Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 14, Number 9 (2018), pp. 1149–1165

http://www.ripublication.com/gjpam.htm

# Mathematical Modelling of Substance Abuse by Commercial Drivers

**Joan Kavuti Kanyaa**[1]
*Department of Mathematics and Actuarial Science,*
*Catholic University of Eastern Africa,*
*Box 62157-00200, Nairobi-Kenya.*

**Shaibu Osman**
*Department of Mathematics and Actuarial Science,*
*Catholic University of Eastern Africa,*
*Box 62157-00200, Nairobi-Kenya.*

**Mary Wainaina**
*Department of Mathematics and Actuarial Science,*
*Catholic University of Eastern Africa,*
*Box 62157-00200, Nairobi-Kenya.*

**Abstract**

In this paper, we formulated a model for substance (drug) abuse that explains the dynamics of the use and abuse of certain substances that are perceived as mood changing by commercial drivers. The drug model model was analysed qualitatively and quantitatively. The threshold for the abuse of substance (drug) was determined. It was found that the drug free equilibrium point was found to be locally asymptotically stable whenever the drug abuse number is less than one and unstable otherwise. The analysis of the contribution of each parameter was performed using sensitivity analysis. The analysis revealed that an increase in the recruitment rate of commercial drivers and the rate at which commercial drivers return to the use and abuse of drugs would cause an increase in the drug abuse number. Numerical simulations was conducted to see the changes in the population dynamics of susceptible drivers, drug users and drug abusers in the population. The results showed that the contact and imitation rates has an impact on the population of commercial drivers. There are impact on interaction among non drug users and drug users in the system with time. An increase in the contact or imitation rate increases the population of drug users.

[1]Corresponding author e-mail: kavutijoan@gmail.com

**AMS subject classification:**
**Keywords:** Drug abuse, drug abuse number, sensitivity analysis, equilibrium points.

## 1. Introduction

Generally, cases of driving under the influence of stimulants and mood changing substances are directly associated with road carnage. Commercial vehicles could be deadly for both the driver and other occupants when the driver is driving under the influence of some substance. The phenomenon of drug driving are usually kept in check by the advanced and developed nations. These are usually enforced by strict drug use evaluation and monitoring mechanisms. Unfortunately, a number developing nations have a very high number of drivers who keep abusing several substances with impunity. Majority of commercial drivers in developing nations use drugs as enhancing substance or stimulants to keep awake when driving long distances and relieve them of fatigue. The highest cases of road traffic incidences and accidents deaths are mostly found in the African continent. There are laws against driving under the influence of drugs. But the effectiveness and enforcement of these laws in combating this phenomenon are questionable.

Authors in [1] conducted a study on the prevalence of alcohol in drivers who are injured in Sweden. Their findings revealed that a number of these drivers were tested positive to alcohol. Moreover, a study carried out by authors in [3] on the levels of Blood Alcohol Concentration by commercial drivers in order to establish the relationship between alcohol concentration and road accidents. Their findings revealed that there exist a relationship between Blood Alcohol Concentration and and the tendency of a driver involving in road accidents. These studies has not used mathematical modelling approach in their findings.

A study by authors in [11] revealed that cannabis (marijuana) is the common illegal substance found in impaired drivers, victims of crashed accidents and serious injured drivers. Moreover, some of other substances are opium amphetamine and cocaine. But this study intends to use a mathematical model to investigate and the various substances that are been abuse by commercial drivers in Kenya and compare the prevailing conditions. The findings by authors in [13] in the United States established that 17 percent of drivers had alcohol concentration in their blood which exceeds the legal limits in the United States.

[13] conducted a study in New Zealand and came out with the findings that about twenty one percent of drivers indicated ever driven at least once under the influence cannabis. However, authors in [2] conducted a study by interviewing some Australian night club attendees. The findings revealed that about sixty percent admitted been driven home on so many occasions by a driver under the influence of drug or driven home by themselves under the influence of Cannabis.

The work done by authors in [6], investigated the use of drugs by commercial drivers in Ghana. They employed statistical concept in determining the factors that leads to substances used and the various types of substances that are been used and abused by

these drivers. This project serves as a departure and an improvement to the existing work by relating the work in a different environment and using different mathematical concepts. A drug use and abused model is developed to compare the findings and make recommendations.

[12] published some work on smoking before driving by drivers. Their findings indicated that a significant number of drivers admitted ever smoking before sitting on a steering wheel in many instances. This is to help them boost their morals as they perceived smoking as a driving enhancing substances. Approximately twenty one percent of all the drivers been sampled admitted to ever smoking before driving. Their data analysis does not involve the use of modelling technique and the use of differential equations in coming out with their findings. They considered only smoking by these drivers and is not realistic as cigarette smoking only forms one aspect of substances that are usually used and abused by drivers. A drunk driving should be looked at in its entity by considering all the substances used by these commercial drivers.

A study by authors in [11] revealed that cannabis (marijuana) is the common illegal substance found in impaired drivers, victims of crashed accidents and serious injured drivers. Moreover, some of other substances are opium amphetamine and cocaine. This findings were supported by the work done by authors in [6]. But this study intends to use a mathematical model to investigate and the various substances that are been abuse by commercial drivers in Kenya and compare the prevailing conditions. The findings by authors in (Williams 2006) in the United States established that 17 percent of drivers had alcohol concentration in their blood which exceeds the legal limits in the United States.

