness of common fixed point of mappings satisfying contractive type condition has been a very active field of research. Obtaining fixed point theorems for self-maps of a metric space by altering distances between the points with the use of certain continuous control functions is an interesting aspect.

The fixed point theorems related to altering distances between points in complete metric spaces have been obtained initially by **D. Delbosco [10] and F. Skof [20]** in 1977. **M. S. Khan et al. [12]** initiated the idea of obtaining fixed point of self maps of a metric space by altering distance between the points with the use of a certain continuous control function.

K. P. R. Sastry and G. V. R. Babu [17] discussed and established the existence of fixed points for the orbits of single self-maps and pairs of self-maps by using a control function.

K. P. R. Sastry et al. [18] proved fixed point theorems in complete metric spaces by using a continuous control function. **B. S. Choudhury et al. [8, 9]**, **G. V. R. Babu et al. [2, 3, 4, 5], S. V. R. Naidu [13, 14], K. P. R. Rao et al. [15, 16]** proved some common fixed point results by altering distances.

The main aim of this paper is to prove the existence and uniqueness of common fixed points on complete metric space for two and four self mappings using generalized altering distance functions in five variables and deficit functions. The results are improvements of results of **K.Sridevi et al. [21]**.

2. PRELIMINARIES:

2.1: Definition [12]: A function $\psi : [0,\infty) \longrightarrow [0,\infty)$ is called an altering distance function, if it satisfies following conditions.

(1) $\psi(t)$ is monotonically increasing and continuous.
(2) $\psi(t) = 0 \quad iff \quad t = 0$.

2.2: Definition [12]: A function $\psi : R^{+^n} \longrightarrow R^+ = [0,\infty)$ is called a generalized altering distance function on R^{+^n} if ψ is continuous, monotone increasing in each variable and $\psi(x_1, x_2, \dots, x_n) = 0 \quad if\ and\ only\ if \quad x_1 = x_2 = \cdots = x_n = 0$.

The collection of all generalized altering distances in n variables is denoted by Ψ_n. Suppose $\psi \in \Psi_n$.

Now we define a function $\varphi_\psi(y)$ by $\varphi_\psi(y) = \psi(y, y, \dots, y)$ for $y \in [0,\infty)$,Clearly $\varphi_\psi(y) = 0 \ \ if\ and\ only\ if\ y = 0$.

2.3: Definition [12]: Two maps $p, q : X \longrightarrow X$ of a metric space (X, d) are called sub compatible if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} p(x_n) = \lim_{n\to\infty} q(x_n) = t, t \in X \ \Rightarrow \ \lim_{n\to\infty} d(pqx_n, qpx_n) = 0.$$

K.Sridevi et al. [21] proved the following Theorem.

2.4: ([21], Theorem 3.1) : Let (X,d) be a complete metric space and $\psi_1,\psi_2 \in \Psi_5$. Suppose $U,\ V : X \longrightarrow X$ are such that for all $x,y \in X$.

$$\varphi_1\big(d(Ux,Vy)\big) \leq \psi_1\begin{pmatrix} d(x,y), d(Ux,x), d(Vy,y), \frac{1}{2}\{d(Vy,x)+d(Ux,y)\}, \\ \frac{1}{2}\{d(x,y)+\max\{d(x,Ux),d(y,Vy)\}\} \end{pmatrix}$$

$$-\psi_2\begin{pmatrix} d(x,y), d(Ux,x), d(Vy,y), \frac{1}{2}\{d(Vy,x)+d(Ux,y)\}, \\ \frac{1}{2}\{d(x,y)+\max\{d(x,Ux),d(y,Vy)\}\} \end{pmatrix}$$

-------- (2.4.1)

where $\varphi_1(\alpha) = \psi_1(\alpha,\alpha,\alpha,\alpha,\alpha),\ \ \alpha \in [0,\infty)$.

Then $U\ \ and\ \ V$ have a unique common fixed point in X.

In this paper we improve the results of K. Sridevi et al.[21] and using the notion of deficit functions.

3. MAIN RESULT:

Now we state and prove our first main result. Before that we first introduce the notion of a deficit function.

