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Applied of $\left(\frac{G'}{G}\right)$– expansion method of coupled nonlinear system of (2+1) dimensional Schrodinger equations

ZHAN Yiheng, CUI Zejian
College of Mathematic and Information, China West Normal University, Nanchong Sichuan 637002, China.

Abstract

In this article, new extension of the generalized and improved homogeneous balance method is proposed for constructing new structure of rich class of exact travelling wave solutions of nonlinear evolution equations by using the $\left(\frac{G'}{G}\right)$– expansion method. In order to demonstrate the innovativeness and motivation of the proposed method, we implement it to the coupled nonlinear system of Schrodinger equations. It indicates that the approach furnishes a powerful mathematical tool for working out the non-linear evolution equations in mathematical physics.

Key word: nonlinear system equations; $\left(\frac{G'}{G}\right)$ – expansion method; Schrodinger equations; travelling wave solution; homogeneous balance method

1. INTRODUCTION

The investigation of exact solutions of nonlinear equations occupies an important position in the research of nonlinear physical phenomena. Pursuing explicit solutions of nonlinear evolution equations by using a variety of methods is the main objectives for many researchers, and considerable effective methods to construct exact solutions of nonlinear evolution equations have been put forward and progressed such as same works [1– 17], the homogeneous balance method [18–19], the F-expansion method [20] and same works [21–26]. One of the most important methods is $\left(\frac{G'}{G}\right)-$ expansion method. The main ideas of the proposed method are that the travelling wave solutions of a nonlinear evolution equation can be conveyed by a polynomial in $\left(\frac{G'}{G}\right)$ where $G=G(\xi)$ satisfies a second order LODE (see[21]), the degree of the polynomial can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms which appears in a given nonlinear evolution equation, and the coefficients of the polynomial can be acquired by solving a set of algebraic equations when we adopt the proposed method. It can be perceived that more travelling wave solutions of many nonlinear evolution equations could be procured by adopting the $\left(\frac{G'}{G}\right)-$ expansion method. It offers the generalized solitary solutions and periodic solutions, as well. We are able to recover some known solutions via existing approaches with the help of the generalized solitary solutions.

In this paper we broaden the homogeneous balance method to a class of nonlinear evolution equations with imaginary number and modulus. We regard the coupled (2 + 1) - dimensional nonlinear system of Schrodinger equations as

$$\begin{cases} iE_t - E_{xx} + E_{yy} + |E|^2 E - 2NE = 0 \\ N_{xx} - N_{yy} - \left(|E|^2\right)_{xx} = 0 \end{cases} \tag{1.1}$$

where $E(x,y,t)$ and $N(x,y,t)$ are complex-valued functions. The type of nonlinear partial differential equation systems given by (1.1) produce main effect in atomic physics and the functions $E(x,y,t)$ and $E(x,y,t)$ enjoy various physical meanings in different branches of physics. For example, the fluid dynamics and plasma physics are well employed in the systems. In the context of water waves, $E(x,y,t)$ is the amplitude of a surface wave packet while $N(x,y,t)$ is the velocity potential of the mean flow interacting with the surface waves. But $E(x,y,t)$ is the envelope of the wave packet and $N(x,y,t)$ is the induced mean flow [27–29] in the hydrodynamic context. Furthermore, Eq.(1.1) are relevant to a number of different physical contexts, describing slow modulation effects of the complex amlitude $N(x,y,t)$, attribute to a small nonlinearity, on a monochromatic wave in a dispersive medium.

2. DESCRIPTION OF THE $\left(\frac{G'}{G}\right)$-EXPANSION METHOD

In this part we describe the $\left(\frac{G'}{G}\right)$-expansion method for finding travelling wave solution of nonlinear evolution equations. Suppose that a nonlinear equation, say in two independent variables x and t, is given by

$$Q(u,u_t,u_x,u_{tt},u_{xt},\cdots)=0 \qquad (2.1)$$

Where $u=u(x,t)$ is an unknown function, Q is a polynomial in $u=u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of the ()-expansion method.

