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# A New Original Vector Ordering based on Search of the Minimum Error Sequence by Recursive Analysis of n-ary trees

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**Abstract**

The elaboration of a vector ordering in vectorial mathematical morphology is tricky. This has been the subject of many works because the extension of a scalar order to the vector case does not seem obvious. Therefore, if we stick to Barnett's classification in 4 groups of orders, we find that the first group called marginal order orders the vectors by component and independently, it introduces false colors and the order is not total. The second group is the partial order, it performs a classification in ordered partitions and does not provide a classification of the vectors of the same partition. The third called reduced order is based on a bijective transformation of vectors into scalars, its implementation requires the choice of a referent that is generally arbitrary. The last group is the conditional order, its principle is based on the lexicographic order used in the dictionaries, and in this case the proximity between the vectors is not guaranteed. Indeed, in our previous work, we contributed to the improvement of the conditional orders by choosing adaptive and non-arbitrary absolute referents. Additional constraints have been proposed to make these orders constitutes a lattice on the space considered. The relevance of the choice of referent is questionable and always makes parametric proposed conditional orders, which weakens their performance. Thus, the purpose of this paper is to propose a new original, simple, natural and non-parametric order to order vector attributes or "color" based on the choice of sequences subject to an error constraint minimal. Our goal is to be able to tend towards a universal order for the ordering of the vector attributes. The new vector order that we present in this work is natural and original. The general idea of its conception is inspired by an observation of the functioning of the order of real numbers, in particular the natural numbers.

Consider a sequence of number $x_1, x_2, ...., x_p$ of ℝ or ℕ of length $p$. Considering all possible permutations of sequences with the number of $p!$, the choice of the ordered sequence on ℝ is that of which the global quadratic error is minimal. This error is obtained by summing the quadratic difference between the consecutive elements of a sequence.

In this paper, we propose a new method, an original approach of vector ordering, by search of global minimum error or minimum cost (*EQMmin*) based on the construction of trees whose branch values are the quadratic differences between consecutive elements of a path of the tree. The use of trees is justified by optimizing the use of RAM memory and reducing algorithmic complexity. The nodes of the tree are represented by the order of the tuples or " color " in the multidimensional compact histogram ordered according to the lexicographic order and a node can have several child nodes. Different paths or sequences can be considered as solutions. In order to build a complete lattice, an unique solution is retained by adding a constraint on the tuples. This additional constraint consists in choosing the order whose gap between the eroded vector and the expanded vector is maximal, and having the first smallest position in the sequence which defines it.

The algorithms of our model have been implemented under MATLAB through the basic morphological operators Gradient and Laplacian. The results obtained on different synthetic and real color images show the relevance of our comparative approach compared to that of reduced order to absolute adaptive referent.

**Keywords:** Vectorial mathematical morphology; Multicomponent image; Vector ordering; Tree; Mean square error; Minimum cost; Morphological operators

## 1. INTRODUCTION

Mathematical morphology is a great success in the community of image analysis and processing. It provides a rigorous framework and effective tools for spatial and non-linear image analysis. Its application to binary images and grayscale images is done very simply by relying on the theory of sets or better, that of lattices. However, its extension to the case of multicomponent images where each pixel is represented by a vector and no longer by a scalar, or in the case of multivariate data is not trivial and remains an open problem. Indeed, there is no single solution and unanimously accepted for ordering vectors [1-6]. Morphological operators cannot therefore be used directly on multivariate data containing for example a spectral or temporal dimension.

Recently, we have developed new orders, one called hybrid based on a reduced order to absolute adaptive referent [1] and the other one of principle almost similar to the first one with integration of a classification of tuples into different numbers of classes [2] whose relevance and robustness to noise were presented. The state of the art made on other articles, allowed us to have an idea of what is done in the literature and to understand their limits [3-9]. This allowed us to propose in this paper a new and original approach.

Unfortunately these methods [1,2] depend strongly on a referent even if it is qualified of absolute adaptive with or without classification. Moreover, the difficulty of discussing the optimality of these parameters weakens the said methods.

In fact, to overcome the limitations of the previous approaches, we propose in this paper a simple, original, algorithmically reduced nonparametric vector order that intuitively exploits the principle of classifying real numbers.