3.1: Definition: A function $\psi : R^{+^n} \longrightarrow R^+ = [0,\infty)$ is called Generalized altering distance function on R^{+^n}, if ψ is continuous monotonically increasing in each variable

$$\psi(x_1,x_2,\dots,x_n) = 0 \quad if\ and\ only\ if \quad x_1 = x_2 = \cdots = x_n = 0$$

3.2: Definition: A function $\psi : R^{+^n} \longrightarrow R^+ = [0,\infty)$ is called a deficit function if ψ is increasing in the first variable.

3.2 Theorem: Let (X,d) be a complete metric space and $\psi_1,\psi_2 \in \Psi_5$. Suppose U, $V : X \longrightarrow X$ are such that for all $x,y \in X$ and for some $\varepsilon > 0$,

$$\varphi_1\big(d(Ux,Vy)\big) \leq \psi_1\begin{pmatrix} d(x,y), d(Ux,x), d(Vy,y), \frac{1}{2}\{d(Vy,x)+d(Ux,y)\}, \\ \frac{1}{2}\{d(x,y)+\max\{d(x,Ux),d(y,Vy)\}\} \end{pmatrix}$$

$$-\psi_2\big((d(x,y)+\varepsilon, d(x,y), d(x,y), d(x,y), d(x,y)\big) - -(3.2.1)$$

where $\varphi_1(\alpha) = \psi_1(\alpha,\alpha,\alpha,\alpha,\alpha),\ \ \alpha \in [0,\infty)$.

Then U *and* V have a unique common fixed point in X.

Proof: Suppose $x_0 \in X.$

$x_1 = Ux_0 \quad , \quad x_2 = Vx_1$

Inductively, $x_{2n+1} = Ux_{2n} \quad , x_{2n+2} = Vx_{2n+1}$

Let $x = x_{2n+2}$ *and* $y = x_{2n+1}$

Substituting for x *and* y *in* (3.2.1), we get,

$$\varphi_1\big(d(Ux_{2n+2}, Vx_{2n+1})\big) \leq \psi_1\begin{pmatrix} d(x_{2n+2}, x_{2n+1}), d(Ux_{2n+2}, x_{2n+2}), d(Vx_{2n+1}, x_{2n+1}), \frac{1}{2}\{d(Vx_{2n+1}, x_{2n+2}) + d(Ux_{2n+2}, x_{2n+1})\}, \\ \frac{1}{2}\{d(x_{2n+2}, x_{2n+1}) + \max\{d(x_{2n+2}, Ux_{2n+2}), d(x_{2n+1}, Vx_{2n+1})\}\} \end{pmatrix}$$
$$-\psi_2\big((d(x_{2n+2}, x_{2n+1}) + \varepsilon, d(x_{2n+2}, x_{2n+1}), d(x_{2n+2}, x_{2n+1}), d(x_{2n+2}, x_{2n+1}), d(x_{2n+2}, x_{2n+1})\big)$$

$$\therefore \varphi_1\big(d(x_{2n+3}, x_{2n+2})\big) \leq \psi_1\begin{pmatrix} d(x_{2n+2}, x_{2n+1}), d(x_{2n+3}, x_{2n+2}), d(x_{2n+2}, x_{2n+1}), \frac{1}{2}\{d(x_{2n+2}, x_{2n+2}) + d(x_{2n+3}, x_{2n+1})\}, \\ \frac{1}{2}[d(x_{2n+2}, x_{2n+1}) + \max\{d(x_{2n+2}, x_{2n+3}), d(x_{2n+1}, x_{2n+2})\}] \end{pmatrix}$$
$$-\psi_2\big((d(x_{2n+2}, x_{2n+1}) + \varepsilon, d(x_{2n+2}, x_{2n+1}), d(x_{2n+2}, x_{2n+1}), d(x_{2n+2}, x_{2n+1}), d(x_{2n+2}, x_{2n+1})\big)$$

Write $d(x_n, x_{n+1}) = a_n$

From the above we get,

$$\varphi_1(a_{2n+2}) \leq \psi_1\begin{pmatrix} a_{2n+1}, a_{2n+2}, a_{2n+1}, \frac{1}{2}\{d(x_{2n+3}, x_{2n+1})\}, \\ \frac{1}{2}\{a_{2n+1} + \max\{a_{2n+2}, a_{2n+1}\}\} \end{pmatrix} - \psi_2(a_{2n+1} + \varepsilon, a_{2n+1}, a_{2n+1}, a_{2n+1}, a_{2n+1})$$
$$\leq \psi_1(\alpha, \alpha, \alpha, \alpha, \alpha) - \psi_2(a_{2n+1} + \varepsilon, a_{2n+1}, a_{2n+1}, a_{2n+1}, a_{2n+1})$$
$$where \quad \alpha = \max\{a_{2n+2}, a_{2n+1}\}$$
$$= \varphi_1(\alpha) - \psi_2(a_{2n+1} + \varepsilon, a_{2n+1}, a_{2n+1}, a_{2n+1}, a_{2n+1})$$

Therefore, $\varphi_1(a_{2n+2}) < \varphi_1(\alpha)$.