Step1.combining the independent variables x and t into one variable $\xi=x-ct$, we suppose that

$$u(x,t)=u(\xi),\quad \xi=x-ct \qquad (2.2)$$

The travelling wave variable (2.2) permits us reducing Eq.(2.1) to an ODE for $u(x,t)=u(\xi)$

$$P\left(u,-cu',u',c^2u'',-cu'',u'',\cdots\right)=0 \qquad (2.3)$$

Step2. Suppose that the solution of ODE(2.3) can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as follows

$$u(\xi)=\sum_{i=0}^{k}a_i\left(\frac{G'}{G}\right)^i \qquad (2.4)$$

Where $G=G(\xi)$ satisfies the second order LODE in the form

$$G''+\lambda G'+\mu G=0 \qquad (2.5)$$

In which $a_i,\cdots,\lambda,\mu$ are constants to be determined later, $a_i\neq 0$,the unwritten part in

(2.4) is also a polynomial in $\left(\frac{G'}{G}\right)$, the positive integer k can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE(2.3).

Step3. By substituting(2.4) into Eq(2.3) and using second order LODE(2.5),collecting all terms with the same order of $\left(\frac{G'}{G}\right)$ together, the left-hand side of Eq.(2.3) is converted into another polynomial in $\left(\frac{G'}{G}\right)$.Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $a_i,\cdots,v,\lambda$ and μ.

Step4. Assuming that the constant $a_i,\cdots,v,\lambda$ and μ can be obtained by solving the algebraic equations in step3. Since the general solutions of the second order LODE(2.5) have been well known for us ,then substituting $a_i,\cdots,v,\lambda$ and the general solution of Eq.(2.5) into (2.4) we have more travelling wave solutions of the nonlinear evolution equation(2.1).

In the subsequent sections we will illustrate the proposed method in detail with various nonlinear evolution equations in mathematical physics.

3. EXACT TRAVELLING WAVE SOLUTION FOR COUPLED NONLINEAR SYSTEM OF SCHRODINGER EQUATIONS

We start with the celebrated coupled nonlinear system of Schrodinger equations in the form

$$\xi=k\left(x+ly+2\left(\alpha-\beta l\right)t\right),\quad \eta=\alpha x+\beta y+\gamma t \qquad (3.1)$$

$$E\left(x,y,t\right)=u\left(\xi\right)\exp\left(i\eta\right),\quad N\left(x,y,t\right)=v\left(\xi\right) \qquad (3.2)$$

Where k; l; α and β are constants to be determined. Note that ξ and η are travelling wave variables, not necessarily in the same direction. That is, ξ and η are independent linear functions of x; y and t. Then u and v are assumed to be rational functions of $\exp\left(\xi\right)$ When u is positive real, u is the modulus of the complex function E, and N is the argument. The modulus and argument are travelling waves but the two waves may be in different directions.

From (1.1), we may obtain the system of ordinary differential equations

$$k^2l^2u'' + \left(\alpha^2 - \beta^2 - \gamma\right)u + u^3 - 2uv = 0 \qquad (3.3)$$

$$\left(1 - l^2\right)v'' - \left(2uu'\right)' = 0 \qquad (3.4)$$

Integrating (3.4) with the respect to ξ and setting the constants of integration equal to zero yield

$$v = \frac{u^2}{1 - l^2} \qquad (3.5)$$

Substituting(3.5)into (3.3), we obtain

$$k^2l^2u'' + \left(\alpha^2 - \beta^2 - \gamma\right)u - \frac{1 + l^2}{1 - l^2}u^3 = 0 \qquad (3.6)$$

