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Codes over $D_n \times C_t$ and $D_n \times D_n$

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Abstract

In this paper we study the codes over semi-simple group algebra FG where the group G = $D_n \times C_t$ is the direct product of dihedral group D_n of order 2n and cyclic group C_t of order t in terms of their generating idempotents.Minimum distance and dimension of the group codes completely described when G = $D_3 \times C_3$ and $D_4 \times C_2$. We also compute the generating idempotents in the group algebra of $D_n \times D_n$ when n is odd.

Keywords: Group algebra, group codes, idempotents.

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1. INTRODUCTION

Let G be finite group and F be a field such that character of F does not divide order of the group G. The group algebra of G over F is the set of all linear combinations $\sum_{g \in G} \alpha_g g$ where $\alpha_g \in$F . The addition and scalar multiplication are defined as follows .For any u = $\sum_{g \in G} \alpha_g g$and v = $\sum_{g \in G} \beta_g g$

u+v = $\sum_{g \in G}(\alpha_g + \beta_g)g$ and λ∈F ,λu =$\sum_{g \in G}(\lambda \alpha_g)g$

And multiplication defined by

$(\sum_{g \in G} \alpha_g g)(\sum_{h \in G} \beta_h h) = \sum_{g,h \in G}(\alpha_g \beta_h)gh$

The *weight* of any element u = $\sum_{g \in G} \alpha_g g$ is equal to number of non zero components in u and is denoted by wt(u).

(Ref.[1]) Let E = $\{e_i\}_{i=1}^{s}$ be the set of all idempotents of FG . If I is any ideal generated by $\{e_j\}_{j=1}^{t} \subseteq$ E and μ=E\\$\{e_j\}_{j=1}^{t}$. then I={ u∈FG : ue = 0 $\forall e \in \mu$ }. Now we denote I by I_μ. Minimum distance of the group code I_μ is defined by d= d(I_μ)= min{wt(u):0≠u∈I_μ}. The length n of a group code I_μ is defined to be order of G. If

I_μ has dimension k and minimum distance d then I_μ is called an (n,k,d) group code. Ref.(Remark, [1]) We know FG= $(\bigoplus_{ei\in E_L} FG_{e_i}) \oplus (\bigoplus_{ej\in E_N} FG_{e_j})$ where $E_L$is the set consists of all linear idempotents in FG and E_N is the set consists of all nonlinear idempotents in FG and FGe_i is minimal ideal generated by e_i. Now E= $E_L \cup E_N$. Note that if $e_i \in E_L$, then $\dim(FG_{ei}) = 1$; and if $e_j \in E_N$, then $\dim(FG_{ej})=m^2$ where m= $\chi_k(1)$ where χ_k is the k^{th} character.

Therefore if $\mu= \mu_L \cup \mu_N$ where $\mu_L \subseteq E_L$ and $\mu_N \subseteq E_N$, then

$\dim(I_\mu) = \dim(FG) - |\mu_L| \dim(FG_{ei}) - |\mu_N| \dim(FG_{ej})$ where $\dim(FG)= |G|$

The product of two characters of G is again a character of G Ref.(cor.19.7,[4]).Let χ be the character of V and ψ be the character of W then the character of V× W is $\chi\times\psi$, where $(\chi\times\psi)(g,h) = \chi(g)\psi(h)$ $(g\in V , h\in W)$Ref.(Pro.19.6,[4])

Let $\chi_1, . . . , \chi_s$ be the distinct irreducible characters of G and let $\psi_1, . . . , \psi_t$ be the distinct irreducible characters of H. then G×H has precisely st distinct irreducible characters, and these are $\chi_i \times \psi_j$ ($1\le i \le s$, $1 \le j \le t$) Ref.(Theo.19.18,[4]).

If $g_1, . . . ,g_s$ are representatives of the conjugacy classes of G and $h_1, . . . , h_t$ are representatives of the conjugacy classes of H, then the elements (g_i, h_j) ($1\le i \le s$,$1 \le j \le t$) are representatives of the conjugacy classes of G×H . In particular , G×H has precisely stconjugacy classes. By (theo.15.3,[4])G×H has exactly st irreducible characters, so the irreducible characters $\chi_i \times \psi_j$ ($1\le i \le s$, $1 \le j \le t$) must be all the irreducible characters of G×H.

2.IDEMPOTENT ELEMENTS IN THE GROUP ALGEBRA $F(D_n\times C_t)$

Consider the group $D_n = \langle a,b : a^n=1, b^2=1, ab=ba^{-1}\rangle$ and $C_t =\langle g\rangle = \{1,g ,g^2, …….g^{t-1}\}$

Define:$x_1 = 1$, $x_2 = a$, $x_3 = a^2$, ….. , $x_n= a^{n-1}$, $x_{n+1} = b$, $x_{n+2} = ab$, ….., $x_{2n} = a^{n-1}b$

$y_1 = 1$, $y_2 = g$, $y_3 = g^2$,…., $y_t = g^{t-1}$

$G = D_n\times C_t = \{(x_i,y_j)\}$ $1\le i\le 2n$, $1\le j\le t$.

