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Further Results On Odd Mean Graphs

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Abstract

Let $G = (V, E)$ be a graph with p vertices and q edges. A graph G is said to have an odd mean labeling if there exists a function $f : V(G) \to \{0, 1, 2, \dots, 2q - 1\}$ satisfying f is $1 - 1$ and the induced map $f^* : E(G) \to \{1, 3, 5, \dots, 2q - 1\}$ defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u) + f(v) \text{ is odd.} \end{cases}$$

is a bijection. A graph that admits an odd mean labeling is called an odd mean graph. Here we study about the odd mean behaviour of some standard graphs.

Keywords: labeling, odd mean labeling, odd mean graph

AMS Mathematics Subject Classification: 05C78

1. INTRODUCTION

All graphs considered in this paper are simple and undirected. Let $G(V, E)$ be a graph with p verticies and q edges. For notation and terminology, we follow [3].

Path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. $K_{1,m}$ is called a *star* and it is denoted by S_m. The bistar $B_{m,n}$ is the graph obtained from K_2 by identifying the central vertices of $K_{1,m}$ and $K_{1,n}$ at the end vertices of K_2 respectively. $B_{m,m}$ is often denoted by $B(m)$. The union of two graphs G_1 and G_2 is a graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. The union of m disjoint copies of a graph G is denoted by mG.

Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices respectively. Then the cartesian product $G_1 \times G_2$ has p_1p_2 vertices which are $\{(u, v)|u \in G_1, v \in G_2\}$. The edges are obtained as follows: (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$. The product $C_m \times P_n$ is called a *prism*. The graph $P_2 \times P_2 \times P_2$ is called a cube and is denoted by Q_3. The H-graph of a path P_n, denoted by H_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even. If m number of pendant vertices are attached at each vertex of G, then the resultant graph obtained from G is the graph $G \odot mK_1$. When $m = 1, G \odot K_1$ is the corona of G.

The graceful labelings of graphs was first introduced by Rosa in 1961 [1] and R.B. Gnanajothi introudced odd graceful graphs [2]. The concept of mean labeling was first introduced by S. Somasundaram and R. Ponraj [7]. The mean labeling of some standard graphs are studied in [5, 7, 8]. Further some more results on mean graphs are discussed in [6, 9, 10]. The concept of odd mean labeling was introduced and studied by K. Manickam and M. Marudai [4].

A graph G is said to have an odd mean labeling if there exists a function $f : V(G) \to \{0, 1, 2, \ldots, 2q-1\}$ satisfying f is $1-1$ and the induced map $f^* : E(G) \to \{1, 3, 5, \ldots, 2q-1\}$ defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

is a bijection. A graph that admits an odd mean labeling is called an odd mean graph [4].

An odd mean labeling of $B_{3,3}$ is given in Figure 1

Figure 1. An odd mean labeling of $B_{3,3}$

In [11], R. Vasuki and A. Nagarajan studied about the odd mean behaviour of the class of graphs $P_{a,b}, P_a^b$ and $P_{\langle 2a \rangle}^b$. In this paper, we prove that $C_m \times P_n$ for $m \equiv 0(mod\ 4), n \geq 1$, $Q_3 \times P_n$, H-graph, corona of a H-graph and $G \odot S_2$ where G is a H-graph are odd mean graphs. Also we prove that if a tree T has an odd mean labeling, then $T_{(n)}$ is an odd mean graph for any $n \geq 1$. Also we establish that union of any number of odd mean graph is an odd mean graph.

2. ODD MEAN GRAPHS

Theorem 2.1. *$C_m \times P_n$ is an odd mean graph for $m \equiv 0(mod\ 4)$ and $n \geq 1$.*

Proof. Let $V(C_m \times P_n) = \{v_{i_j} : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(C_m \times P_n) = \{e_{i_j} : e_{i_j} = v_{i_j} v_{(i+1)_j}, 1 \leq j \leq n, 1 \leq i \leq m\} \cup \{E_{i_j} : E_{i_j} = v_{i_j} v_{i_{j+1}}, 1 \leq j \leq n-1, 1 \leq i \leq m\}$ where $i+1$ is taken modulo m.

Let C_m^j denote the j^{th} copy of C_m in $C_m \times P_n$. Let the vertices of C_m^j be $v_{1_j}, v_{2_j}, \ldots, v_{m_j}$ for $1 \leq j \leq n$. Label the vertices of $C_m, m \equiv 0(mod\, 4)$ as follows:

$$f(v_{i_j}) = \begin{cases} 4i - 4 & \text{if } 1 \leq i \leq \frac{m}{2} + 1 \text{ and } i \text{ is odd} \\ 4i - 6 & \text{if } 2 \leq i \leq \frac{m}{2} \text{ and } i \text{ is even} \\ 4m + 3 - 4i & \text{if } \frac{m}{2} + 1 < i < n \text{ and } i \text{ is odd} \\ 4m + 6 - 4i & \text{if } \frac{m}{2} < i \leq m \text{ and } i \text{ is even.} \end{cases}$$

If the vertices of C_m^{j-1} are labeled then the vertices of C_m^j are labeled as follows:

$f(v_{i_j}) = f(v_{(i-1)_{(j-1)}}) + 4m$ where $i-1$ and $j-1$ are taken modulo m.