Suppose that the solution of ODE(3.6) can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as follows:

$$u(\xi) = \sum_{i=0}^{k} a_i \left(\frac{G'}{G}\right)^i \qquad (3.7)$$

Where $G = G(\xi)$ satisfies the second order LODE in the form $G'' + \lambda G' + \mu G = 0$

It's easy to obtain that $k = 1$ by using the homogeneous balance between u^3 and u'' in Eq.(3.6)which can write as $3k = k + 2$. so we can write (3.7) as

$$u(\xi) = \sum_{i=0}^{1} a_i \left(\frac{G'}{G}\right)^i \quad a_1 \neq 0 \qquad (3.8)$$

And therefore

$$u'(\xi)=a_1\left[\left(\frac{G'}{G}\right)^2-\lambda\left(\frac{G'}{G}\right)-\mu\right] \qquad (3.9)$$

$$u''(\xi)=2a_1\left(\frac{G'}{G}\right)^3-3a_1\lambda\left(\frac{G'}{G}\right)^2+a_1\left(\lambda^2-2\mu\right)\left(\frac{G'}{G}\right)+a_1\lambda\mu \qquad (3.10)$$

$$u^3(\xi)=a_0{}^3+3a_0{}^2a_1\left(\frac{G'}{G}\right)+3a_0a_1{}^2\left(\frac{G'}{G}\right)^2+a_1{}^3\left(\frac{G'}{G}\right)^3 \qquad 3.11)$$

By substituting (3.8)-(3.11)into Eq.（3.6）and collecting all terms with the same power of $\left(\frac{G'}{G}\right)$. We can write as:

$$k^2l^2\left[2a_1\left(\frac{G'}{G}\right)^3-3a_1\lambda\left(\frac{G'}{G}\right)^2+a_1\left(\lambda^2-2\mu\right)\left(\frac{G'}{G}\right)+a_1\lambda\mu\right]+\left(\alpha^2-\beta^2-\gamma\right)\left(a_0+a_1\left(\frac{G'}{G}\right)\right)$$
$$-\frac{1+l^2}{1-l^2}\left(a_0{}^3+3a_0{}^2a_1\left(\frac{G'}{G}\right)+3a_0a_1{}^2\left(\frac{G'}{G}\right)^2+a_1\left(\frac{G'}{G}\right)^3\right)=0$$

Equating each coefficient of this polynomial to zero, yields a set of simultaneous algebraic equations for $a_0,a_1,k,l,\alpha,\beta,\gamma,\lambda$ and μ as the follows:

$$2a_1k^2l^2-\frac{1+l^2}{1-l^2}a_1{}^3=0 \qquad (3.12)$$

$$3a_1\lambda k^2l^2+3a_0a_1{}^2=0 \qquad (3.13)$$

$$a_1k^2l^2\left(\lambda^2-2\mu\right)+a_1\left(\alpha^2-\beta^2-\gamma\right)-3\frac{1+l^2}{1-l^2}a_0a_1=0 \qquad (3.14)$$

$$a_1k^2l^2\lambda\mu+\left(\alpha^2-\beta^2-\gamma\right)a_0-\frac{1+l^2}{1-l^2}a_0{}^3=0 \qquad (3.15)$$

Solving the algebraic equations above, yields

$$a_1 = \pm kl\sqrt{2\cdot\frac{1-l^2}{1+l^2}} \tag{3.16}$$

$$a_0 = \pm \lambda kl\sqrt{\frac{\left(1+l^2\right)}{2\left(1-l^2\right)}} \tag{3.17}$$

$$\mu = \frac{\lambda^2}{2} + \frac{\alpha^2-\beta^2-\gamma}{2k^2l^2} - \frac{3\left(1+l^2\right)\lambda^2}{4\left(1-l^2\right)^2} \tag{3.18}$$

$k, l, \alpha, \beta, \gamma, \lambda$ are arbitrary constant.