The general principle of our approach is as follows: consider a sequence of $p$ ($p> = 3$) given real numbers; the order on $\mathbb{R}$ will impose a unique solution, however we will be able to count $p!$ orders corresponding to the number of possible permutations. This unique solution on $\mathbb{R}$ is characterized by a global minimal error (*EQMmin*), this error is obtained by the sum of the quadratic differences of the adjacent numbers of the permutation considered.

In order to determine the solution on the space of the attribute vectors or "colors" with optimal algorithmic complexity and with less memory capacity, we use respectively the tree structure and the multidimensional compact histogram [10-12]. Thus the algorithmic elaboration of our approach is based on the following stages: (i) computation of the multidimensional compact histogram; (ii) construction of recursive trees; (iii) search for minimum cost path or minimum overall mean quadratic error from constructed trees. Our algorithms have been implemented under MATLAB and a study compared to the reduced order with absolute adaptive reference shows well the proven performances of our approach.

## 2. MATERIALS AND METHODS

This work was carried out at the laboratory of signals and electrical systems (L2SE) of the National Polytechnic Institute Houphouët-Boigny (INP-HB) during the academic year 2017-2018 following the research of our PhD student KOUASSI Adles Francis. Concerning the algorithmic aspect, we realized our own programming codes of our modeling under the Matlab scientific software environment using its development language.

### 2.1 Principle of our new vector ordering by N-ary trees analysis based on the research of sequences with EQMmin

The principle of the proposed vector order consists first, from an image $I$ considered, to compute its multidimensional compact histogram (*nD*) [10-12] where $n$ denotes here the number of components of the image, denoted *Hc_nD*. In the compact histogram *nD*, the image attribute vectors or tuples are ordered according to the lexicographic order [10-12]. Then a first tree is built recursively, its parent node corresponds to the first tuple or " color " of *Hc_nD*. This father node has for son the rest of the tuples of the compact histogram *nD*. A son is characterized by the quadratic error calculated between him and his father. The process is reiterated for sons who become father until they get leaves knowing on a path from the top of the tree to a leaf, the tuples are unique and the length of all paths correspond to tuples of *Hc_nD*. The path (s) of minimal cost or minimum squared errors (*EQMmin*) are retained as best temporary orders. So, this first tree is deleted for optimizing RAM. The other trees are created consecutively taking as nodes fathers respectively the other tuples in ascending order in *Hc_nD*. The best final paths are those that have the same minimum quadratic error (*EQMmin*) on all the trees built. The Quadratic Error EQM is defined by Eq. 1 below. Let *Cond1* be the minimum error constraint.

$$EQM = \sum_{i=1}^{p}(x_i - x_{i+1})^2 \qquad (1)$$

To obtain a total order, an additional constraint is realized on the previous order solutions. This constraint consists in choosing as a best order first the path (s) whose difference between eroded and dilated is maximal (*EcartMax*), then choose the path whose father node is the smallest relative to the lexicographic order. In case of equality of the tuples of this rank 1, the tuples of the same rank on the different paths solutions relative to the rank 1 are compared in ascending order until retaining as the best order the path whose tuple is the smallest compared to the lexicographic order at the smallest rank, hence the realization of a complete lattice on the space of the tuples of the histogram *nD* compact. Let *Cond2* be this additional constraint for obtaining a single order.

Let *PERM* be the set of *p!* Permutations possible on the tuples of $x_1, ..., x_p$ of the histogram *nD* compact. An element of *PERM* will be considered as a possible path or order of all permutations. Let $PERM_k$ be the *k-th* order of the *PERM* set. The best order $O_u$ is defined by Eq. 2:

$$O_u = \text{ArgMax}\,(PERM_k, Cond1, Cond2), \mid k \text{ varying from 1 to } p! \qquad (2)$$

The unique order solution is obtained for $k = u$. The determination of the unique order becomes prohibitive when it comes to generating all possible permutations because we are confronted with the problem of limitation of the RAM memory; this is why in this article, we opted for an algorithmic structure of tree array to generate sequentially the potential orders solutions.

## 2.2 Algorithm of our new vector ordering

We propose in this section the algorithms to generate the best orders. An additional constraint on these orders produces a unique solution.

Since our different developments were made in the Matlab environment and the language used is close to the C language, for the implementation of the n-ary tree we used as a type a structure or record table.