$(since\ \psi_2(a_{2n+1} + \varepsilon, a_{2n+1}, a_{2n+1}, a_{2n+1}, a_{2n+1}) > 0\ )$

$\therefore\ a_{2n+2} < \alpha \quad \Longrightarrow \quad a_{2n+2} < a_{2n+1}$

Now let $x = x_{2n}$ *and* $y = x_{2n+1}$, in (3.2.1). Then we get

$$\varphi_1(a_{2n+1}) \leq \psi_1\begin{pmatrix} a_{2n}, a_{2n}, a_{2n+1}, \frac{1}{2}\{d(x_{2n}, x_{2n+2})\}, \\ \frac{1}{2}\{a_{2n} + \max\{a_{2n+1}, a_{2n}\}\} \end{pmatrix} - \psi_2(a_{2n} + \varepsilon, a_{2n}, a_{2n}, a_{2n}, a_{2n})$$

$$\varphi_1(a_{2n+1}) \;\leq\; \psi_1\begin{pmatrix} a_{2n}, a_{2n}, a_{2n+1}, \frac{1}{2}\{a_{2n} + a_{2n+1}\}, \\ \frac{1}{2}\{a_{2n} + \max\{a_{2n+1}, a_{2n}\}\} \end{pmatrix} - \psi_2(a_{2n} + \varepsilon, a_{2n}, a_{2n}, a_{2n}, a_{2n})$$

$$\leq \psi_1(\beta_{2n}, \beta_{2n}, \beta_{2n}, \beta_{2n}, \beta_{2n}) - \psi_2(a_{2n} + \varepsilon, a_{2n}, a_{2n}, a_{2n}, a_{2n}) \quad \text{- - - - - - (A)}$$

$where \;\; \beta_{2n} = \max\{a_{2n}, a_{2n+1}\}$

$$< \varphi_1(\beta_{2n})$$

$\therefore \varphi_1(a_{2n+1}) < \varphi_1(\beta_{2n})$ $\quad$ ($since\ \psi_2(a_{2n} + \varepsilon, a_{2n}, a_{2n}, a_{2n}, a_{2n}) > 0$) $\Rightarrow$ $a_{2n+1} < \beta_{2n} \;\Rightarrow\; a_{2n+1} < a_{2n}$

Similarly, we can show that

$$a_{2n+2} < a_{2n+1}$$

Hence, $\quad a_{n+1} < a_n$ for $n = 1,2, \dots \dots$

so that $\{a_n\}$ strictly decreasing , say, to δ

also $\{\varphi_1(a_n)\}$ strictly decreases, say, to γ

On letting $n \longrightarrow \infty$ in (A), we get

$$\varphi_1(\delta) \leq \psi_1(\delta, \delta, \delta, \delta, \delta) - \psi_2(\delta + \varepsilon, \delta, \delta, \delta, \delta)$$

Therefore, $\quad \varphi_1(\delta) < \varphi_1(\delta) \;\; if \;\; \delta > 0$

a contradiction.

Therefore, $\delta = 0$.

But $\gamma = \lim_{n\longrightarrow\infty} \varphi_1(a_n) = \varphi_1(\delta) \;\Rightarrow\; \gamma = 0$

Therefore, $a_n = d(x_n, x_{n+1}) \longrightarrow 0$.

Now, we show that the sequence $\{x_n\}$ is a Cauchy sequence in X.

For this we first show that $\{x_{2n}\}$ is a Cauchy sequence.