Ferguson et al., (2008) conducted a study in New Zealand and came out with the findings that about twenty one percent of drivers indicated ever driven at least once under the influence cannabis. However, authors in (Degenhardt et al., 2006) conducted a study by interviewing some Australian night club attendees. The findings revealed that about sixty percent admitted been driven home on so many occasions by a driver under the influence of drug or driven home by themselves under the influence of Cannabis.

[12] conducted a study on smoking before driving by drivers. Their findings indicated that a significant number of drivers admitted ever smoking before sitting on a steering wheel in many instances. This is to help them boost their morals as they perceived smoking as a driving enhancing substances. Approximately twenty one percent of all the drivers been sampled admitted to ever smoking before driving. Their data analysis does not involve the use of mathematical modelling as well as the use of differential equations in coming out with their findings. However, the work by [9] indicated that fifteen percent of drivers usually drive under the influence of one or more drugs. Drivers who usually smoke have the tendency of causing accidents than non smokers. These drugs are capable of causing poor vision and thereby leading to poor vehicle control. They concluded that, generally, substance used by drivers largely affect driving performance and can lead to road accidents when used in multiples or combinations.

A study by [10] in Australia to investigate the relationship between the presence of marijuana in a drivers blood and accidents caused by drivers showed that the presence of marijuana in drivers has the tendency of causing. The issue at stake is not the rising

number of drug use by drivers but the associated factors which usually trigger the use of these drugs. The objective of their study is to analyse the factors associated with the use of some substances by drivers. Their emphasis were on the social factors associated with these substance use by drivers. Their work do not employ the use of mathematical models in the analysis of their results.

In a work conducted by [1] on drivers injured under the influence of alcohol in Swedish, their findings showed that approximately 38 percent of deadly injured drivers have been tested to be positive to alcohol. However, alcoholic substances are usually considered to be among the recreational beverages with intention that they are consumed in small quantities for relaxation and enjoyment during social settings and entertainment gatherings. But, when it is consumed mainly for the purposes of physical mood activation, it therefore becomes substance abuse. These substances generally lower physical responses and continuously impairs physical and mental functioning.

## 2. Drug model description and formulation

The drug model divides the commercial drivers into four compartments depending on their substance use status. The susceptible, $S(t)$ comprises of all drivers that are at risk of using any substance (drug). All commercial drivers who use substance (drug) of any form are grouped under drug users, $D(t)$, The compartment of all drivers who abuse drug of any form are classified under, $A(t)$and those who stopped using drugs either by abstinence or through rehabilitation are under the class, $R(t)$. The rate at which drivers are recruited into the susceptible class is $\Lambda$, the rate at which drivers imitate their colleagues who use any substance is $\alpha$and $\beta$ is the rate at which drivers interact in the population. $\delta$ is the natural recovery rate of drug users and $\rho$ is the rate at which commercial drivers return to the use of drugs. The natural death rate of all commercial drivers is represented by $\mu$and $\sigma$ is the rate at which all drug users abuse drugs. Where $r$ and $\theta$are the death rates of drug users and drug abusers respectively.

The total population of the drivers becomes;

$$N(t) = S(t) + D(t) + A(t) + R(t) \tag{2.1}$$

The following system of equations are obtained from the model flow diagram in figure 1;

$$\left.\begin{aligned} \frac{dS}{dt} &= \Lambda - \beta SD(1+\alpha D) - \mu S \\ \frac{dD}{dt} &= \beta SD(1+\alpha D) - (\sigma + \delta + r + \mu)\, D + \rho R \\ \frac{dR}{dt} &= \gamma A + \delta D - (\rho + \mu)\, R \\ \frac{dA}{dt} &= \sigma D - (\theta + \gamma + \mu)\, A \end{aligned}\right\} \tag{2.2}$$

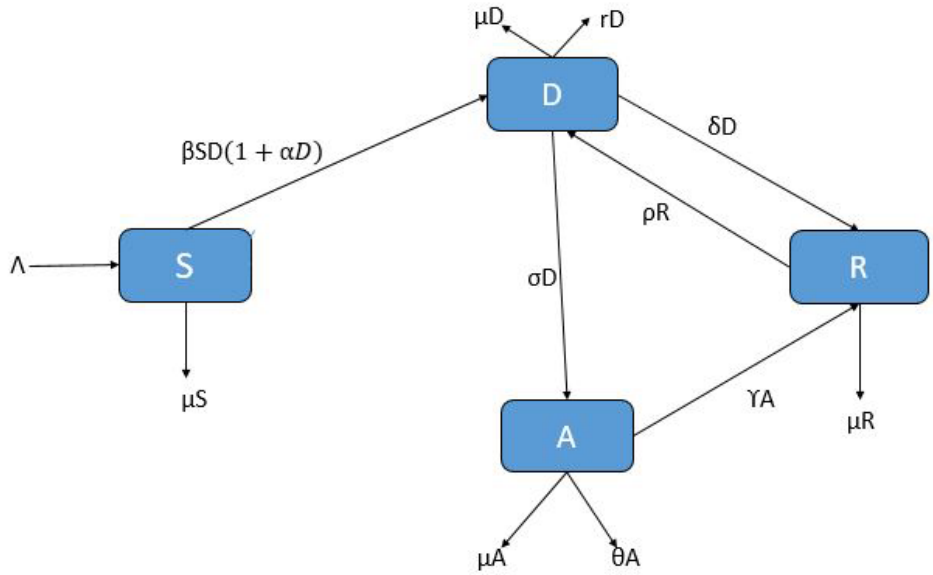


Figure 1: Flow diagram of substance use and abuse.