By using (3.16)-(3.18) into (3.8),the travelling wave solution can be written as

$$u(\xi) = \pm kl\sqrt{2\cdot\frac{1-l^2}{1+l^2}}\left(\frac{G'}{G}\right) \pm \lambda kl\sqrt{\frac{\left(1+l^2\right)}{2\left(1-l^2\right)}} \tag{3.19}$$

$$v(\xi) = \frac{2k^2l^2}{1+l^2}\left(\frac{G'}{G}\right)^2 \pm \frac{\lambda k^2l^2}{1-l^2}\left(\frac{G'}{G}\right) + 2\lambda^2k^2l^2\,\frac{1+l^2}{\left(1-l^2\right)^2} \tag{3.20}$$

Where $\xi = k\left(x+ly+2\left(\alpha-\beta l\right)t\right)$, $\eta = \alpha x+\beta y+\gamma t$.

Case1: $\lambda^2-4\mu>0$ 时

$$\left(\frac{G'}{G}\right) = -\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\cdot\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + C_2\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + C_2\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}$$

$$\left(\frac{G'}{G}\right)^2 = \frac{\lambda^2-4\mu}{4}\cdot\frac{\left(C_1\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + C_2\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)^2}{\left(C_1\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + C_2\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)^2} - \frac{\lambda}{2}\sqrt{\lambda^2-4\mu}\,\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + C_2\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + C_2\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi} + \frac{\lambda^2}{4}$$

$$u(\xi) = \pm kl\sqrt{2\cdot\frac{1-l^2}{1+l^2}}\left(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\cdot\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + C_2\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + C_2\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}\right) \pm \lambda kl\sqrt{\frac{\left(1+l^2\right)}{2\left(1-l^2\right)}}$$

$$v(\xi)=\frac{2k^2l^2}{1+l^2}\left(=\frac{\lambda^2-4\mu}{4}\cdot\frac{\left(C_1\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)^2}{\left(C_1\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)^2}-\frac{\lambda}{2}\sqrt{\lambda^2-4\mu}\,\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}+\frac{\lambda^2}{4}\right)$$

$$\pm\frac{\lambda k^2l^2}{1-l^2}\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\cdot\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}\right)+2\lambda^2k^2l^2\,\frac{1+l^2}{\left(1-l^2\right)^2}$$

$$E(x,y,t)=\left(\pm kl\sqrt{2\cdot\frac{1-l^2}{1+l^2}}\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\cdot\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}\right)\pm\lambda kl\sqrt{\frac{\left(1+l^2\right)}{2\left(1-l^2\right)}}\right)\exp(i\eta)$$

$$N(x,y,t)=\frac{2k^2l^2}{1+l^2}\left(\frac{\lambda^2-4\mu}{4}\cdot\frac{\left(C_1\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)^2}{\left(C_1\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)^2}-\frac{\lambda}{2}\sqrt{\lambda^2-4\mu}\,\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}+\frac{\lambda^2}{4}\right)$$

$$\pm\frac{\lambda k^2l^2}{1-l^2}\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\cdot\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}\right)+2\lambda^2k^2l^2\,\frac{1+l^2}{\left(1-l^2\right)^2}$$

Case2: $\lambda^2-4\mu<0$

$$\left(\frac{G'}{G}\right)=-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\cdot\frac{C_1\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{C_1\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}$$

$$\left(\frac{G'}{G}\right)^2=\frac{4\mu-\lambda^2}{4}\cdot\frac{\left(C_1\cos\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\sin\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)^2}{\left(C_1\sin\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\cos\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)^2}-\frac{\lambda}{2}\sqrt{4\mu-\lambda^2}\,\frac{C_1\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{C_1\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}+\frac{\lambda^2}{4}$$

$$u(\xi)=\pm kl\sqrt{2\cdot\frac{1-l^2}{1+l^2}}\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\cdot\frac{C_1\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{C_1\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}\right)\pm\lambda kl\sqrt{\frac{\left(1+l^2\right)}{2\left(1-l^2\right)}}$$