It can be verified that the label of the edges are $1, 3, 5, \ldots, 2q-1$. Then f is an odd mean labeling of $C_m \times P_n$ for $n \geq 1$ and $m \equiv 0(mod\, 4)$. Hence $C_m \times P_n$ is an odd mean graph for $n \geq 1$ and $m \equiv 0(mod\, 4)$. □

For example, an odd mean labeling of $C_8 \times P_4$ is shown in Figure 2.

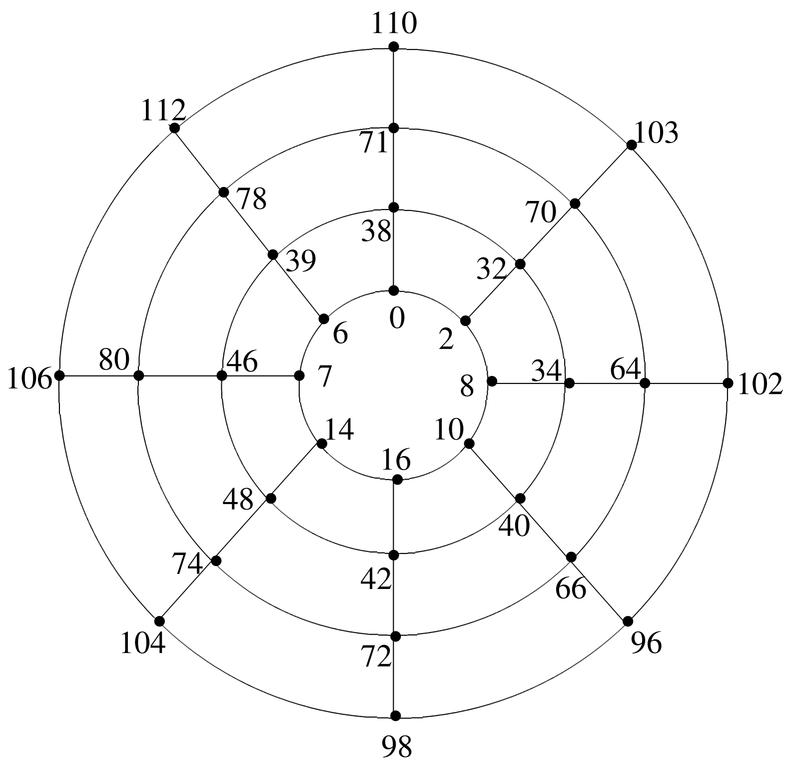

Figure 2. An odd mean labeling of $C_8 \times P_4$

Theorem 2.2. *$Q_3 \times P_n$ is an odd mean graph.*

Proof. Let Q_3^j denote the j^{th} copy of Q_3 in $Q_3 \times P_n$ and for $1 \leq i \leq 8$, let v_{i_j} denote the i^{th} vertex in Q_3^j, where $1 \leq j \leq n$.

The vertices and their labels of $Q_3 \times P_2$ are shown in Figure 3.

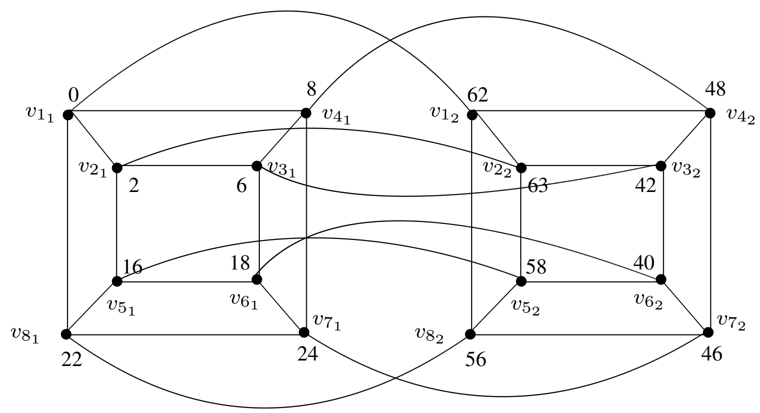

Figure 3. An odd mean labeling of $Q_3 \times P_2$

If the vertices of Q_3^{j-2} are labeled by f, then the vertices of Q_3^j are labeled as follows:

$f(v_{i_j}) = f(v_{i_{j-2}}) + 80$, for $1 \leq i \leq 8$ and $3 \leq j \leq n$.