2.1. Explicit expression for the t(n+3)/2 irreducible idempotentsin the group algebra $F(D_n\times C_t)$ when n is an odd number.

Character table of $D_n\times C_t$.

For $1\le j\le n-1/2$ and k= 1,2,…,t-1and $1\le r \le n-1/2$

	(1,1)	(1,g)	$(1,g^2)$	…	$(1,g^{t-1})$	$(a^r,1)$	(a^r,g)	….	(a^r,g^{t-1})	(b,1)	(b,g)	…	$(b\ g^{t-1})$
χ_1	1	1	1	…	1	1	1	. . .	1	1	1	. . .	1
χ_2	1	1	1	…	1	1	1	. . .	1	−1	−1	. . .	−1
φ_k	1	α^k	$(\alpha^k)^2$	…	$(\alpha^k)^{t-1}$	1	α^k	. . .	$(\alpha^k)^{t-1}$	1	α^k	. . .	$(\alpha^k)^{t-1}$

ψ_k	1	α^k	$(\alpha^k)^2$ …	$(\alpha^k)^{t-1}$	1	α^k. . .	$(\alpha^k)^{t-1}$	-1	$-\alpha^k$ …$-(\alpha^k)^{t-1}$
ζ_j	2	2	2 …	2	$2(\varepsilon^{jr}+\varepsilon^{-jr})$	. . .	$2(\varepsilon^{jr}+\varepsilon^{-jr})$	0	0 . . . 0
γ_{jk}	2	$2\alpha^k$	$2(\alpha^k)^2$ …	$2(\alpha^k)^{t-1}$	$2(\varepsilon^{jr}+\varepsilon^{-jr})$	. . .	$2(\varepsilon^{jr}+\varepsilon^{-jr})\,(\alpha^k)^{t-1}$	0	0 . . . 0

where $\varepsilon = e^{\frac{2\pi i}{n}}$ and α is t^{th} root of unity.

We define

$\overline{c_1} = \sum_{i=1}^{n}(x_{i}, y_1)$

$\overline{c_2} = \sum_{i=1}^{n}(x_{i}, y_2)$

$\overline{c_3} = \sum_{i=1}^{n}(x_{i}, y_3)$, ... $,\overline{c_t} = \sum_{i=1}^{n}(x_{i}, y_t)$

$\overline{c_{t+1}} = \sum_{i=n+1}^{2n}(x_{i}, y_1)$

$\overline{c_{t+2}} = \sum_{i=n+1}^{2n}(x_{i}, y_2)$ . . . .$\overline{c_{2t}} = \sum_{i=n+1}^{2n}(x_{i}, y_t)$

And the idempotents of FG are given by $e = \frac{1}{|G|}\sum_{g\in G}\chi(g^{-1})g$ ref.(pro.14.10, [4])

Linear idempotents are

$e_1 = \frac{1}{2nt}\sum_{\substack{1\le i\le 2n\\ 1\le j\le t}}(x_i, y_j)$

$e_2 = \frac{1}{2nt}\left(\sum_{\substack{1\le i\le n\\ 1\le j\le t}}(x_i, y_j) - \sum_{\substack{n+1\le i\le 2n\\ 1\le j\le t}}(x_i, y_j)\right)$

$\theta_k = \frac{1}{2nt}[\overline{c_1} + (\alpha^k)^{t-1}\overline{c_2} + \dots + \alpha^k\overline{c_t} + \overline{c_{t+1}} + (\alpha^k)^{t-1}\overline{c_{t+2}} + \dots\dots + \alpha^k\overline{c_{2t}}\ ]$

$\eta_k = \frac{1}{2nt}[\overline{c_1} + (\alpha^k)^{t-1}\overline{c_2} + \dots + \alpha^k\overline{c_t} - \overline{c_{t+1}} - (\alpha^k)^{t-1}\overline{c_{t+2}} - \dots\dots - \alpha^k\overline{c_{2t}}\ ]$

where $k=1,2,\dots,t-1$.

and the non linear idempotents are given by

$\vartheta_j = \frac{1}{nt}[\ 2\sum_{j=1}^{t}(x_1, y_j) + 2\sum_{r=1}^{n-1}\cos\frac{2\pi jr}{n}\{(a^r,1) + (a^r,g) + \dots + (a^r,g^{t-1})\}]$

$\delta_{jk} = \frac{1}{nt}\ [2(1,1) + 2(\alpha^k)^{t-1}(1,g) + \dots + 2\alpha^k(1,g^{t-1}) + 2\sum_{r=1}^{n-1}\cos\frac{2\pi jr}{n}(a^r,1) + 2(\alpha^k)^{t-1}\sum_{r=1}^{n-1}\cos\frac{2\pi jr}{n}(a^r,g) + \dots + 2\alpha^k\sum_{r=1}^{n-1}\cos\frac{2\pi jr}{n}(a^r, g^{t-1})\ ]$

where $1\le j\le n-1/2$ and $k=1,2,\dots,t-1$.