$$v(\xi)=\frac{2k^2l^2}{1+l^2}\left(\left(\frac{G'}{G}\right)^2=\frac{4\mu-\lambda^2}{4}\cdot\frac{\left(C_1\cos\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\sin\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)^2}{\left(C_1\sin\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\cos\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)^2}-\frac{\lambda}{2}\sqrt{4\mu-\lambda^2}\,\frac{C_1\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{C_1\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}+\frac{\lambda^2}{4}\right)$$

$$\pm\frac{\lambda k^2l^2}{1-l^2}\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\cdot\frac{C_1\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{C_1\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}\right)+2\lambda^2k^2l^2\frac{1+l^2}{\left(1-l^2\right)^2}$$

$$E(x,y,t)=\left(\pm kl\sqrt{2\cdot\frac{1-l^2}{1+l^2}}\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\cdot\frac{C_1\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{C_1\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}\right)\pm\lambda kl\sqrt{\frac{\left(1+l^2\right)}{2\left(1-l^2\right)}}\right)\exp(i\eta)$$

$$N(x,y,t)=\frac{2k^2l^2}{1+l^2}\left(\left(\frac{G'}{G}\right)^2=\frac{4\mu-\lambda^2}{4}\cdot\frac{\left(C_1\cos\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\sin\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)^2}{\left(C_1\sin\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+C_2\cos\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)^2}-\frac{\lambda}{2}\sqrt{4\mu-\lambda^2}\,\frac{C_1\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{C_1\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}+\frac{\lambda^2}{4}\right)$$

$$\pm\frac{\lambda k^2l^2}{1-l^2}\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\cdot\frac{C_1\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{C_1\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+C_2\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}\right)+2\lambda^2k^2l^2\frac{1+l^2}{\left(1-l^2\right)^2}$$

Case3: $\lambda^2-4\mu=0$ 时

$$\left(\frac{G'}{G}\right)=-\frac{\lambda}{2}+\frac{C_2}{C_1+C_2\xi}$$

$$\left(\frac{G'}{G}\right)^2=\frac{{C_2}^2}{\left(C_1+C_2\xi\right)^2}-\frac{\lambda C_2}{C_1+C_2\xi}+\frac{\lambda^2}{4}$$

$$u(\xi)=\pm kl\sqrt{2\cdot\frac{1-l^2}{1+l^2}}\left(\frac{C_2}{C_1+C_2\xi}-\frac{\lambda}{2}\right)\pm\lambda kl\sqrt{\frac{\left(1+l^2\right)}{2\left(1-l^2\right)}}$$

$$v(\xi)=\frac{2k^2l^2}{1+l^2}\left(\frac{{C_2}^2}{\left(C_1+C_2\xi\right)^2}-\frac{\lambda C_2}{C_1+C_2\xi}+\frac{\lambda^2}{4}\right)\pm\frac{\lambda k^2l^2}{1-l^2}\left(\frac{C_2}{C_1+C_2\xi}-\frac{\lambda}{2}\right)+2\lambda^2k^2l^2\frac{1+l^2}{\left(1-l^2\right)^2}$$

$$E(x,y,t)=\left(\pm kl\sqrt{2\cdot\frac{1-l^2}{1+l^2}}\left(\frac{C_2}{C_1+C_2\xi}-\frac{\lambda}{2}\right)\pm\lambda kl\sqrt{\frac{\left(1+l^2\right)}{2\left(1-l^2\right)}}\right)\exp(i\eta)$$

$$N(x,y,t)=\frac{2k^2l^2}{1+l^2}\left(\frac{C_2{}^2}{(C_1+C_2\xi)^2}-\frac{\lambda C_2}{C_1+C_2\xi}+\frac{\lambda^2}{4}\right)\pm\frac{\lambda k^2l^2}{1-l^2}\left(\frac{C_2}{C_1+C_2\xi}-\frac{\lambda}{2}\right)+2\lambda^2k^2l^2\frac{1+l^2}{\left(1-l^2\right)^2}$$

4. CONCLUSIONS

In Case 1:while $C_1=0$ the exact solution can be in the terms of solution which is solved by the algorithm of homogeneous balance method Case 1 and Case 2[30].