If $\{x_{2n}\}$ is not a Cauchy sequence then there exist $\varepsilon > 0$ and natural numbers $\{2m(k),\ 2n(k)\}$ such that $k < m(k) < n(k)$,

$$d(x_{2m(k)}, x_{2n(k)}) \geq \varepsilon \quad and \quad d(x_{2m(k)}, x_{2n(k)-2}) < \varepsilon$$

$$\varepsilon \leq d(x_{2m(k)}, x_{2n(k)})$$

$$\leq d(x_{2m(k)}, x_{2n(k)-2}) + d(x_{2n(k)-2}, x_{2n(k)})$$

$$\leq \varepsilon + d(x_{2n(k)}, x_{2n(k)-1}) + d(x_{2n(k)-1}, x_{2n(k)-2})$$

Taking $k \longrightarrow \infty$ in the inequality, it follows that

$$\lim_{k\longrightarrow\infty} d(x_{2m(k)}, x_{2n(k)}) = \varepsilon$$

Consider, $d(x_{2n(k)+1}, x_{2m(k)}) \leq d(x_{2n(k)+1}, x_{2n(k)}) + d(x_{2n(k)}, x_{2m(k)})$

$d(x_{2n(k)}, x_{2m(k)}) \leq d(x_{2n(k)}, x_{2n(k)+1}) + d(x_{2n(k)+1}, x_{2m(k)})$

On letting $k \longrightarrow \infty$ we get,

$$\limsup_{k \longrightarrow \infty} d(x_{2n(k)+1}, x_{2m(k)}) \leq \varepsilon \quad and$$

$$\liminf_{k \longrightarrow \infty} d(x_{2n(k)+1}, x_{2m(k)}) \geq \varepsilon \quad \Rightarrow \quad \lim_{k \longrightarrow \infty} d(x_{2n(k)+1}, x_{2m(k)}) = \varepsilon.$$

Consider, $d(x_{2n(k)}, x_{2m(k)-1}) \leq d(x_{2n(k)}, x_{2m(k)}) + d(x_{2m(k)}, x_{2m(k)-1})$

$d(x_{2n(k)}, x_{2m(k)}) \leq d(x_{2n(k)}, x_{2m(k)-1}) + d(x_{2m(k)-1}, x_{2m(k)})$

On letting $k \longrightarrow \infty$ we get,

$$\limsup_{k \longrightarrow \infty} d(x_{2n(k)}, x_{2m(k)-1}) \leq \varepsilon \quad and$$

$$\liminf_{k \longrightarrow \infty} d(x_{2n(k)}, x_{2m(k)-1}) \geq \varepsilon$$

$$\Rightarrow \quad \lim_{k \longrightarrow \infty} d(x_{2n(k)}, x_{2m(k)-1}) = \varepsilon.$$

Consider, $d(x_{2n(k)+1}, x_{2m(k)-1}) \leq d(x_{2n(k)+1}, x_{2m(k)}) + d(x_{2m(k)}, x_{2m(k)-1})$

$d(x_{2n(k)+1}, x_{2m(k)}) \leq d(x_{2n(k)+1}, x_{2m(k)-1}) + d(x_{2m(k)-1}, x_{2m(k)})$

On letting $k \longrightarrow \infty$ we get,

$$\limsup_{k \longrightarrow \infty} d(x_{2n(k)+1}, x_{2m(k)-1}) \leq \varepsilon \quad and \quad \liminf_{k \longrightarrow \infty} d(x_{2n(k)+1}, x_{2m(k)-1}) \geq \varepsilon$$

$$\Rightarrow \quad \lim_{k \longrightarrow \infty} d(x_{2n(k)+1}, x_{2m(k)-1}) = \varepsilon.$$

Substituting $x = x_{2n(k)}$ *and* $y = x_{2m(k)-1}$ *in* (3.2.1), we get

$$\varphi_1\left(d(x_{2n(k)+1}, x_{2m(k)})\right)$$

$$\leq \psi_1 \begin{pmatrix} d(x_{2n(k)}, x_{2m(k)-1}), d(x_{2n(k)+1}, x_{2n(k)}), d(x_{2m(k)}, x_{2m(k)-1}), \\ \frac{1}{2}\{d(x_{2m(k)}, x_{2n(k)}) + d(x_{2n(k)+1}, x_{2m(k)-1})\}, \\ \frac{1}{2}[d(x_{2n(k)}, x_{2m(k)-1}) + \max\{d(x_{2n(k)}, x_{2n(k)+1}), d(x_{2m(k)-1}, x_{2m(k)})\}] \end{pmatrix}$$

$$-\psi_2 \begin{pmatrix} d(x_{2n(k)}, x_{2m(k)-1}) + \varepsilon, d(x_{2n(k)}, x_{2m(k)-1}), d(x_{2n(k)}, x_{2m(k)-1}), d(x_{2n(k)}, x_{2m(k)-1}), \\ d(x_{2n(k)}, x_{2m(k)-1}), \end{pmatrix}$$