Let E_j be the set of all edges in Q_3^j and $E_{j_{j+1}}$ be the set of all edges having one end in Q_3^j and the other in Q_3^{j+1}.

Denote the set of edge labels for the edges of E by $f^*(E)$. Then, it is observed that

$f^*(E_j) = \{40 + f^*(e) : e \in E_{j-1}\}, 2 \leq j \leq n$

$f^*(E_{j_{j+1}}) = \{40 + f^*(e) : e \in E_{(j-1)_j}\}, 2 \leq j \leq n-1.$

Then, f is an odd mean labeling of $Q_3 \times P_n$. □

For example, an odd mean labeling of $Q_3 \times P_4$ is shown in Figure 4.

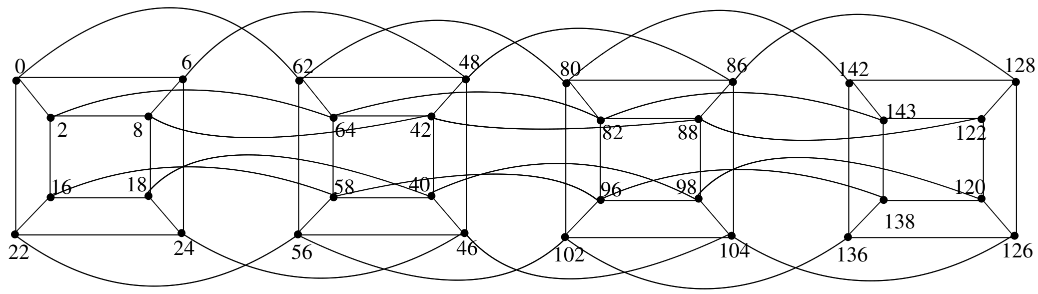

Figure 4. An odd mean labeling of $Q_3 \times P_4$

Let T be any tree. Denote the tree, obtained from T by considering two copies of T and adding an edge between them, by $T_{(2)}$ and in general, the graph obtained from $T_{(n-1)}$ and T by adding an edge between them is denoted by $T_{(n)}$. Note that $T_{(1)}$ is nothing but T.

Theorem 2.3. *If a tree T has an odd mean labeling, then $T_{(n)}$ is an odd mean graph for any $n \geq 1$.*

Proof. We prove this result by induction on n.

When $n = 1$, the result is obvious. Let $n = 2$. Assume that $f : V(T) \to \{0, 1, 2, \ldots, 2q-1\}$ is an odd mean labeling of T. Let T_1 and T_2 be two copies of T in $T_{(2)}$. Define a labeling l of $T_{(2)}$ as follows:

$$l(v) = \begin{cases} f(v) & \text{if } v \in T_1 \\ f(v) + 2p & \text{if } v \in T_2 \text{ where } p \text{ is the number of vertices in } T. \end{cases}$$

Then, l is an odd mean labeling and hence the result is true when $n = 2$.

Assume that $T_{(n)}$ is an odd mean graph for any $n \geq 1$. Let g be an odd mean labeling of $T_{(n)}$. To complete the induction process, it is enough to prove that $T_{(n+1)}$ is an odd mean graph.

Define a labeling l of $T_{(n+1)}$ as follows:

$$l(v) = \begin{cases} g(v) & \text{if } v \in T_{(n)} \\ f(v) + 2np & \text{if } v \in T_{n+1} \text{ where } T_{n+1} \text{ is a} \\ & (n+1)^{th} \text{ copy of } T \text{ in } T_{(n+1)} \end{cases}$$

Clearly, l is an odd mean labeling of $T_{(n+1)}$. Hence, $T(n)$ is an odd mean graph for any $n \geq 1$. □

For example, an odd mean labelings of $T, T_{(2)}$ and $T_{(3)}$ are shown in Figure 5.

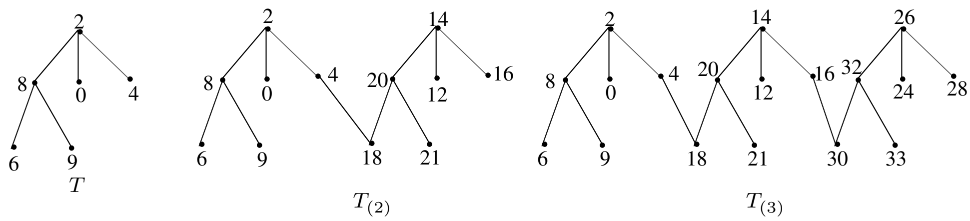

Figure 5. An odd mean labelings of $T, T_{(2)}$ and $T_{(3)}$

Corollary 2.4. *$B(m)_{(n)}$ is an odd mean graph for any $m \geq 0$ and $n \geq 1$.*

Proof. It is enough to show that $B(m)$ has an odd mean labeling. Let the vertices of $B(m)$ be $v_0, v_1, \ldots, v_m$ and $u_0, u_1, \ldots, u_m$. Label the vertices of $B(m)$ by

$f(v_0) = 0$

$f(v_i) = 4i - 2, 1 \leq i \leq m$

$f(u_0) = 4m + 2$

$f(u_i) = 4i, 1 \leq i \leq m.$

Then, f is an odd mean labeling of $B(m)$. Therefore, by Theorem 2.3, $B(m)_{(n)}$ is an odd mean graph. □

For example, an odd mean labeling of $B(5)_{(3)}$ is illustrated in Figure 6.