Type Arbre : Record Table

**Node** : Record

Pos_Hc : *Integer*

Predecesseurs : *Array of integers*

Pere : *Integer*

Erreur : *Real*

**EndNode**

- ✓ ***Function of Tree construction***

```
// --------------------------------------------------------------
[Sommet,Arbre,k] = ArbreConstructionOrdre (Sommet,Arbre,Histo,k)

TailleArbreInit = Size (Arbre);   // Return the size of the tree Arbre
TailleArbre=TailleArbreInit ;
IndexArbreFilsFilles=[];
Index=FindLeaves (Arbre); //  Return the leaves of the tree Arbre
ErreurTab=[];
If (Taille (Index) ≠ 0)
   For  i=1 to Size (Index)
[VoisinsImmediats,Erreur]=FindNeighbours
(Arbre(Index(i)).PosHisto,Histo,Arbre(Index(i)).Predecesseurs);
      Arbre(Index(i)).FilsFilles=VoisinsImmediats;
      If (Arbre(Index(i)).FilsFilles = 0)  Then
         k=k+1
         Sommet(k).PosHisto=Arbre(Index(i)).PosHisto;
         Sommet(k).Predecesseurs=Arbre(Index(i)).Predecesseurs;
         Sommet(k).Erreur=Arbre(Index(i)).Erreur;
         Sommet(k).FilsFilles=Arbre(Index(i)).FilsFilles;
```

```
        Else
            IndexArbreFilsFilles =[Index(i),IndexArbreFilsFilles];
            ErreurTab=[ErreurTab,Erreur];
        EndIf
    EndFor
EndIf
TailleIndexArbreFilsFilles = Size (IndexArbreFilsFilles);
If ( TailleIndexArbreFilsFilles ≠ 0 )
    For j=1 to TailleIndexArbreFilsFilles
        T=Arbre(IndexArbreFilsFilles(j)).FilsFilles;
        TailleT =Size (T);
        For m=1 to TailleT
            TailleArbre=TailleArbre+1;
            Arbre(TailleArbre).PosHisto=T(m);
            Arbre(TailleArbre).FilsFilles=0;

Arbre(TailleArbre).Predecesseurs=[Arbre(IndexArbreFilsFilles(j)).Predecesseurs,T(
m)];
            Arbre(TailleArbre).Pere=Arbre(IndexArbreFilsFilles(j)).PosHisto;

Arbre(TailleArbre).Erreur=Arbre(IndexArbreFilsFilles(j)).Erreur+ErreurTab(j);
        EndFor
    EndFor
    [Sommet,Arbre,k] = ArbreConstructionOrdre (Sommet,Arbre,Histo,k);
EndIf
```

- ✓ ***Main Program***

```
// ---------------------------------------------
NomImage='ImageSavoise08.tiff';
Img = ReadMatrixImage (NomImage);
[H,counts] = HistoCompact-nD (img);  // Return the Compact nD Histogram
TailleHisto = SizeX (H); // Return the first size of H
// -------------------------------------------
//  Initialization of the tree Arbre
```

```
// -------------------------------------------
For ChoixNoeudInitialVal =1 to TailleHisto
    Arbre(1).PosHisto=ChoixNoeudInitialVal;
    Arbre(1).FilsFilles=0;
    Arbre(1).Predecesseurs=Arbre(1).PosHisto;
    Arbre(1).Pere=[];
    Arbre(1).Erreur=0;
    // ----------------------------------
    //  Initialization of the tree Arbre
    // ------------------------------------
    k=0;
    Sommet=[ ];
    [Sommet,Arbre,k]= ArbreConstructionOrdre (Sommet,Arbre,H,k);
    OrdreArbreNumSommet=FindBestPaths (Sommet); // Minimal error Paths
SolutionsPossiblesTaille_OrdreArbreNumSommet=Size (OrdreArbreNumSommet)
    Ordre=Sommet(OrdreArbreNumSommet).Predecesseurs;
    OrdrePossiblePereVariable(ChoixNoeudInitialVal).Ordre = Ordre;
    ErreurMinimale =Sqrt (Sommet(OrdreArbreNumSommet(1)).Erreur)/(-
1+size(H,1));
    OrdrePossiblePereVariable(ChoixNoeudInitialVal).ErreurMinimale=ErreurMini
male;
EndFor
    ErreurMinimaleAbsolue=min([OrdrePossiblePereVariable.ErreurMinimale])
    PereAbsolueDuMinError = FindBestOrderPaths (OrdrePossiblePereVariable)
For i=1 to Size (PereAbsolueDuMinError)
  SommetAbsolueDuMinError(i)=OrdrePossiblePereVariable(PereAbsolueDuMinErr
or(i)).Ordre(TailleHisto))
  OrdreOptimal(i,:)=OrdrePossiblePereVariable(PereAbsolueDuMinError(i)).Ordre;
EndFor
```