## 3. Drug model analysis

### 3.1. The drug invariant region

The solution of the system remains positive at any point in time if the initial values of all the variables are positive.

**Theorem 3.1.** Considering $\psi = \{(S(t), D(t), R(t), A(t)) \in R_+^4 : S(0) > 0, D(0) > 0, R(0) > 0, A(0) > 0\}$, then the solution of $\{S(t), D(t), R(t), A(t)\}$are positive for $t \geq 0$.

Boundedness refers to the region in which solutions of the model or system is uniformly bounded in the proper subset $\psi \subset R_+^4$.

Considering the total population at any time, $(t)$:

$$N(t) = S(t) + D(t) + R(t) + A(t), \tag{3.1}$$

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dD}{dt} + \frac{dR}{dt} + \frac{dA}{dt}$$

$$\left.\begin{aligned} \frac{dN}{dt} &= \Lambda - \beta SD(1+\alpha D) - \mu S + \beta SD(1+\alpha D) - (\sigma + \delta + r + \mu)\, D \\ &\quad + \rho R + \gamma A + \delta D - (\rho + \mu)\, R + \sigma D - (\theta + \gamma + \mu)\, A \end{aligned}\right\}$$

By simplification;

$$\frac{dN}{dt} = \Lambda - \mu S - (r + \mu)\, D - (\theta + \mu)\, A \tag{3.2}$$

There are no drug addiction and recovery in the absence of drug. Hence, $D = 0, R = 0, A = 0$. It becomes;

$$\frac{dN}{dt} = \Lambda - \mu S \tag{3.3}$$

If the total population $N$ is equal to the number of Susceptible $S$, it implies that $N = S$, such that;

$$\frac{dN}{dt} = \Lambda - \mu N$$

Integrating the above equation

$$\int \frac{dN}{\Lambda - \mu N} \leq \int dt$$

$$-\frac{\ln(\Lambda - \mu N)}{\mu} \leq t + B$$

$$\ln(\Lambda - \mu N) \geq -\mu(t + B)$$

$$(\Lambda - \mu N) \geq e^{-\mu(t+B)}$$

$$(\Lambda - \mu N) \geq e^{-\mu t} e^{-\mu B},$$

where $e^{-\mu B} = C$. Then, $\Lambda - \mu N \geq Ce^{-\mu t}$.

Applying the conditions at $N(0) = S(0)$, $\Lambda - \mu N(0) = C$. Therefore, $\Lambda - \mu N = (\Lambda - \mu N(0))e^{-\mu t}$

Rearranging the above equation and simplifying it;

$$N \leq \frac{\Lambda}{\mu} - \left(\frac{(\Lambda - \mu N(0)}{\mu}\right) e^{-\mu t}$$

As $t \to \infty$, the size of population $N \to \frac{\Lambda}{\mu}$ This implies that; $0 \leq N \leq \frac{\Lambda}{\mu}$ and $N(t) \leq \frac{\Lambda}{\mu}$ Therefore,

$$\psi = \left\{(S, D, R, A) \in R_+^4 : S + D + R + A \leq \frac{\Lambda}{\mu}\right\} \tag{3.4}$$

### 3.2. Drug free equilibrium

The drug free equilibrium is obtained when the system of differential equations are set to zero. At this point there are no drug users,no addicted and recovered individuals.

$$\left.\begin{array}{l} D = 0 \\ R = 0 \\ A = 0 \end{array}\right\}$$

Equating the systems of equations from the model in figure 1 to zero;

$$\Lambda - \beta SD(1 + \alpha D) - \mu S = 0 \tag{3.5}$$

$\mu S = \Lambda - \beta SD(1 + \alpha D)$ therefore;

$$S^* = \frac{\Lambda}{\mu} \tag{3.6}$$

$$(S*, 0, 0, 0) = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right)$$

The drug free equilibrium is given by the relation;

$$\left(\frac{\Lambda}{\mu}, 0, 0, 0\right) \tag{3.7}$$

### 3.3. Stability of the drug free equilibrium

## 4. Drug abuse number

Considering the system of equations from the model in figure 1, using the approach in [8] to obtain the threshold value for substance abuse.

$$\left.\begin{aligned} \frac{dS}{dt} &= \Lambda - \beta SD(1 + \alpha D) - \mu S \\ \frac{dD}{dt} &= \beta SD(1 + \alpha D) - (\sigma + \delta + r + \mu)\, D + \rho R \\ \frac{dR}{dt} &= \gamma A + \delta D - (\rho + \mu)\, R \\ \frac{dA}{dt} &= \sigma D - (\theta + \gamma + \mu)\, A \end{aligned}\right\}$$