2.2. Explicit expression for the t(n+6)/2 irreducible idempotents in the group algebra F($D_n \times C_t$) when n is an even number.

In this case n is even say n=2m.

Then Idempotents of FG are given by [ref(Theo. 19.18 and 18.3 and 14.10, [4])]

$e_1 = \frac{1}{2nt}\sum_{g\in Dn\times Ct} g$

$e_2 = \frac{1}{2nt}\,[\sum_{\substack{1\le i\le n\\1\le j\le t}}(x_i, y_j) - \sum_{\substack{n+1\le i\le 2n\\1\le j\le t}}(x_i, y_j)]$

$e_3 = \frac{1}{2nt}[\sum_{1\le j\le t}(x_1, y_j) + \sum_{\substack{1\le r\le n-1\\1\le j\le t}}(-1)^r(x_{r+1}, y_j) + \sum_{\substack{1\le i\le n-1\\i=odd\\1\le j\le t}}(x_{n+i}, y_j) - \sum_{\substack{2\le i\le n\\i=even\\1\le j\le t}}(x_{n+i}, y_j)\,]$

$e_4 = \frac{1}{2nt}[\sum_{1\le j\le t}(x_1, y_j) + \sum_{\substack{1\le r\le n-1\\1\le j\le t}}(-1)^r(x_{r+1}, y_j) - \sum_{\substack{1\le i\le n-1\\i=odd\\1\le j\le t}}(x_{n+i}, y_j) + \sum_{\substack{2\le i\le n\\i=even\\1\le j\le t}}(x_{n+i}, y_j)\,]$

$\theta_k = \frac{1}{2nt}[\overline{c_1} + (\alpha^k)^{t-1}\overline{c_2} + \ldots.\ \alpha^k\overline{c_t} + \overline{c_{t+1}} + (\alpha^k)^{t-1}\overline{c_{t+2}} + \ldots\ \ldots. + \alpha^k\overline{c_{2t}}\,]$

$\eta_k = \frac{1}{2nt}[\overline{c_1} + (\alpha^k)^{t-1}\overline{c_2} + \ldots.\ \alpha^k\overline{c_t} - \overline{c_{t+1}} - (\alpha^k)^{t-1}\overline{c_{t+2}} - \ldots\ \ldots. - \alpha^k\overline{c_{2t}}\,]$

$\zeta_k = \frac{1}{2nt}[\,(x_1,y_1) + (\alpha^k)^{t-1}(x_1,y_2) + \ldots.. + \alpha^k(x_1,y_t) + (\alpha^k)^{t-1}\sum_{r=1}^{n-1}(-1)^r(x_{r+1}, y_1) + \ldots. + \alpha^k \sum_{r=1}^{n-1}(-1)^r(x_{r+1}, y_t) + \sum_{\substack{1\le i\le n-1\\i=odd}}(x_{n+i}, y_1) - \sum_{\substack{2\le i\le n\\i=even}}(x_{n+i}, y_1) + (\alpha^k)^{t-1}\{\sum_{\substack{1\le i\le n-1\\i=odd}}(x_{n+i}, y_2) - \sum_{\substack{2\le i\le n\\i=even}}(x_{n+i}, y_2)\} + \ldots\ldots + \alpha^k\{\sum_{\substack{1\le i\le n-1\\i=odd}}(x_{n+i}, y_t) - \sum_{\substack{2\le i\le n\\i=even}}(x_{n+i}, y_t)\}\,]$

$\delta_k = \frac{1}{2nt}[(x_1,y_1) + (\alpha^k)^{t-1}(x_1,y_2) + \ldots.. + \alpha^k(x_1,y_t) + (\alpha^k)^{t-1}\sum_{r=1}^{n-1}(-1)^r(x_{r+1}, y_1) + \ldots. + \alpha^k \sum_{r=1}^{n-1}(-1)^r(x_{r+1}, y_t) - \sum_{\substack{1\le i\le n-1\\i=odd}}(x_{n+i}, y_1) + \sum_{\substack{2\le i\le n\\i=even}}(x_{n+i}, y_1) + (\alpha^k)^{t-1}\{\sum_{\substack{2\le i\le n\\i=even}}(x_{n+i}, y_2) - \sum_{\substack{1\le i\le n-1\\i=odd}}(x_{n+i}, y_2)\,\} + \ldots\ldots + \alpha^k\{\sum_{\substack{2\le i\le n\\i=even}}(x_{n+i}, y_t) + \sum_{\substack{1\le i\le n-1\\i=odd}}(x_{n+i}, y_t)\}]$

where k =1,2,…,t−1

and the non linear idempotents are given by

$\vartheta_j = \frac{1}{nt}[\,2\sum_{1\le j\le t}(x_1, y_j) + 2(-1)^j\sum_{1\le l\le t}(x_{m+1}, y_l) + 2\sum_{r=1}^{n-1}\cos\frac{2\pi jr}{n}\{(a^r,1) + (a^r,g) + \ldots.. + (a^r,g^{t-1})\}\,]$