In Case 1:while $C_1=0$ the exact solution can be in the terms of solution which is solved by the algorithm of homogeneous balance method Case 4[30].

In Case 1:while $C_1=0$ the exact solution can be in the terms of solution which is solved by the algorithm of homogeneous balance method Case 3[30].

We can see that, the exact solution solved by $\left(\frac{G'}{G}\right)-$ expansion method which contains the exact solution solved by the algorithm of homogeneous balance method.

Exact solutions can serve as a basis for perfecting and testing computer algebra software packages for solving NLEEs. It is of great significance that many equations of physics, chemistry, and biology contain empirical parameters or empirical functions. Exact solutions allow researchers to design and run experiments, by creating appropriate natural conditions, to determine these parameters or functions .

In this paper, the homogeneous balance method is employed along with a computerized symbolic computation to obtain the single and combined generalized solutions of coupled (2 +1)-dimensional nonlinear system of Schrodinger equations. The results show that the homogeneous balance method is a powerful and promising new method to solve nonlinear evolution equations. On comparing this method with the other methods, we see that the homogeneous balance method is much simpler than these methods. Also we deduce that the homogeneous balance method is direct and effective and can be applied to many other nonlinear evolution equations.

5.REFERENCE

[1] Kamruzzaman Khan, M. Ali Akbar, Exact and solitary wave solutions for the Tzitzeica-Dodd-Bullough and the modified KdV-Zakharov-Kuznetsov equations using the modified simple equation method, Ain Shams Eng. J. 4 (4) (2013) 903–909.

[2] Kamruzzaman Khan, M. Ali Akbar, Traveling wave solutions of the nonlinear Drinfel'd-Sokolov-Wilson equation and modified Benjamin-Bona-Mahony equations, J. Egypt. Math. Soc. 21 (3) (2013) 233–240.

[3] Kamruzzaman Khan, M. Ali Akbar, Traveling wave solutions of the (2 + 1)-dimensional Zoomeron equation and the Burgers equations via the MSE

method and the Exp-function method, Ain Shams Eng. J. 5 (1) (2014) 247–256.

[4] Kamruzzaman Khan, M. Ali Akbar, Exact solutions of the (2+1)-dimensional cubic Klein-Gordon equation and the (3+1)-dimensional Zakharov-Kuznetsov equation using the modified simple equation method, J. Assoc. Arab Univ. Basic Appl. Sci. 15 (1) (2014) 74–81.

[5] Hasibun Naher, Farah Aini Abdullah, M. Ali Akbar, New traveling wave solutions of the higher dimensional nonlinear partial differential equation by the exp-function method, J. Appl. Math. 2012 (2012).

[6] Hasibun Naher, Farah Aini Abdullah, M. Ali Akbar, Generalized and Improved (G0/G)-expansion method for (3+ 1)-dimensional modified KdV-Zakharov-Kuznetsev Equation, PLoS ONE 8 (5) (2013) 1–7.

[7] K. Khan, M.A. Akbar, Traveling wave solutions of nonlinearevolution equations via the enhanced (G0/G)-expansion method, J. Egypt. Math. Soc. 22 (2) (2014) 220–226.

[8] E.G. Fan, Extended tanh-method and its applications to nonlinear equations, Phy. Lett. A 277 (2000) 212–218.

[9] G. Adomian, Solving Frontier Problems of Physics: The Decomposition Method, Kluwer Academic, Boston, MA, 1994.

[10] E.J. Parkes, B.R. Duffy, An automated tanh-function method for finding solitary wave solutions to non-linear evolution equations, Comput. Phys. Commun. 98 (1996) 288–300.