The jacobian matrix of the system of differential equations of the model in figure 1 becomes;

$$J\,(S*, D*, R*, A*) = \begin{pmatrix} \frac{\partial S}{\partial S} & \frac{\partial S}{\partial D} & \frac{\partial S}{\partial R} & \frac{\partial S}{\partial A} \\ \frac{\partial D}{\partial S} & \frac{\partial D}{\partial D} & \frac{\partial D}{\partial R} & \frac{\partial D}{\partial A} \\ \frac{\partial R}{\partial S} & \frac{\partial R}{\partial D} & \frac{\partial R}{\partial R} & \frac{\partial R}{\partial A} \\ \frac{\partial A}{\partial S} & \frac{\partial A}{\partial D} & \frac{\partial A}{\partial R} & \frac{\partial A}{\partial A} \end{pmatrix} \tag{4.1}$$

$$J = \begin{pmatrix} -\beta D(1 + \alpha D) - \mu & -\beta S(1 + \alpha D) - \beta \alpha SD & 0 & 0 \\ \beta D(1 + \alpha D) & \beta S(1 + \alpha D) + \beta \alpha SD - (\mu + r + \delta + \sigma) & 0 & \rho \\ 0 & \sigma & -(\mu + \gamma) & 0 \\ 0 & \delta & \gamma & -(\mu + \gamma + \theta) \end{pmatrix}$$

The jacobian matrix at the drug free equilibrium point, $\left(\frac{\Lambda}{\mu}, 0, 0, 0\right)$;

$$J\left(\frac{\Lambda}{\mu}, 0, 0, 0\right) = \begin{pmatrix} -\mu & -\frac{\beta\Lambda}{\mu} & 0 & 0 \\ 0 & \frac{\beta\Lambda}{\mu} - (\mu + r + \delta + \sigma) & 0 & \rho \\ 0 & \sigma & -(\mu + \gamma) & 0 \\ 0 & \delta & \gamma & -(\mu + \gamma + \theta) \end{pmatrix} \tag{4.2}$$

Finding the determinant of the Jacobian matrix at drug free equilibrium;

$$\begin{vmatrix} -\mu - \lambda & -\frac{\beta\Lambda}{\mu} & 0 & 0 \\ 0 & \left(\frac{\beta\Lambda}{\mu} - (\sigma + \delta + r + \mu)\right) - \lambda & 0 & \rho \\ 0 & \sigma & -(\gamma + \mu) - \lambda & 0 \\ 0 & \delta & 0 & -(\mu + \gamma + \theta) - \lambda \end{vmatrix} = 0$$

Where $\lambda$ is the eigenvalue.

By using the approach in [7], the threshold for drug abuse is given by:

$$\frac{(\beta\Lambda\,(\mu + \rho)\,(\mu + \gamma + \theta)\,)}{\mu\left[(\mu + \gamma + \theta)\,(\mu(\mu + r + \delta + \sigma) + \rho(\mu + r)) + \mu\rho\sigma\right]}.$$

Hence, the threshold for substance abuse, $\Re_A$ is given by:

$$\Re_A = \frac{(\beta\Lambda\,(\mu + \rho)\,(\mu + \gamma + \theta)\,)}{\mu\left[(\mu + \gamma + \theta)\,(\mu(\mu + r + \delta + \sigma) + \rho(\mu + r)) + \mu\rho\sigma\right]} \tag{4.3}$$

## 5. Drug endemic equilibrium

This is the point where substance use exist within the susceptible. All the compartments in the model in this case will be considered since there are those prone to getting introduced to the use of substances (drugs) within the set of these drivers. All the four equations obtained from model developed in figure 1 are equated to zero;

$$\frac{dS}{dt} = \Lambda - \beta SD(1 + \alpha D) - \mu S = 0$$

$$S(\beta D(1 + \alpha D) + \mu) = \Lambda$$

$$S* = \frac{\Lambda}{\beta D^*(1 + \alpha D^*) + \mu} \tag{5.1}$$

$$\frac{dD}{dt} = \beta SD(1+\alpha D) - (\sigma+\delta+r+\mu)\,D + \rho R = 0$$
$$(\sigma+\delta+r+\mu)\,D - \beta SD(1+\alpha D) = \rho R$$
$$R* = \frac{(\sigma+\delta+r+\mu)\,D^* - \beta S^* D*(1+\alpha D^*)}{\rho} \tag{5.2}$$
$$\frac{dR}{dt} = \gamma A + \delta D - (\rho+\mu)\,R = 0$$
$$\delta D = (\rho+\mu)\,R - \gamma A$$
$$D* = \frac{(\rho+\mu)\,R^* - \gamma A^*}{\delta} \tag{5.3}$$
$$\frac{dA}{dt} = \sigma D - (\theta+\gamma+\mu)\,A = 0$$
$$(\theta+\gamma+\mu)\,A = \sigma D$$
$$A* = \frac{\sigma D^*}{\theta+\gamma+\mu} \tag{5.4}$$