$\gamma_{jk} = \frac{1}{nt}\,[2(1,1) + 2(\alpha^k)^{t-1}(1,g) + \ldots.. + 2\alpha^k(1,g^{t-1}) + 2(-1)^j\{a^m,1) + (\alpha^k)^{t-1}(a^m,g) + \ldots. + \alpha^k(a^m,g^{t-1})\} + 2\sum_{r=1}^{n-1}\cos\frac{2\pi jr}{n}(a^r,1) + 2(\alpha^k)^{t-1}\sum_{r=1}^{n-1}\cos\frac{2\pi jr}{n}(a^r,g) + \ldots.. + 2\alpha^k\sum_{r=1}^{n-1}\cos\frac{2\pi jr}{n}(a^r, g^{t-1})\,]$

where $1\le j \le n\text{-}1/2$ and k=1,2,…,t-1.

3.MINIMUM DISTANCE AND DIMENSION OF CODES

In this section we find the minimum distance of the codes I_μ generated by the idempotents which is described above.

Case 1.When n is odd number

(i) $d(I_{\{e_i\}})$=2 and $dim(I_{\{e_i\}})$ = 2nt−1 for i=1,2

(ii) $d(I_{\{\theta_k\}})$=2 and $dim(I_{\{\theta_k\}})$= 2nt−1

(iii) $d(I_{\{\eta_k\}})$=2 and $dim(I_{\{\eta_k\}})$= 2nt−1

(iv) $d(I_{\{e_1,e_2\}})$=2 and $dim(I_{\{e_1,e_2\}})$=2nt−2

(v) $d(I_{\{e_i,\theta_k\}})$=2and $dim(I_{\{e_i,\theta_k\}})$=2nt−2 for i=1,2

(vi) $d(I_{\{e_i,\eta_k\}})$=2 and $dim(I_{\{e_i,\theta_k\}})$=2nt−2 for i=1,2

(vii) $d(I_{\{\theta_k,\eta_k\}})$=2 and $dim(I_{\{\theta_k,\eta_k\}})$=2nt−2

(viii) $d(I_{\{e_1,e_2,\theta_k,\eta_k\}})$=2 and $dim(I_{\{e_1,e_2,\theta_k,\eta_k\}})$=2nt−4

Let u = $\sum_{i=1}^{n} \lambda_i(x_i,y_1) + \sum_{i=1}^{n} \lambda_{n+i}(x_i,y_2) + \sum_{i=1}^{n} \lambda_{2n+i}(x_i,y_3) + \ldots + \sum_{i=1}^{n} \lambda_{nt+i}(x_{n+i},y_1) + \ldots + \sum_{i=1}^{n} \lambda_{(2t-1)n+i}(x_{n+i},y_t)$ be any element of F($D_n \times C_t$) then

$ue_1 = (\sum_{i=1}^{2nt} \lambda_i)\, e_1$(3.1)

$ue_2 = (\sum_{i=1}^{nt} \lambda_i - \sum_{i=nt+1}^{2nt} \lambda_i)\, e_2$ (3.2)

$u\theta_k = (\sum_{i=1}^{n} \lambda_i + \alpha^k \sum_{i=n+1}^{2n} \lambda_i + (\alpha^k)^2 \sum_{i=2n+1}^{3n} \lambda_i + \ldots + (\alpha^k)^{n-1} \sum_{i=(t-1)n+1}^{nt} \lambda_i + \sum_{i=nt+1}^{n(t+1)} \lambda_i + \alpha^k \sum_{i=(t+1)n+1}^{(t+2)n} \lambda_i + \ldots + (\alpha^k)^{n-1} \sum_{i=(2t-1)n+1}^{2nt} \lambda_i)\theta_k$ (3.3)

$u\eta_k = (\sum_{i=1}^{n} \lambda_i + \alpha^k \sum_{i=n+1}^{2n} \lambda_i + (\alpha^k)^2 \sum_{i=2n+1}^{3n} \lambda_i + \ldots + (\alpha^k)^{n-1} \sum_{i=(t-1)n+1}^{nt} \lambda_i - \sum_{i=nt+1}^{n(t+1)} \lambda_i - \alpha^k \sum_{i=(t+1)n+1}^{(t+2)n} \lambda_i - \ldots - (\alpha^k)^{n-1} \sum_{i=(2t-1)n+1}^{2nt} \lambda_i)\eta_k$ (3.4)