On letting $k \longrightarrow \infty$ we get,

$$\varphi_1(\varepsilon) \leq \psi_1\left(\varepsilon, 0, 0, \varepsilon, \frac{\varepsilon}{2}\right) - \psi_2(2\varepsilon, \varepsilon, \varepsilon, \varepsilon, \varepsilon)$$

Therefore, $\varphi_1(\varepsilon) \leq \varphi_1(\varepsilon) - \psi_2(2\varepsilon, \varepsilon, \varepsilon, \varepsilon, \varepsilon)$

Therefore, $\varphi_1(\varepsilon) < \varphi_1(\varepsilon)$ (since $\varepsilon > 0$ and hence $\psi_2(2\varepsilon, \varepsilon, \varepsilon, \varepsilon, \varepsilon) \neq 0$) a contradiction.

Therefore, $\{x_{2n}\}$ is a Cauchy sequence.

Similarly, we can show that $\{x_{2n+1}\}$ is a Cauchy sequence.

Since $a_n = d(x_n, x_{n+1}) \longrightarrow 0$ as $n \longrightarrow \infty$, now follows that

$\therefore \{x_n\}$ is a Cauchy sequence.

Suppose $\{x_n\} \longrightarrow l$

Substituting, $x = x_{2n}$ *and* $y = l$ *in* (3.2.1), we get

$$\varphi_1\big(d(Ux_{2n}, Vl)\big) \leq \psi_1\begin{pmatrix} d(x_{2n}, l), d(Ux_{2n}, x_{2n}), d(Vl, l), \frac{1}{2}\{d(Vl, x_{2n}) + d(Ux_{2n}, l)\}, \\ \frac{1}{2}\{d(x_{2n}, l) + \max\{d(x_{2n}, Ux_{2n}), d(l, Vl)\}\} \end{pmatrix} - \psi_2\big(d(x_{2n}, l) + \varepsilon, d(x_{2n}, l), d(x_{2n}, l), d(x_{2n}, l), d(x_{2n}, l)\big)$$

On letting $n \longrightarrow \infty$ we get,

$$\begin{aligned} \varphi_1\big(d(l, Vl)\big) &\leq \psi_1\begin{pmatrix} d(l,l), d(l,l), d(Vl, l), \frac{1}{2}\{d(Vl, l) + d(l,l)\}, \\ \frac{1}{2}\{d(l,l) + \max\{d(l,l), d(l, Vl)\}\} \end{pmatrix} \\ &\quad - \psi_2\big(d(l,l) + \varepsilon, d(l,l), d(l,l), d(l,l), d(l,l)\big) \\ &= \psi_1\left(0, 0, d(Vl, l), \tfrac{1}{2}\{d(Vl, l)\}, \tfrac{1}{2}\{d(l, Vl)\}\right) \\ &\quad - \psi_2(0 + \varepsilon, 0, 0, 0, 0) \\ &< \psi_1\big(d(Vl, l), d(Vl, l), d(Vl, l), d(Vl, l), d(Vl, l)\big) \\ &= \varphi_1\big(d(Vl, l)\big) \end{aligned}$$

Therefore, $\varphi_1\big(d(Vl, l)\big) < \varphi_1\big(\mathrm{d}(Vl, l)\big)$ $(if \;\; Vl \neq l)$

a contradiction. Therefore, $Vl = l$. Therefore, l is a fixed point of V.