Therefore, the drug endemic equilibrium is given as;

$$DEE = (S*, D*, A*, R*)$$
$$\left(\frac{\Lambda}{\beta D^*(1+\alpha D^*)+\mu}, \frac{(\rho+\mu)\,R^* - \gamma A^*}{\delta}, \frac{(\sigma+\delta+r+\mu)\,D^* - \beta S^* D*(1+\alpha D^*)}{\rho}, \frac{\sigma D^*}{\theta+\gamma+\mu}\right) \tag{5.5}$$

### 5.1. Stability of drug endemic equilibrium

**Theorem 5.1.** If $R_0 > 1$, then the drug endemic equilibrium is globally asymptotically stable.

*Proof.* By considering a Lyapunov function defined by;

$$L(S^*, D^*, R^*, A^*) = \left(S - S^* - S^* \ln\left(\frac{S^*}{S}\right)\right) + \left(D - D^* - D^* \ln\left(\frac{D^*}{D}\right)\right) + \left(R - R^* - R^* \ln\left(\frac{R^*}{R}\right)\right) + \left(A - A* - A^* \ln\left(\frac{A^*}{A}\right)\right)$$

By computing the derivative of the $L$ along the solutions of the system of equations directly;

$$\frac{dL}{dt} = \left(\frac{S-S^*}{S}\right)\frac{dS}{dt} + \left(\frac{D-D^*}{D}\right)\frac{dD}{dt} + \left(\frac{R-R^*}{R}\right)\frac{dR}{dt} + \left(\frac{A-A^*}{A}\right)\frac{dA}{dt} \tag{5.6}$$

By substitution;

$$\left.\begin{aligned}\frac{dL}{dt} &= \left(\frac{S-S^*}{S}\right)[\Lambda-\beta SD(1+\alpha D)-\mu S]\\ &+\left(\frac{D-D^*}{D}\right)[\beta SD(1+\alpha D)-(\sigma+\delta+r+\mu)\,D+\rho R]\\ &+\left(\frac{R-R^*}{R}\right)\left[\gamma A+\delta D-(\rho+\mu)\,R\right]\\ &+\left(\frac{A-A^*}{A}\right)\left[\sigma D-(\theta+\gamma+\mu)\,A\right]\end{aligned}\right\}$$

By expansion;

$$\left.\begin{aligned}\frac{dL}{dt} &= \Lambda-\beta SD(1+\alpha D)-\mu S-\frac{\Lambda S^*}{S}+\beta S^*D(1+\alpha D)+\mu S^*\\ &+\beta SD(1+\alpha D)-(\sigma+\delta+r+\mu)\,D+\rho R\\ &-\beta SD^*(1+\alpha D)+(\sigma+\delta+r+\mu)\,D*\\ &-\frac{\rho RD^*}{D}+\gamma A+\delta D-(\rho+\mu)\,R\\ &-\frac{\gamma AR^*}{R}-\frac{\delta DR^*}{R}+(\rho+\mu)\,R^*+\sigma D-(\theta+\gamma+\mu)\,A\\ &-\frac{\sigma DA^*}{A}+(\theta+\gamma+\mu)\,A^*\end{aligned}\right\}$$

from;

$$\frac{dL}{dt}=M-N \tag{5.7}$$

where $M$ are the positive terms and $N$ are the negative terms;

$$M=\Lambda+\beta S^*D(1+\alpha D)+\mu S^*+(\sigma+\delta+r+\mu)\,D*+(\rho+\mu)\,R^*+(\theta+\gamma+\mu)\,A^*$$

$$N=\mu S+\frac{\Lambda S^*}{S}+(r+\mu)\,D+\beta SD^*(1+\alpha D)+\frac{\rho RD^*}{D}+(\rho+\mu)\,R+\frac{(\gamma A+\delta D)R^*}{R}+\theta A-\frac{\sigma DA^*}{A}$$

If $M<N$, then $\frac{dL}{dt}\leq 0$. $\frac{dL}{dt}=0$, if and only if $S=S^*$,$D=D^*$, $R=R^*$, and $A=A^*$.

The largest compact invariant set in

$$\left\{(S,D,R,A)\in\psi:\frac{dL}{dt}=0\right\}$$

is a singleton $E^*$,where $E^*$ is the endemic equilibrium.

Therefore, the endemic equilibrium is globally asymptotically stable in the invariant $\psi$if $M<N$ by [4, 8].

## 6. Parameter contribution to drug abuse number

Parameter contribution analysis are usually meant to determine the effectiveness of the reproduction number. Usually, parameter values and model assumptions can be influenced as a result of changes and errors in the process of formulation and computing the reproduction number. Therefore sensitivity analysis are conducted to determine those changes and sources of error and their impact to the model [9]. It is a technique that is mostly used by modelers whose objectives are to assist and support decision makers by providing with them informed decisions base on analysis of dynamic of the model. However, authors in [9], presented similar analogue that explains that models and parameters usually are uncertain. This analysis are usually required to determine on how sensitive they are to parameter values and determine which is the most sensitive parameter of the reproduction number. Hence, the uncertainty effects of the parameter is solely the cause sensitivity analysis.

The sensitivity analysis of a model parameter is normally evaluated by relating each parameter to the reproduction number, $(R_A)$. The sensitivity of a variable $m$ s given by the relation;

$$S_m^{R_0} = \frac{\partial R_0}{\partial_m} * \frac{m}{R_0} \tag{6.1}$$

By using the approach in [7, 5], the sensitivity indices of all the parameters in table 1were obtained.