## 3. RESULTS AND DISCUSSIONS

To understand the principle of the proposed new vector order, we first apply our different algorithms to synthetic images in particular those of Figure 1 and their *nD* compact histograms presented in Table 1. Then the relevance of our approach was highlighted on the morphological vector operators gradient and Laplacian using quantitative evaluation methods relative [9] to the hybrid vector order with absolute adaptive referent. The visual relevance of the order is illustrated on color images through the gradient and Laplacian operators in Figures 2 and 3.

Let us first consider the Savoise synthetic image, this image contains exactly six (06) colors as indicated by the compact histogram (*Hc_3D*) of Table 1, also indicating the order number (N°) of the colors according to the lexicographical order. When the first constraint ***Cond1*** of our algorithm is applied to its *Hc_3D*, two (02) possible orders are produced which are **$O_1$ = (1, 2, 3, 4, 5, 6)** and **$O_2$ = (6, 5, 4, 3, 2, 1)** with a minimum mean squared error (***EQMmin***) estimate at **0.7135**. When the second constraint ***Cond2*** is applied to the previous solutions in order to have the unique solution $O_u$, we obtain **$O_u$ = O1 = (1, 2, 3, 4, 5, 6)** of maximum deviation (***EcartMax***) of value **126.3369**.

Then our algorithm is applied to Synt_Sie1 synthetic image. Through condition 1 namely Cond1, three (03) possible order solutions are generated. It is **$O_1$ = (1,2,3,6,4,5)**, **$O_2$ = (3,2,6,4,5,1)** and **$O_3$ = (6,2,3,1,4,5**) with an ***EQMmin*** value of **1.5733**. The ***Cond2*** Condition applied to the preceding solutions gives the unique solution **$O_u$ = $O_2$ = (3, 2, 6, 4, 5, 1)** whose parameter ***EcartMax*** is **257.5966**.

Finally the application of our algorithm to the Synt_Sie5ou7 synthetic image of compact histogram with seven (07) colors gives two (02) possible solutions through the ***Cond1*** constraint that are **$O_1$ = (5, 4, 6, 3, 2 , 1, 7)** and **$O_2$ = (7, 6, 3, 2, 1, 4, 5)** of error ***EQMmin*** equal to **0.7135.** The single solution is **$O_u$ = $O_1$ = (5, 4, 6, 3, 2, 1, 7),** taking the value ***EcartMax*** equal to **249.8179**.

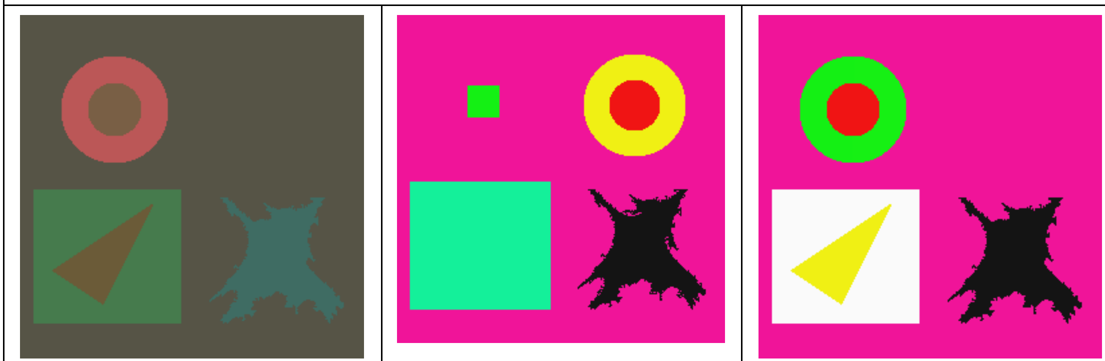
| **Synthetic images** | | |
|---|---|---|
|  | | |
| Savoise | Synt_Sie1 | Synt_Sie5ou7 |