Let u = $\lambda g \in$ F($D_n \times C_t$) and $\lambda \neq 0$ then wt(u)=1. By equation (3.1) $ue_1 = \lambda e_1 \neq 0$. Hence we conclude that u = $\lambda g \notin I_{\{e_1\}}$ and so d ($I_{\{e_1\}}$) ≥2. If we take u = $\lambda_1 g + \lambda_2 h \in$ F($D_n \times C_t$) then ue_1 =0 iff $\lambda_1 = -\lambda_2$. So u$\in I_{\{e_1\}}$ hence d$(I_{\{e_1\}})$=2.

Case 2. when n is an even number

(i) $d(I_{\{e_i\}})$=2 and $dim(I_{\{e_i\}})$ = 2nt−1 for i=1,2,3,4

(ii) $d(I_{\{\theta_k\}})$=2 and $d(I_{\{\eta_k\}})$=2 and $dim(I_{\{\theta_k\}}) = dim(I_{\{\eta_k\}})$= 2nt−1

(iii) $d(I_{\{\zeta_k\}})$=2 and $d(I_{\{\delta_k\}})$=2 and $dim(I_{\{\zeta_k\}}) = dim(I_{\{\delta_k\}})$= 2nt−1

(iv) $d(I_{\{e_i,e_j\}})$ = 2 and $dim(I_{\{e_i,e_j\}})$ = 2nt−2 for 1≤ i,j≤ 4 , i≠j

(v) $d(I_{\{\theta_k,\eta_k\}})$=2 and $dim(I_{\{\theta_k,\eta_k\}})$=2nt−2

(vi) $d(I_{\{\zeta_k,\delta_k\}})$= 2 and $dim(I_{\{\zeta_k,\delta_k\}})$= 2nt−2

(vii) $d(I_\beta)$=2 and $dim(I_\beta)$= 2nt−8 where β= $\{e_1, e_2, e_3, e_4, \theta_k, \eta_k, \zeta_k, \delta_k\}$

Let u be any element of F($D_n \times C_t$) then

$ue_1 = (\sum_{i=1}^{2nt} \lambda_i)\, e_1$ (3.5)

$ue_2 = (\sum_{i=1}^{nt} \lambda_i - \sum_{i=nt+1}^{2nt} \lambda_i)\, e_2$ (3.6)

$ue_3 = (\sum_{i=1}^{nt} \lambda_i (-1)^{i-1} + \sum_{i=nt+1}^{2nt} \lambda_i (-1)^{i+1})\, e_3$ (3.7)

$ue_4 = (\sum_{i=1}^{nt} \lambda_i (-1)^{i-1} + \sum_{i=nt+1}^{2nt} \lambda_i (-1)^{i})\, e_4$ (3.8)

$u\theta_k = (\sum_{i=1}^{n} \lambda_i + \alpha^k \sum_{i=n+1}^{2n} \lambda_i + (\alpha^k)^2 \sum_{i=2n+1}^{3n} \lambda_i + \ldots + (\alpha^k)^{n-1} \sum_{i=(t-1)n+1}^{nt} \lambda_i + \sum_{i=nt+1}^{n(t+1)} \lambda_i + \alpha^k \sum_{i=(t+1)n+1}^{(t+2)n} \lambda_i + \ldots + (\alpha^k)^{n-1} \sum_{i=(2t-1)n+1}^{2nt} \lambda_i)\theta_k$ (3.9)

$u\eta_k = (\sum_{i=1}^{n} \lambda_i + \alpha^k \sum_{i=n+1}^{2n} \lambda_i + (\alpha^k)^2 \sum_{i=2n+1}^{3n} \lambda_i + \ldots + (\alpha^k)^{n-1} \sum_{i=(t-1)n+1}^{nt} \lambda_i - \sum_{i=nt+1}^{n(t+1)} \lambda_i$

$-\ \alpha^k\sum_{i=(t+1)n+1}^{(t+2)n}\lambda_i - \ldots\ldots - (\alpha^k)^{n-1}\sum_{i=(2t-1)n+1}^{2nt}\lambda_i\ )\eta_k$ (3.10)

$u\zeta_k = (\sum_{i=1}^{n}\lambda_i\ (-1)^{i-1} + \alpha^k\sum_{i=n+1}^{2n}\lambda_i(-1)^{i-1} + (\alpha^k)^2\sum_{i=2n+1}^{3n}\lambda_i(-1)^{i-1} + \ldots .. + (\alpha^k)^{n-1}\sum_{i=(t-1)n+1}^{nt}\lambda_i(-1)^{i-1} + \sum_{i=nt+1}^{n(t+1)}\lambda_i(-1)^{i-1} + \ldots\ldots. + (\alpha^k)^{n-1}\sum_{i=(2t-1)n+1}^{2nt}\lambda_i(-1)^{i-1}\ )\ \zeta_k$