Table 1: Sensitivity indices of parameter values.

| Parameter | Sensitivity index (-ve/+ve) |
|---|---|
| $\mu$ | -ve |
| $\beta$ | +ve |
| $\gamma$ | -ve |
| $\delta$ | -ve |
| $\sigma$ | -ve |
| $\rho$ | +ve |
| $\Lambda$ | +ve |
| $\theta$ | -ve |
| $r$ | -ve |

The sign of the sensitivity index in table 1 indicates the contribution of each parameter to the drug abuse number. The essence of this project is to help reduce the use and abuse of drugs by commercial drivers. Therefore, the contribution of each parameter is an important factor in determining which parameter contributes to the spread of drug use among commercial drivers. Figure 1 shows a significant contribution of the recruitment of individuals to the susceptible compartment. The implication is that, as more drivers become susceptible to the use of drugs, the use and abuse of drugs would increase and

this would lead to the continuous use of drugs by commercial drivers. However, the sign of the values of the natural recovery rate of drug users and the recovery rate of drivers who abuse drugs are indications that, they help reduce the value of the drug abuse number. This would help curb the use of drugs by commercial drivers.

## 7. Numerical solutions

In this section, we perform qualitative analysis of the system of differential equations of the model in figure 1 using Range-Kutta fourth order scheme [8]. The duration of the spread of the use of drug thorough interaction between drivers at risk of using drugs and the drug users or abusers was taken to be five years. The period was assumed to be the duration at which imitation can have impact of the susceptible population.

### 7.1. Susceptible drug users

Figure 2 shows what happens in the susceptible population in the system with time. As the number of those who continue to use and abuse substances increases with time, so does the population size of susceptible drivers who are at risk of getting into the use of substances increases with time. Since this happenings take a lot of time to realised, the period of the simulation is taken as five years. The susceptible population decreases steadily with time. This explains why the graph in figure 5 increases with as time goes on. Moreover, it can also be vied as the recruitment of drivers into the susceptible population is lower as compared to the rate at which drivers interact with those who use substances as the are quickly involved in the use of drugs through imitation.

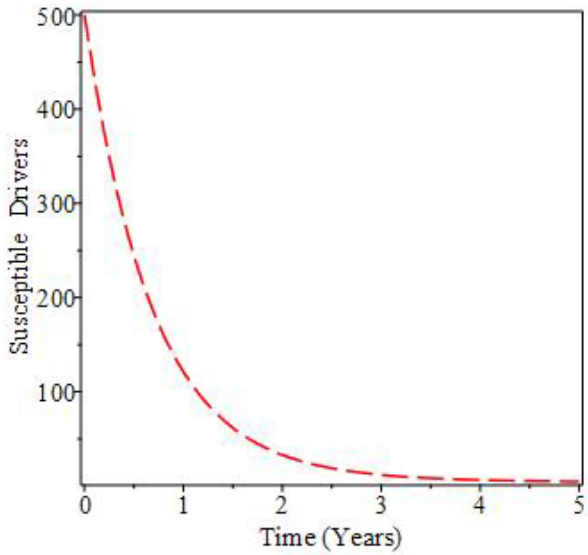


Figure 2: Population of susceptible drivers who are at risk of drug use.

### 7.2. Drug users

The rate at which commercial drivers imitate those who use drugs increase exponentially. Figure 5 explains this happenings. There is an increase in the number of drivers who use substance. At a point in time the number of those who use drugs reduces. The graph in figure 3 shows why the population decreases. As the number of recovered (drivers who avoid or stop the use of drugs) begins to increase at a point in time. A decrease in the number of drug users corresponds to an increase in the number of drivers avoiding the use of drugs.

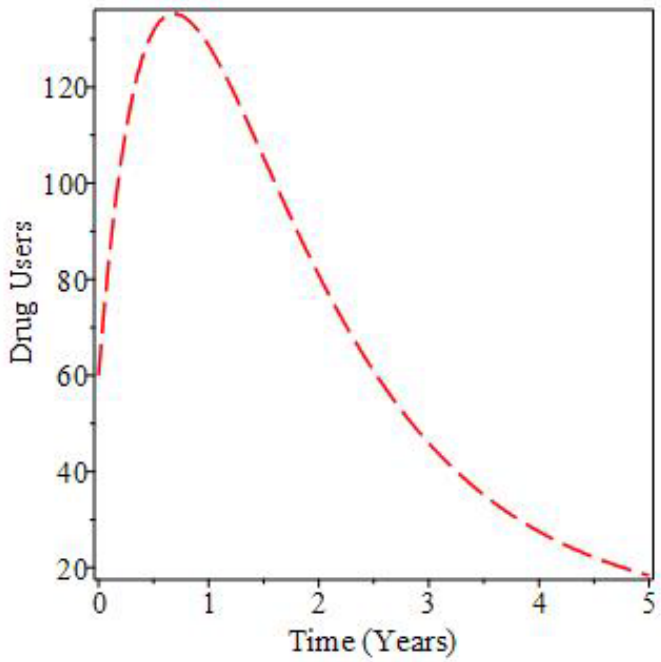


Figure 3: Population of drivers who are involved in substance use.