**Figure 1:** Color synthetic images

**Table 1:** 3D compact Histograms of color synthetic images

| **Savoise** | | | | **Synt_Sie1** | | | | **Synt_Sie5ou7** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **N°** | ***R*** | ***G*** | ***B*** | **N°** | ***R*** | ***G*** | ***B*** | **N°** | ***R*** | ***G*** | ***B*** |
| **1** | *63* | *108* | *99* | **1** | *20* | *20* | *20* | **1** | *20* | *20* | *20* |
| **2** | *70* | *123* | *77* | **2** | *20* | *240* | *20* | **2** | *20* | *240* | *20* |
| **3** | *86* | *84* | *70* | **3** | *20* | *240* | *154* | **3** | *22* | *240* | *20* |
| **4** | *107* | *91* | *56* | **4** | *240* | *20* | *20* | **4** | *240* | *20* | *20* |
| **5** | *121* | *95* | *69* | **5** | *240* | *20* | *153* | **5** | *240* | *20* | *153* |
| **6** | *187* | *87* | *87* | **6** | *240* | *240* | *20* | **6** | *240* | *240* | *20* |
| | | | | | | | | **7** | *240* | *240* | *20* |

We have also calculated on different color images the vector morphological operators that are the gradient and the Laplacian resulting from this new order with minimum mean square error.

The visual results of the Gradient and Laplacian illustrated in Figure 2 on synthetic images and in Figure 3 on real images show the relevance of the proposed new vector ordering.

We also quantitatively evaluated the visual results of Gradient and Laplacian images from natural images, as shown in Tables 2 and 3 from the Vinet and Pratt measures with two metrics that are maximum (Max) and norm (Norm). These images come from the BSD300 Benchmark database, each possessing a ground truth and evaluated in [1] through the vector hybrid ordering recently developed by our research team. More the criterion value decreases, better is the quality of contours detection.

The evaluation results in Tables 2-3 on images 113044.tiff and 323016.tiff show that with Vinet's criterion, we have a better detection of contours with the Laplacian with respect to the Gradient. This is the same for the Pratt criterion. Also, we can notice that the metric *Max* gives better results than *Norm*. For the two criteria used, we notice that in some cases the proposed order gives good results than the hybrid order and vice versa. This can be related to the content of the image and the properties of the edge detection operators.

**Table 2:** Evaluation of the performances of the proposed order through the Vinet criterion

| | Image 113044.tiff | | Image 323016.tiff | |
|---|---|---|---|---|
| | Gradient | Laplacian | Gradient | Laplacian |
| Hybrid Order (Norm) | **0,1867** | 0,2980 | **0,4592** | 0,1953 |
| Proposed order (Norm) | **0,1911** | **0,2972** | 0,4757 | **0,1950** |
| Proposed order (Max) | 0,2200 | **0,0870** | **0,1305** | **0,0528** |

**Table 3:** Evaluation of the performances of the proposed order through the Pratt criterion

| | Image 113044.tiff | | Image 323016.tiff | |
|---|---|---|---|---|
| | Gradient | Laplacian | Gradient | Laplacian |
| Hybrid Order (Norm) | 0,8151 | **0,2991** | 0,5455 | **0,1956** |
| Proposed order (Norm) | **0,8083** | 0,3066 | **0,5404** | 0,1984 |
| Proposed order (Max) | **0,2229** | **0,0751** | **0,1240** | **0,0406** |