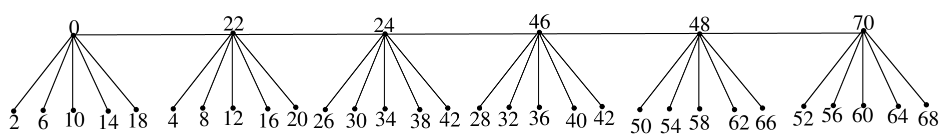

Figure 6. An odd mean labeling of $B(5)_{(3)}$

Corollary 2.5. *$P_{n_{(m)}}$ is an odd mean graph for any $n \geq 1, m \geq 1$.*

Proof. It is enough to show that P_n has an odd mean labeling. Let the vertices of P_n be $v_1, v_2, \ldots, v_n$. Label the vertices of P_n by $f(v_i) = 2i - 2$ for $1 \leq i \leq n$. Then, f is an odd mean labeling of P_n. Hence, by Theorem 2.3, $P_{n_{(m)}}$ is an odd mean graph. □

For example, an odd mean labeling of $P_6, P_{6_{(2)}}$ and $P_{6_{(3)}}$ are shown in Figure 7.

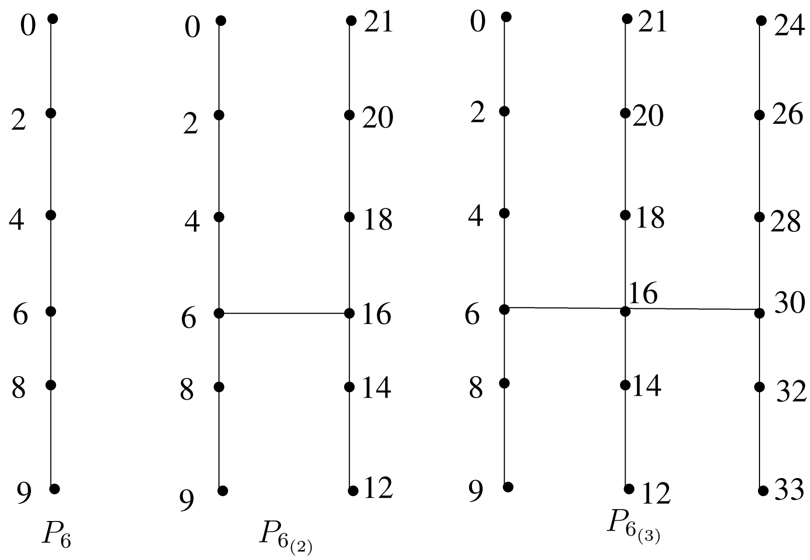

Figure 7. An odd mean labeling of $P_6, P_{6_{(2)}}$ and $P_{6_{(3)}}$

Theorem 2.6. *The H-graph G is an odd mean graph.*

Proof. Let $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ be the vertices of the H-graph G. Define $f : V(G) \to \{0, 1, 2, \ldots, 2q-1\}$ as follows:

$$\begin{array}{lll} f(v_i) & = 2i-2, & 1 \leq i \leq n \\ f(u_i) & = 2n+2i-2, & 1 \leq i \leq n-1 \\ f(u_n) & = 4n-3. & \end{array}$$

The induced edge labels are given by

$$\begin{array}{lll} f^*(v_i v_{i+1}) & = 2i-1, & 1 \leq i \leq n-1 \\ f^*(u_i u_{i+1}) & = 2n+2i-1, & 1 \leq i \leq n-1 \\ f^*(v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}) & = 2n-1 & \text{if } n \text{ is odd} \\ f^*(v_{\frac{n}{2}+1} u_{\frac{n}{2}}) & = 2n-1 & \text{if } n \text{ iseven.} \end{array}$$

Then, f is an odd mean labeling. Hence, the H-graph G is an odd mean graph. □

For example, an odd mean labeling of H_7 and H_6 are shown in Figure 8.