(3.11)

$u\delta_k = (\sum_{i=1}^{n}\lambda_i\ (-1)^{i-1} + \alpha^k\sum_{i=n+1}^{2n}\lambda_i(-1)^{i-1} + (\alpha^k)^2\sum_{i=2n+1}^{3n}\lambda_i(-1)^{i-1} + \ldots .. + (\alpha^k)^{n-1}\sum_{i=(t-1)n+1}^{nt}\lambda_i(-1)^{i-1} + \sum_{i=nt+1}^{n(t+1)}\lambda_i(-1)^{i} + \ldots\ldots. + (\alpha^k)^{n-1}\sum_{i=(2t-1)n+1}^{2nt}\lambda_i(-1)^{i}\ )\delta_k$

(3.12)

3.2.Idempotent elements in the group algebra F(D_3×C_3)

Consider $D_3 = \{1,a,a^2,b,ab,a^2b\}$ and $C_3 = \{1,x,x^2\}$.The group algebra F(D_3×C_3) has 9 idempotents.

Notations :

$\overline{C_1}$ = (1,1) + (a,1) + (a^2,1)

$\overline{C_2}$ = (1,x) + (a,x) + (a^2,x)

$\overline{C_3}$ = (1,x^2) + (a,x^2) + (a^2,x^2)

$\overline{C_4}$ = (b,1) + (ab,1) + (a^2b,1)

$\overline{C_5}$ = (b,x) + (ab,x) + (a^2b,x)

$\overline{C_6}$ = (b,x^2) + (ab,x^2) + (a^2b,x^2)

$e_1 = \frac{1}{18}\sum_{g\in D_3\times C_3} g$

$e_2 = \frac{1}{18}[\overline{C_1} + \overline{C_2} + \overline{C_3} - \overline{C_4} - \overline{C_5} - \overline{C_6}]$

$e_3 = \frac{1}{18}[\overline{C_1} + \omega^2\overline{C_2} + \omega\overline{C_3} + \overline{C_4} + \omega^2\overline{C_5} + \overline{C_6}]$

$e_4 = \frac{1}{18}[\overline{C_1} + \omega\overline{C_2} + \omega^2\overline{C_3} + \overline{C_4} + \omega\overline{C_5} + \omega^2\overline{C_6}]$

$e_5 = \frac{1}{18}[\overline{C_1} + \omega^2\overline{C_2} + \omega\overline{C_3} - \overline{C_4} - \omega^2\overline{C_5} - \overline{C_6}]$

$e_6 = \frac{1}{18}[\overline{C_1} + \omega\overline{C_2} + \omega^2\overline{C_3} - \overline{C_4} - \omega\overline{C_5} - \omega^2\overline{C_6}]$

$e_7 = \frac{1}{9}[\ 2(1,1) + 2(1,x) + 2(1,x^2) - (a,1) - (a^2,1) - (a,x) - (a^2,x) - (a,x^2) - (a^2,x^2)]$

$e_8 = \frac{1}{9}[2(1,1) + 2\omega^2(1,x) + 2\omega(1,x^2) - (a,1) - (a^2,1) - \omega^2\{(a,x) + (a^2,x)\} - \omega\{(a,x^2) + (a^2,x^2)\}]$

$e_9 = \frac{1}{9}[\ 2(1,1) + 2\omega(1,x) + 2\omega^2(1,x^2) - (a,1) - (a^2,1) - \omega\{(a,x) + (a^2,x)\} - \omega^2\{(a,x^2) + (a^2,x^2)\}]$

whereω is cube root of unity.

3.3. Codes over F(D_3×C_3)

The minimum distance and dimension of the codes which is generated by the idempotents

(i) $d(I_{\{e_i\}})=2$ for $i=1,2,\ldots,7$ and $dim(I_{\{e_i\}})=17$ for $1\le i\le 6$

(ii) $d(I_{\{e_i\}})=3$ for $i=8,9$ and $dim(I_{\{e_j\}})=14$ for $7\le j\le 9$

(iii) $d(I_{\{e_i,e_j\}})=2$ and $dim(I_{\{e_i,e_j\}})=16$ for $1\le i,j\le 6$, $i\ne j$

(iv) $d(I_{\{e_i,e_j,e_k\}})=2$ and $dim(I_{\{e_i,e_j,e_k\}})=15$ for $1\le i,j,k\le 6$, $i\ne j\ne k$

(v) $d(I_{\{e_i,e_j,e_k,e_l\}})=2$ and $dim(I_{\{e_i,e_j,e_k,e_l\}})=14$ for $1\le i,j,k,l\le 6$, $i\ne j\ne k\ne l$