### 7.3. Effects of contact and imitation rates

Naturally, the interaction between commercial drivers as well as imitation would obvious have an impact on some commercial drivers as far as the use of substances are concern. An analysis of the imitation rate and contact rate among drivers is analysed. This to confirm whether or not this interaction has an effect on the drivers. Figure 4 shows the effects of the contact and imitation rates among the commercial drivers. The analysis confirmed the impact of interaction among non drug users and drug users in the system with time. An increase in the contact or imitation rate increases the number or population of drug users.

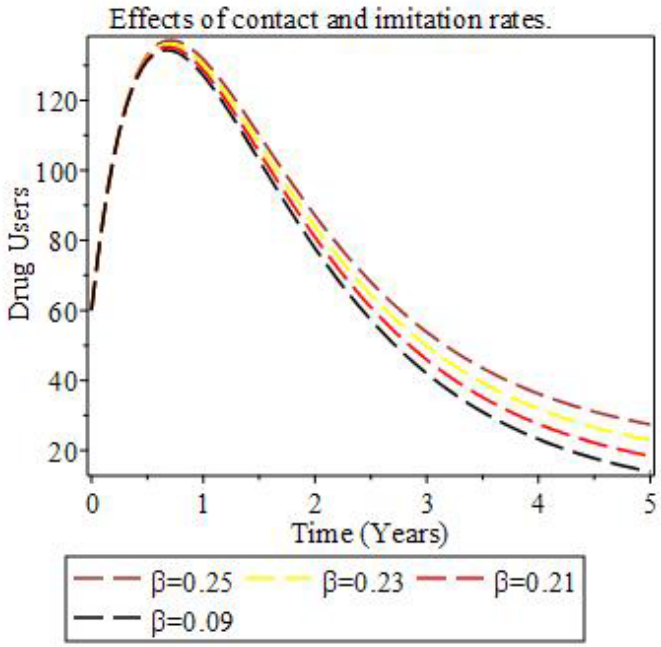


Figure 4: Effects of contact and imitation rates among drivers.

### 7.4. Drug abusers

Figure 5 shows the population dynamics of drug abusers by commercial drivers. The population of drivers who used substances while driving continue to increase with time.

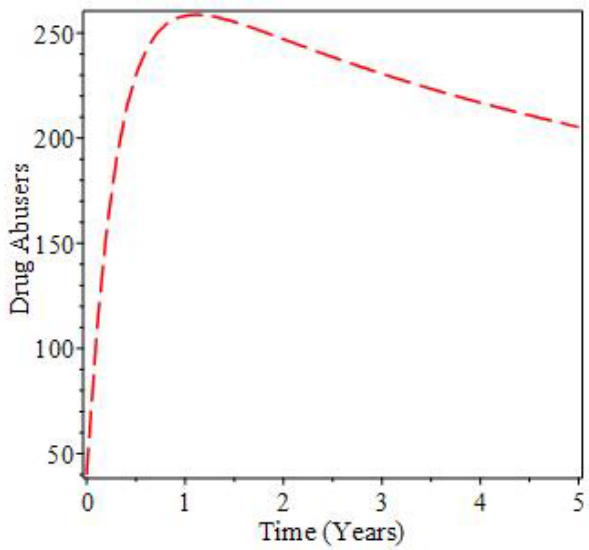


Figure 5: Population of drivers who abuse drugs.

As the population of drivers at risk of using substances either through imitation or contact with those who used substance reduces, the population of drivers who use or abuse substances would continue to increase as indicated in figure 5.

### 7.5. Recovered drug users

Figure 6 shows the population of drivers who have either decided to stop the use of substances or avoid substances. This could either be through education or self realisation of the effects of the use of these substances. The number of drivers who avoid the use of substances reduces completely at a point in time. This could either be through intensive education and campaign against the use of drugs by the transport ministry. Moreover, the transport unit of the police unit sometimes intensify regular checks on drunk driving. This could also be one of the reasons for the reduction or avoidance of the use of substances by these drivers.

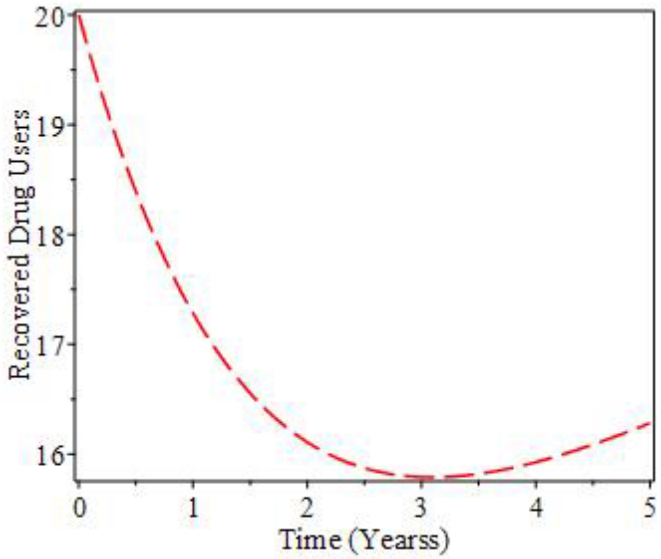


Figure 6: Population of drivers who have avoided the use of drugs.