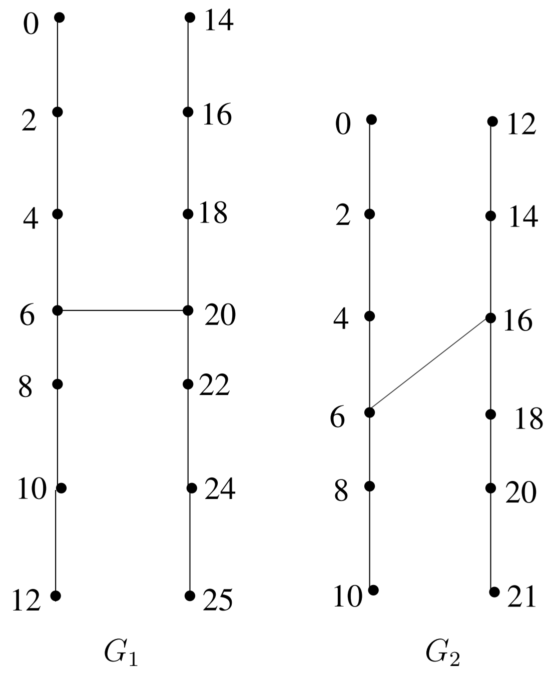

Figure 8. An odd mean labeling of H_7 and H_6

Theorem 2.7. *For a H-graph G, $G \odot K_1$ is an odd mean graph.*

Proof. By Theorem 2.6, there exists an odd mean labeling f for G. Let $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ be the vertices of G.

Let $V(G \odot K_1) = V(G) \cup \{v'_1, v'_2, \ldots, v'_n\} \cup \{u'_1, u'_2, \ldots, u'_n\}$ and $E(G \odot K_1) = E(G) \cup \{v_i v'_i, u_i u'_i : 1 \leq i \leq n\}$.

Define $g : V(G \odot K_1) \rightarrow \{0, 1, 2, \ldots, 2q-1\}$ as follows:

$$
\begin{aligned}
g(v_i) &= f(v_i) + 2i - 1, && 1 \leq i \leq n \\
g(u_i) &= f(u_i) + 2n + 2i - 1, && 1 \leq i \leq n-1 \\
g(u_n) &= f(u_n) + 4n \\
g(v'_i) &= f(v_i) + 2i - 2, && 1 \leq i \leq n \\
g(u'_i) &= f(u_i) + 2n + 2i - 2, && 1 \leq i \leq n-1 \\
g(u'_n) &= f(u_n) + 4n - 1.
\end{aligned}
$$

The induced edge labeling g^* is obtained as follows:

$$
\begin{array}{lll}
g^*(v_iv_{i+1}) & = f^*(v_iv_{i+1}) + 2i, & 1 \le i \le n-1 \\
g^*(u_iu_{i+1}) & = f^*(u_iu_{i+1}) + 2n + 2i, & 1 \le i \le n-1 \\
g^*(v_iv_i') & = f(v_i) + 2i - 1, & 1 \le i \le n \\
g^*(u_iu_i') & = f(u_i) + 2n + 2i - 1, & 1 \le i \le n \\
g^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}) & = 2f^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}) + 1 & \text{if } n \text{ is odd} \\
g^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}}) & = 2f^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}}) + 1 & \text{if } n \text{ is even.}
\end{array}
$$

Then, g is an odd mean labeling and hence $G \odot K_1$ is an odd mean graph. □

For example, an odd mean labelings of $H_5 \odot K_1$ and $H_4 \odot K_1$ for the H-graphs H_5 and H_4 are shown in Figure 9.