(vi) $d(I_{\{e_7,e_8,e_9\}})=2$ and $dim(I_{\{e_7,e_8,e_9\}})=6$

(vii) $d(I_{\{e_4,e_8\}})=6$ and $dim(I_{\{e_4,e_8\}})=13$

Let $u=\sum_{\substack{j=0,i=0,1,2\\ j=1,i=0,1,2\\ j=2,i=0,1,2}}\lambda_{i+3j+1}(a^i,x^j)+\sum_{\substack{j=0,i=0,1,2\\ j=1,i=0,1,2\\ j=2,i=0,1,2}}\lambda_{i+3j+10}(a^ib,x^j)$ be any element of $F(D_3\times C_3)$. Then

$ue_1=(\sum_{i=1}^{18}\lambda_i)e_1$ (3.13)

$ue_2=(\sum_{i=1}^{9}\lambda_i-\sum_{i=10}^{18}\lambda_i)e_2$ (3.14)

$ue_3=(\sum_{i=1}^{3}\lambda_i+\omega\sum_{i=4}^{6}\lambda_i+\omega^2\sum_{i=7}^{9}\lambda_i+\sum_{i=10}^{12}\lambda_i+\omega\sum_{i=13}^{15}\lambda_i+\omega^2\sum_{i=16}^{18}\lambda_i)\,e_3$ (3.15)

$ue_4=(\sum_{i=1}^{3}\lambda_i+\omega^2\sum_{i=4}^{6}\lambda_i+\omega\sum_{i=7}^{9}\lambda_i+\sum_{i=10}^{12}\lambda_i+\omega^2\sum_{i=13}^{15}\lambda_i+\omega\sum_{i=16}^{18}\lambda_i)e_4$ (3.16)

$ue_5=(\sum_{i=1}^{3}\lambda_i+\omega\sum_{i=4}^{6}\lambda_i+\omega^2\sum_{i=7}^{9}\lambda_i-\sum_{i=10}^{12}\lambda_i-\omega\sum_{i=13}^{15}\lambda_i-\omega^2\sum_{i=16}^{18}\lambda_i)\,e_5$ (3.17)

$ue_6=(\sum_{i=1}^{3}\lambda_i+\omega^2\sum_{i=4}^{6}\lambda_i+\omega\sum_{i=7}^{9}\lambda_i-\sum_{i=10}^{12}\lambda_i-\omega^2\sum_{i=13}^{15}\lambda_i-\omega\sum_{i=16}^{18}\lambda_i)e_6$ (3.18)

$ue_7=\frac{1}{9}[\{2\lambda_1-\lambda_2-\lambda_3+2\lambda_4-\lambda_5-\lambda_6+2\lambda_7-\lambda_8-\lambda_9\}\{(1,1)+(1,x)+(1,x^2)\}+\{-\lambda_1+2\lambda_2-\lambda_3-\lambda_4+2\lambda_5-\lambda_6-\lambda_7+2\lambda_8-\lambda_9\}\{(a,1)+(a,x)+(a,x^2)\}+\{-\lambda_1-\lambda_2+2\lambda_3-\lambda_4-\lambda_5+2\lambda_6-\lambda_7-\lambda_8+2\lambda_9\}\{(a^2,1)+(a^2,x)+(a^2,x^2)\}+\{2\lambda_{10}-\lambda_{11}-\lambda_{12}+2\lambda_{13}-\lambda_{14}-\lambda_{15}+2\lambda_{16}-\lambda_{17}-\lambda_{18}\}\{(b,1)+(b,x)+(b,x^2)\}+\{-\lambda_{10}+2\lambda_{11}-\lambda_{12}-\lambda_{13}+2\lambda_{14}-\lambda_{15}-\lambda_{16}+2\lambda_{17}-\lambda_{18}\}\{(ab,1)+(ab,x)+(ab,x^2)\}+\{-\lambda_{10}-\lambda_{11}+2\lambda_{12}-\lambda_{13}-\lambda_{14}+2\lambda_{15}-\lambda_{16}-\lambda_{17}+2\lambda_{18}\}\{(a^2b,1)+(a^2b,x)+(a^2b,x^2)\}]$ (3.19)