Put $x = y = l$ in (3.2.1).Then

$$\varphi_1\big(d(Ul, Vl)\big) \leq \psi_1\begin{pmatrix} d(l,l), d(Ul, l), d(Vl, l), \frac{1}{2}\{d(Vl, l) + d(Ul, l)\}, \\ \frac{1}{2}\{d(l,l) + \max\{d(l, Ul), d(l, Vl)\}\} \end{pmatrix} - \psi_2\big(d(l,l) + \varepsilon, d(l,l), d(l,l), d(l,l), d(l,l)\big)$$

$$\therefore \varphi_1\big(d(Ul,l)\big) < \psi_1\begin{pmatrix} d(l,l), d(Ul,l), d(l,l), \frac{1}{2}\{d(l,l)+d(Ul,l)\}, \\ \frac{1}{2}\{d(l,l)+\max\{d(l,Ul), d(l,l)\}\} \end{pmatrix}$$

Since $\psi_2\big(d(l,l)+\varepsilon, d(l,l), d(l,l), d(l,l), d(l,l)\big) > 0.$

$$\therefore \varphi_1\big(d(Ul,l)\big) < \psi_1\left(0, d(Ul,l), 0, \tfrac{1}{2}d(Ul,l), \tfrac{1}{2}d(l,Ul)\right) \qquad (\text{ if } Ul \neq l\,)$$

$$\leq \psi_1(d(Ul,l), d(Ul,l), d(Ul,l), d(Ul,l), d(l,Ul)) = \varphi_1\big(d(Ul,l)\big)$$

$\therefore \varphi_1\big(d(Ul,l)\big) < \varphi_1\big(d(Ul,l)\big)$ a contradiction. $\therefore Ul = l.$

Therefore, $Ul = Vl = l$

Therefore, l is a common fixed point of U *and* V.

Suppose, h *and* l are common fixed point of U *and* V.

Put $x = h$ and $y = l$ in (3.2.1) we get,

$$\varphi_1\big(d(Uh,Vl)\big) \leq \psi_1\begin{pmatrix} d(h,l), d(Uh,h), d(Vl,l), \frac{1}{2}\{d(Vl,h)+d(Uh,l)\}, \\ \frac{1}{2}\{d(h,l)+\max\{d(h,Uh), d(l,Vl)\}\} \end{pmatrix} - \psi_2\big(d(h,l)+\varepsilon, d(h,l), d(h,l), d(h,l), d(h,l)\big)$$

Therefore, $= \psi_1\left(d(h,l), 0, 0, d(h,l), \tfrac{1}{2}d(h,l)\right) - \psi_2\big(d(h,l)+\varepsilon, d(h,l), d(h,l), d(h,l), d(h,l)\big)$

$$< \varphi_1\big(d(h,l)\big) - \psi_2\big(d(h,l)+\varepsilon, d(h,l), d(h,l), d(h,l), d(h,l)\big)$$

Therefore, $\varphi_1\big(d(h,l)\big) < \varphi_1\big(d(h,l)\big) \qquad (\,if\;\; h \neq l\,)$

a contradiction. Therefore, $h = l$.

Therefore, U *and* V have a unique common fixed point in X.

3.3 Corollary: Let (X,d) be a complete metric space and $\psi_1, \psi_2 \in \Psi_5$. Suppose $U, V : X \longrightarrow X$ are such that for all $x, y \in X$

$$\varphi_1\big(d(Ux,Vy)\big) \leq \psi_1\begin{pmatrix} d(x,y), d(Ux,x), d(Vy,y), \frac{1}{2}\{d(Vy,x)+d(Ux,y)\}, \\ \frac{1}{2}\{d(x,y)+d(x,Ux)\} \end{pmatrix} - \psi_2\big(d(x,y)+\varepsilon, d(x,y), d(x,y), d(x,y), d(x,y)\big) \qquad -\ (3.3.1)$$

where $\varphi_1(\alpha) = \psi_1(\alpha,\alpha,\alpha,\alpha,\alpha),\;\; \alpha \in [0,\infty)$.

Then U *and* V have a unique common fixed point in X.

Proof: Since (3.3.1) $\Longrightarrow$ (3.2.1), the result follows from Theorem 3.2.

Now we show that the result of K. Sridevi et al.[21] follows from our Theorem 3.2.

3.4. Corollary: (Theorem 2.4)

Proof: Since (2.4.1) $\Longrightarrow$ (3.2.1),The result follows from Theorem 3.2.

ACKNOWLEDGEMENTS: The second author gratefully acknowledges

(i) The financial support from university grants commission (U.G.C) through UGC grant No .MRP-6045/15(SERO/UGC) to carry on this research.

(ii) The Principal and Management of Gayatri Vidya Parishid College for Degree and P.G Courses (A) for giving necessary permissions and for providing necessary facilities to carry on this research.

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