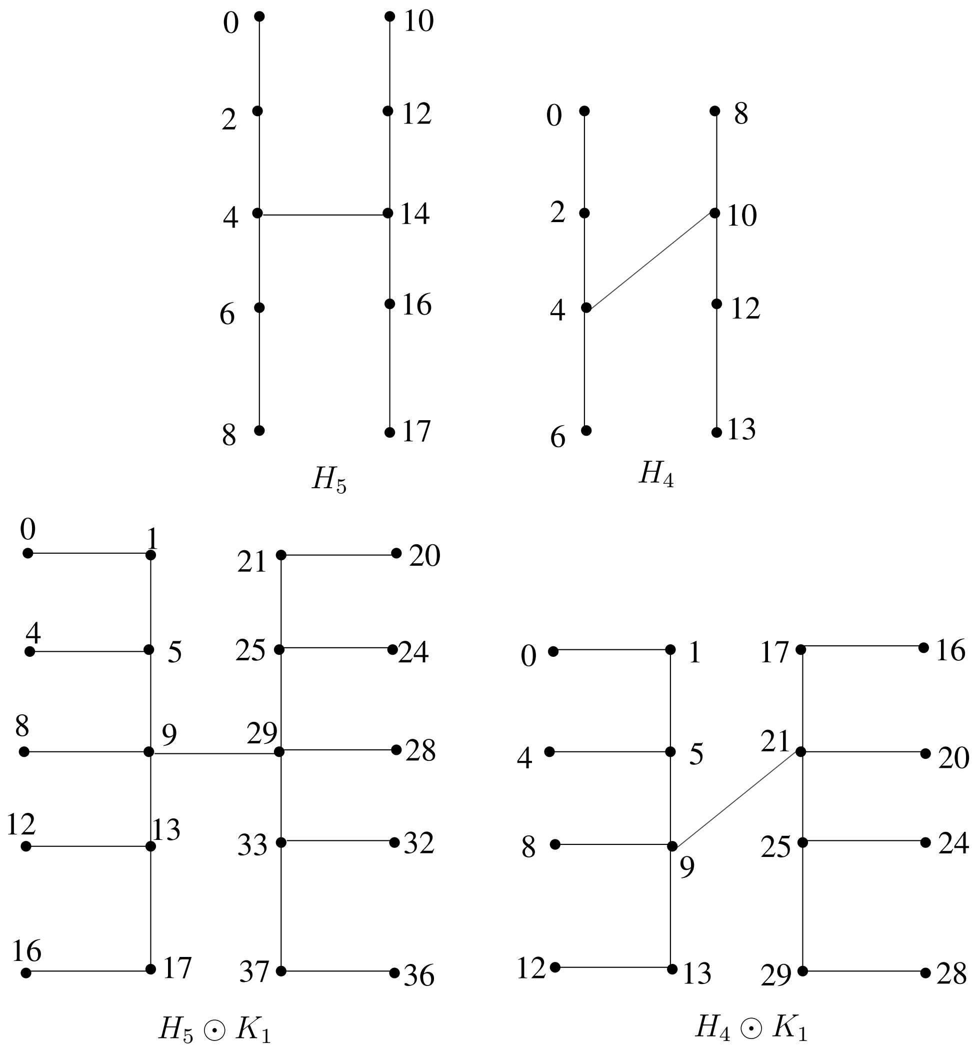

Figure 9. An odd mean labeling of $H_5, H_4, H_5 \odot K_1$ and $H_4 \odot K_1$

Theorem 2.8. *For a H-graph G, $G \odot S_2$ is an odd mean graph.*

Proof. By Theorem 2.6, there exists an odd mean labeling f for G. Let $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ be the vertices of G. Let $V(G)$ together with $v'_1, v'_2, \ldots, v'_n, v''_1, v''_2, \ldots, v''_n$, $u'_1, u'_2, \ldots, u'_n$ and $u''_1, u''_2, \ldots, u''_n$ form the vertex set of $G \odot S_2$ and the edge set is $E(G)$ together with $\{v_iv'_i, v_iv''_i, u_iu'_i, u_iu''_i : 1 \leq i \leq n\}$.

Define $g : V(G \odot S_2) \to \{0, 1, 2, \ldots, 2q-1\}$ as follows:

$$
\begin{aligned}
g(v_i) &= f(v_i) + 4i - 2, && 1 \leq i \leq n \\
g(v'_i) &= f(v_i) + 4i - 4, && 1 \leq i \leq n \\
g(v''_i) &= f(v_i) + 4i, && 1 \leq i \leq n \\
g(u_i) &= f(u_i) + 4n + 4i - 2, && 1 \leq i \leq n \\
g(u'_i) &= f(u_i) + 4n + 4i - 4, && 1 \leq i \leq n \\
g(u''_i) &= f(u_i) + 4n + 4i, && 1 \leq i \leq n.
\end{aligned}
$$

The induced edge labeling f^* is given as follows:

$$
\begin{aligned}
g^*(v_iv_{i+1}) &= f^*(v_iv_{i+1}) + 4i, && 1 \leq i \leq n-1 \\
g^*(v_iv'_i) &= f(v_i) + 4i - 3, && 1 \leq i \leq n \\
g^*(v_iv''_i) &= f(v_i) + 4i - 1, && 1 \leq i \leq n \\
g^*(u_iu_{i+1}) &= f^*(u_iu_{i+1}) + 4n + 4i, && 1 \leq i \leq n-1 \\
g^*(u_iu'_i) &= f(u_i) + 4n + 4i - 3, && 1 \leq i \leq n \\
g^*(u_iu''_i) &= f(u_i) + 4n + 4i - 1, && 1 \leq i \leq n.
\end{aligned}
$$

$$
\begin{aligned}
g^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}) &= 3f^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}) + 2 && \text{if } n \text{ is odd} \\
g^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}}) &= 3f^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}}) + 2 && \text{if } n \text{ is even}
\end{aligned}
$$

Then, g is an odd mean labeling and hence $G \odot S_2$ is an odd mean graph. □

For example, an odd mean labelings of $H_7 \odot S_2$ and $H_6 \odot S_2$ for the H-graphs H_7 and H_6 are shown in Figure 10.