## 8. Conclusion

The drug model was qualitatively and quantitatively analysed to give an account of drug spread among commercial drivers. The threshold number for the use and abuse of all the substances has been computed. The use and abuse of substances would continue to spread among commercial drivers if the drug abuse number is less than one and the abuse of drugs would die out of the system if the abuse number is greater than one. Sensitivity analysis was therefore carried out to determine the most sensitive parameter that would contribute to the abuse number in order to make a decision on the factors that leads to these spread. Figure 1 shows a significant contribution of the recruitment

of individuals to the susceptible compartment. The implication is that, as more drivers become susceptible to the use of drugs, the use and abuse of drugs would increase and this would lead to the continuous use of drugs by commercial drivers. However, the sign of the values of the natural recovery rate of drug users and the recovery rate of drivers who abuse drugs are indications that, they help reduce the value of the drug abuse number. This would help curb the use of drugs by commercial drivers.

Numerical simulations of the drug model was performed to explain the dynamics of the all commercial drivers who are at risk of drug use as a result of imitations. Figure 2 shows that the susceptible population in the system decreases with time. As the number of those who continue to use and abuse substances increases with time, so does the population size of susceptible drivers who are at risk of getting into the use of substances increases with time. Moreover, figure 5 shows an increase in the number of drivers who use substance. As the number of recovered (drivers who avoid or stop the use of drugs) begins to increase at a point in time. A decrease in the number of drug users corresponds to an increase in the number of drivers avoiding the use of drugs. An analysis of the imitation rate and contact rate among drivers was analysed. This confirms whether imitation and interaction among drivers have an effect on the drivers with respect to the use and abuse of drugs. Figure 4 shows the effects of the contact and imitation rates among the commercial drivers. It revealed that the impact of interaction among non drug users and drug users in the system with time. An increase in the contact or imitation rate increases the number or population of drug users.

## References

[1] Kristin Ahlm, Ulf Björnstig, and Mats Öström. Alcohol and drugs in fatally and non-fatally injured motor vehicle drivers in northern sweden. *Accident Analysis & Prevention*, 41(1): 129–136, 2009.

[2] Louisa Degenhardt, Paul Dillon, Cameron Duff, and Joanne Ross. Driving, drug use behaviour and risk perceptions of nightclub attendees in victoria, australia. *International Journal of Drug Policy*, 17 (1):41–46, 2006.

[3] Hallvard Gjerde, Per T Normann, Bjørg S Pettersen, Terje Assum, Magne Aldrin, Unni Johansen, Lena Kristoffersen, Elisabeth L Øiestad, Asbjørg S Christophersen, and Jørg Mørland. Prevalence of alcohol and drugs among norwegian motor vehicle drivers: a roadside survey. *Accident Analysis & Prevention*, 40(5): 1765–1772, 2008.

[4] JP LaSalle. The stability of dynamical systems, regional conference series in appl. *Math., SIAM, Philadelphia*, 1976.

[5] Shaibu Osman and Oluwole Daniel Makind. A mathematical model for co-infection of listeriosis and anthrax diseases. *International Journal of Mathematics and Mathematical Sciences*, 2018.

[6] Shaibu Osman, AY Omari-Sasu, and RK Boadi. Logit model for the determinants of drug driving in ghana. *International Journal of Statistics and Applications*, 6(6):339–346, 2016.

[7] Shaibu Osman, Oluwole Daniel Makinde, and David Mwangi Theuri. Mathematical modelling of transmission dynamics of anthrax in human and animal population. *Mathematical Theory and Modelling*, 2018a.

[8] Shaibu Osman, Oluwole Daniel Makinde, and David Mwangi Theuri. Stability analysis and modelling of listeriosis dynamics in human and animal populations. *Global Journal of Pure and Applied Mathematics*, 14 (1):115–137, 2018b.

[9] Renske Penning, Janet L Veldstra, Anne P Daamen, Berend Olivier, and Joris C Verster. Drugs of abuse, driving and traffic safety. *Current drug abuse reviews*, 3(1):23–32, 2010.

[10] Johannes G Ramaekers, Günter Berghaus, Margriet van Laar, and Olaf H Drummer. Dose related risk of motor vehicle crashes after cannabis use. *Drug and alcohol dependence*, 73(2): 109–119, 2004.

[11] Carl A Soderstrom, Patricia C Dischinger, Timothy J Kerns, Joseph A Kufera, Kimberly A Mitchell, and Thomas M Scalea. Epidemic increases in cocaine and opiate use by trauma center patients: documentation with a large clinical toxicology database. *Journal of Trauma and Acute Care Surgery*, 51 (3):557–564, 2001.

[12] Chris Urmson, Joshua Anhalt, Drew Bagnell, Christopher Baker, Robert Bittner, MN Clark, John Dolan, Dave Duggins, Tugrul Galatali, Chris Geyer, et al. Autonomous driving in urban environments: Boss and the urban challenge. *Journal of Field Robotics*, 25(8):425–466, 2008.

[13] Allan F Williams. Alcohol-impaired driving and its consequences in the united states: the past 25 years. *Journal of safety research*, 37(2): 123–138, 2006.