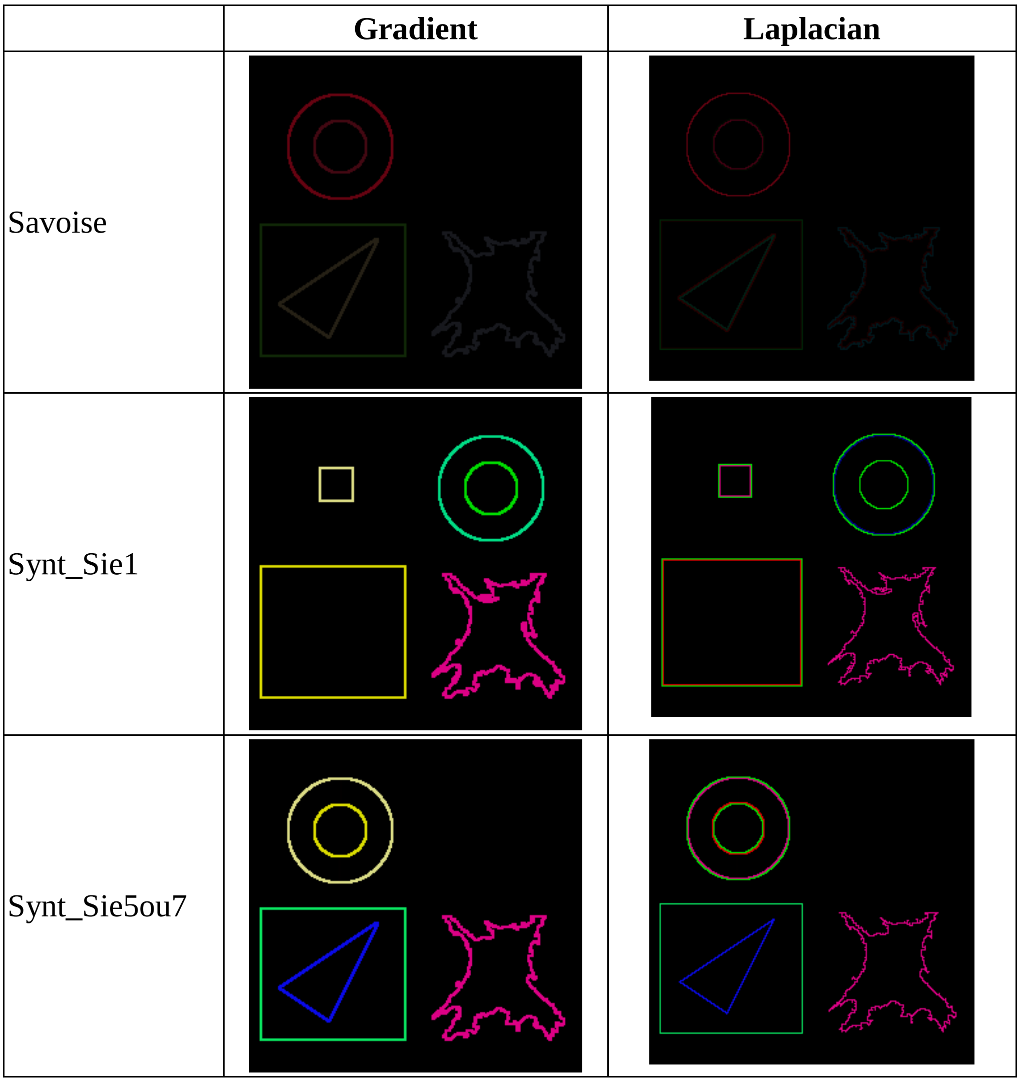



**Figure 2:** Gradient and Laplacian contour images from synthetic images

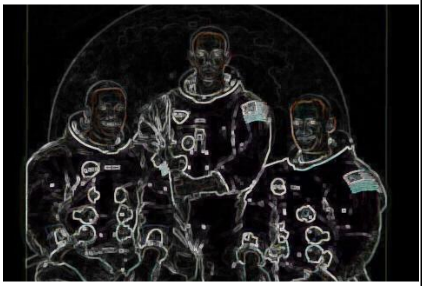
| | **Gradient** | **Laplacian** |
|---|---|---|
|  House |  |  |
|  113044 |  |  |
|  323016 |  |  |

**Figure 3:** Gradient and Laplacian contour images from real images

## 4. CONCLUSION

In this work, we have developed a new vector morphological ordering. It is original and its principle is based on the combinatorial search by n-ary tree analysis of order sequences of 'color' tuples whose mean squared error is minimal. This logic is inspired by the construction of natural numbers or real numbers.

The results of this new approach have made it possible to highlight the visual and quantitative quality of the detection of multicomponent image contours through the morphological gradient and Laplacian operators, thus showing that there is no absolute method of detection.

The choice of an additional constraint for the construction of a lattice remains subjective. A thorough reflection for future work should allow us to elucidate this concern.

Also, this order can be integrated with grayscale image processing methods for segmenting multicomponent images.

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