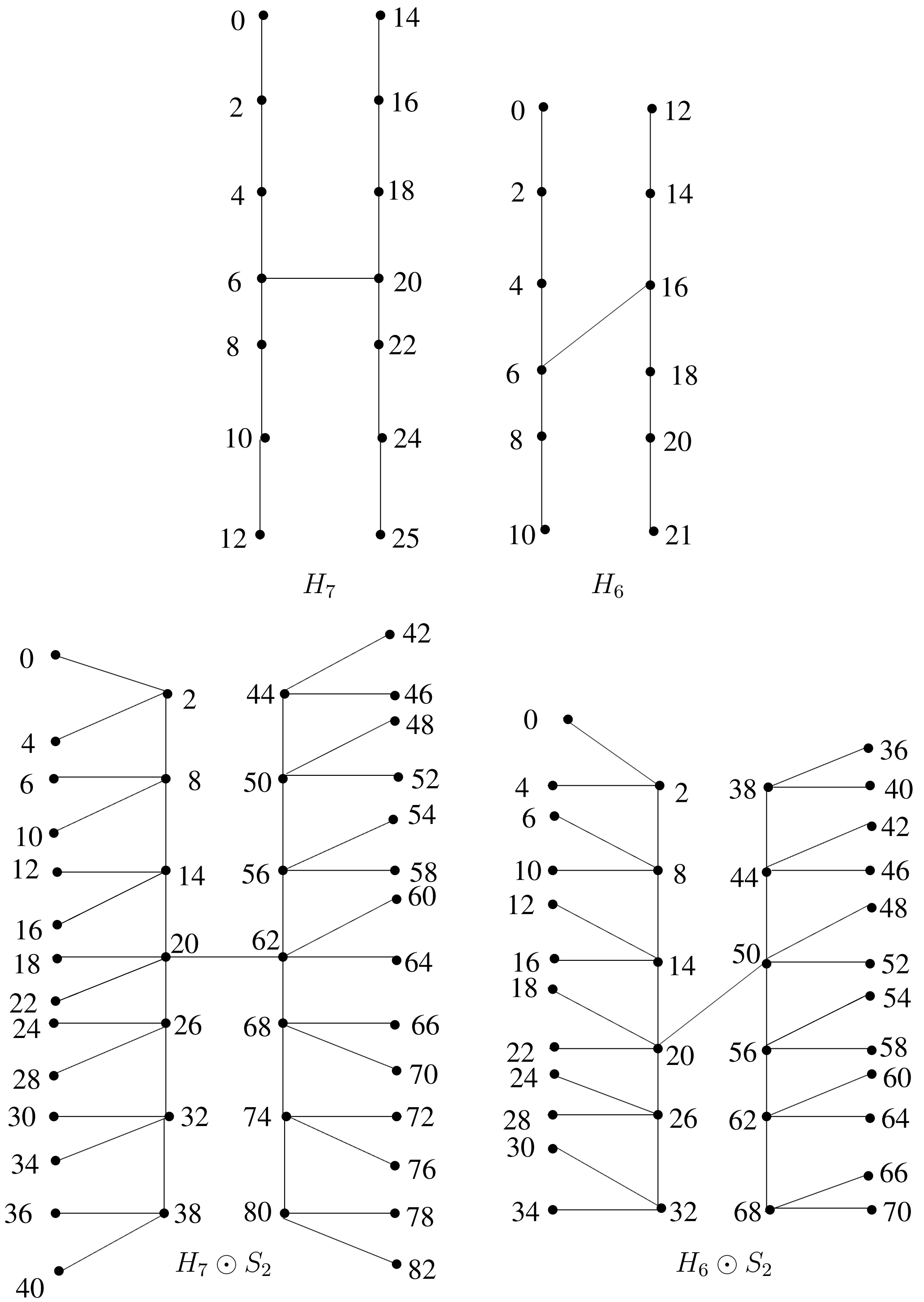

Figure 10. An odd mean labeling of $H_7, H_6, H_7 \odot S_2$ and $H_6 \odot S_2$

Theorem 2.9. *If $G_1, G_2, G_3, \ldots, G_m$ are odd mean graphs, then $G_1 \cup G_2 \cup G_3 \cdots \cup G_m$ is an odd mean graph.*

Proof. If $G_1 = (p_1, q_1), G_2 = (p_2, q_2), G_3 = (p_3, q_3), \ldots, G_m = (p_m, q_m)$ are any m odd mean graphs with odd mean labelings $f_1, f_2, \ldots, f_m$ respectively, then $G_1 \cup G_2 \cup G_3 \cdots \cup G_m$ has $p_1 + p_2 + \cdots + p_m$ vertices and $q_1 + q_2 + \cdots + q_m$ edges. Let $u_{1_i}(1 \leq i \leq p_1)$, $u_{2_i}(1 \leq i \leq p_2), \ldots, u_{m_i}$ $(1 \leq i \leq p_m)$ and $e_{1_i}(1 \leq i \leq q_1), e_{2_i}(1 \leq i \leq q_2), \ldots, e_{m_i}(1 \leq i \leq q_m)$ be the vertices and edges of the graphs $G_1, G_2, G_3, \ldots, G_m$ respectively.