[11] A. Neirameh, Topological soliton solutions to the coupled Schrodinger-Boussinesq equation by the SEM, Optik – Int. J. Light Electron Opt. 126 (23) (2015) 4179–4183.

[12] A. Neirameh, Binary simplest equation method to the generalized Sinh-Gordon equation, Optik – Int. J. Light Electron Opt. 126 (23) (2015) 4763–4770.

[13] N. Taghizadeh, A. Neirameh, New complex solutions for some special nonlinear partial differential systems, Comput. Math. Appl. 62 (4) (2011) 2037–2044.

[14] A. Neirameh, New analytical solutions for the coupled nonlinear Maccari's system, Alexandria Eng. J. 55 (3) (2016) 2839–2847.

[15] A. Neirameh, Exact solutions of the generalized Sinh-Gordon equation, Comput. Math. Math. Phys. 56 (7) (July 2016) 1336– 1342.

[16] Thabet Abdeljawad, On conformable fractional calculus, J. Comput. Appl. Math. 279 (1) (2015) 57–66.

[17] J.F. Zhang, Homogeneous balance method and chaotic and fractal solutions for the Nizhnik–Novikov–Veselov equation, Phys. Lett. A 313 (2003) 401–

407.

[18] J.F. Zhang, Homogeneous balance method and chaotic and fractal solutions for the Nizhnik– Novikov-Veselov equation, Phys. Lett. A 313 (2003) 401–407.

[19] X. Zhao, L. Wang, W. Sun, The repeated homogeneous balance method and its applications to nonlinear partial differential equations, Chaos Solitons Fractals 28 (2006) 448–453.

[20] S. Zhang, New exact solutions of the KdV–Burgers–Kuramoto equation, Phys. Lett. A 358 (2006) 414–420.

[21] M.L. Wang, X.Z. Li, J. Zhang, The (G0/G)-expansion method and traveling wave solutions of nonlinear evolution equations in mathematical physics, Phys. Lett. A 372 (2008) 417–423.

[22] Abdul-Majid Wazwaz, The tanh method for travelling wave solutions of nonlinear equations, Appl. Math. Comput. 154 (3)(2004) 713–723.¨

[23] Ahmet Bekir, Ozkan Gu¨ner, Topological (dark) soliton solutions for the Camassa-Holm type equations, Ocean Eng. 74 (2013) 276–279.

[24] Ahmet Bekir, Ahmet Boz, Exact solutions for nonlinear evolution equations using exp-function method, Phys. Lett. A 372 (2008) 1619–1625.¨

[25] Ahmet Bekir, Ozkan Gu¨ner, Bright and dark soliton solutions of the (3 + 1)-dimensional generalized Kadomtsev-Petviashvili equation and generalized Benjamin equation, Pramana-J. Phys. 81 (2) (2013) 203–214.

[26] Ozkan Gu¨ner, Bright and dark soliton solutions of thegeneralized Zakharov-Kuznetsov-Benjamin-Bona-Mahony nonlinear evolution equation, in: Proceedings of the Romanian Academy, Series A, vol. 16(3), 2015, pp. 422–429.

[27] M.J. Ablowitz, H. Segur, Solitons and the Inverse Scattering Transform, SIAM, Philadelphia, 1981.

[28] A. Davey, K. Stewartson, On three-dimensional packets of surface waves, Proc. Roy. Soc. Lond. Ser. A 338 (1974) 101–110.

[29] K. Nishinari, K. Abe, J. Satsuma, A new type of soliton behavior in a two dimensional plasma system, J. Phys. Soc. Jpn. 62 (1993) 2021–2029.

[30] A. Neirameh, M.Eslami , S. Shokooh, New solution algorithm of coupled nonlinear system of Schrodinger equations , Alexandria Engineering Journal(2018) 57.247-253