Define $g : V(G_1 \cup G_2 \cup \cdots \cup G_m) \to \{0, 1, 2, 3, \ldots, 2(q_1 + q_2 + \cdots + q_m) - 1\}$ as follows:

$$
\begin{aligned}
g(u_{1_i}) &= f_1(u_{1_i}) \\
g(u_{2_i}) &= f_2(u_{2_i}) + 2q_1, 1 \leq i \leq p_2 \\
g(u_{3_i}) &= f_3(u_{3_i}) + 2(q_1 + q_2), 1 \leq i \leq p_3 \\
g(u_{4_i}) &= f_4(u_{4_i}) + 2(q_1 + q_2 + q_3), 1 \leq i \leq p_4 \\
&\cdots\cdots\cdots \\
&\cdots\cdots\cdots \\
g(u_{m_i}) &= f_m(u_{m_i}) + 2(q_1 + q_2 + q_3 + \cdots + q_{m-1}), 1 \leq i \leq p_m
\end{aligned}
$$

The induced edge labels are given by

$$
\begin{aligned}
g^*(e_{1_i}) &= f_1^*(e_{1_i}), 1 \leq i \leq q_1 \\
g^*(e_{2_i}) &= f_2^*(e_{2_i}) + 2q_1, 1 \leq i \leq q_2 \\
g^*(e_{3_i}) &= f_3^*(e_{3_i}) + 2(q_1 + q_2), 1 \leq i \leq q_3 \\
g^*(e_{4_i}) &= f_4^*(e_{4_i}) + 2(q_1 + q_2 + q_3), 1 \leq i \leq q_4 \\
&\cdots\cdots\cdots \\
&\cdots\cdots\cdots \\
g^*(e_{m_i}) &= f_m^*(e_{m_i}) + 2(q_1 + q_2 + q_3 + \cdots + q_{m-1}), 1 \leq i \leq q_m.
\end{aligned}
$$

Then, g is an odd mean labeling. Hence, $G_1 \cup G_2 \cup G_3 \cdots \cup G_m$ is an odd mean graph. □

For example, an odd mean labelings of G_1, G_2, G_3, G_4 and $G_1 \cup G_2 \cup G_3 \cup G_4$ are shown in Figure 11.

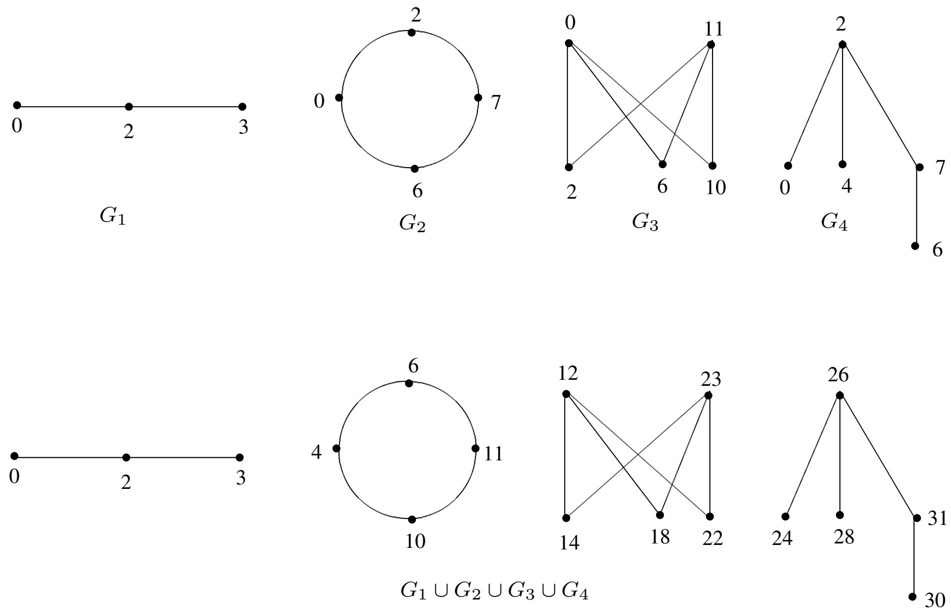

Figure 11. An odd mean labeling of G_1, G_2, G_3, G_4 and $G_1 \cup G_2 \cup G_3 \cup G_4$

Corollary 2.10. *If G is an odd mean graph, then mG is also an odd mean graph, for all $m \geq 1$.*

Proof. The proof follows from Theorem 2.9, by taking $G_1 = G_2 = G_3 =, \ldots, G_m = G$. □

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