$ue_8=\frac{1}{9}[\{2\lambda_1-\lambda_2-\lambda_3+2\omega\lambda_4-\omega(\lambda_5+\lambda_6)+2\omega^2\lambda_7-\omega^2(\lambda_8+\lambda_9)\}(1,1)+\{2\omega^2\lambda_1-\omega^2\lambda_2-\omega^2\lambda_3+2\lambda_4-\lambda_5-\lambda_6+2\omega\lambda_7-\omega\lambda_8-\omega\lambda_9\}(1,x)+\{2\omega\lambda_1-\omega\lambda_2-\omega\lambda_3+2\omega^2\lambda_4-\omega^2\lambda_5-\omega^2\lambda_6+2\lambda_7-\lambda_8-\lambda_9\}(1,x^2)+\{-\lambda_1+2\lambda_2-\lambda_3-\omega\lambda_4+2\omega\lambda_5-\omega\lambda_6-\omega^2\lambda_7+2\omega^2\lambda_8-\omega^2\lambda_9\}(a,1)+\{-\omega^2\lambda_1+2\omega^2\lambda_2-\omega^2\lambda_3-\lambda_4+2\lambda_5-\lambda_6-\omega\lambda_7+2\omega\lambda_8-\omega\lambda_9\}(a,x)+\{-\omega\lambda_1+2\omega\lambda_2-\omega\lambda_3-\omega^2\lambda_4+2\omega^2\lambda_5-\omega^2\lambda_6-\lambda_7+2\lambda_8-\lambda_9\}(a,x^2)+\{-\lambda_1-\lambda_2+2\lambda_3-\omega\lambda_4-\omega\lambda_5+2\omega\lambda_6-\omega^2\lambda_7-\omega^2\lambda_8+2\omega^2\lambda_9\}(a^2,1)+\{-\omega^2\lambda_1-\omega^2\lambda_2+2\omega^2\lambda_3-\lambda_4-\lambda_5+2\lambda_6-\omega\lambda_7-\omega\lambda_8+2\omega\lambda_9\}(a^2,x)+\{-\omega\lambda_1-\omega\lambda_2+2\omega\lambda_3-\omega^2\lambda_4-\omega^2\lambda_5+2\omega^2\lambda_6-\lambda_7-\lambda_8+2\lambda_9\}(a^2,x^2)+\{2\lambda_{10}-\lambda_{11}-\lambda_{12}+2\omega\lambda_{13}-\omega(\lambda_{14}+\lambda_{15})+2\omega^2\lambda_{16}-\omega^2(\lambda_{17}+\lambda_{18})\}(b,1)+\{2\omega^2\lambda_{10}-\omega^2\lambda_{11}-\omega^2\lambda_{12}+2\lambda_{13}-\lambda_{14}-\lambda_{15}+2\omega\lambda_{16}-\omega\lambda_{17}-\omega\lambda_{18}\}(b,x)+\{2\omega\lambda_{10}-\omega\lambda_{11}-\omega\lambda_{12}+2\omega^2\lambda_{13}-\omega^2\lambda_{14}-\omega^2\lambda_{15}+2\lambda_{16}-\lambda_{17}-\lambda_{18}\}(b,x^2)+\{-\lambda_{10}+2\lambda_{11}-\lambda_{12}-\omega\lambda_{13}+2\omega\lambda_{14}-\omega\lambda_{15}-\omega^2\lambda_{16}+2\omega^2\lambda_{17}-\omega^2\lambda_{18}\}(ab,1)+\{-\omega^2\lambda_{10}+2\omega^2\lambda_{11}-\omega^2\lambda_{12}-\lambda_{13}+2\lambda_{14}-\lambda_{15}-\omega\lambda_{16}+2\omega\lambda_{17}-\omega\lambda_{18}\}(ab,x)+\{-\omega\lambda_{10}+2\omega\lambda_{11}-\omega\lambda_{12}-\omega^2\lambda_{13}+2\omega^2\lambda_{14}-\omega^2\lambda_{15}-\lambda_{16}+2\lambda_{17}-\lambda_{18}\}(ab,x^2)+\{-\lambda_{10}-\lambda_{11}+2\lambda_{12}-\omega\lambda_{13}-\omega\lambda_{14}+2\omega\lambda_{15}$

$-\omega^2\lambda_{16}-\omega^2\lambda_{17}+2\omega^2\lambda_{18}\}(a^2b,1)$ + $\{-\omega^2\lambda_{10}-\omega^2\lambda_{11}+2\omega^2\lambda_{12}-\lambda_{13}-\lambda_{14}+2\lambda_{15}-\omega\lambda_{16}-\omega\lambda_{17}+2\omega\lambda_{18}\}(a^2b, x)$ $+\{-\omega\lambda_{10}-\omega\lambda_{11}+2\omega\lambda_{12}-\omega^2\lambda_{13}-\omega^2\lambda_{14}+2\omega^2\lambda_{15}-\lambda_{16}-\lambda_{17}+2\lambda_{18}\}(a^2b, x^2)]$ (3.20)

$ue_9 = \frac{1}{9}[\{2\lambda_1-\lambda_2-\lambda_3+2\omega^2\lambda_4-\omega^2(\lambda_5+\lambda_6)+2\omega\lambda_7-\omega(\lambda_8+\lambda_9)\}(1,1) + \{2\omega\lambda_1-\omega\lambda_2-\omega\lambda_3+2\lambda_4-\lambda_5-\lambda_6+2\omega^2\lambda_7-\omega^2\lambda_8-\omega^2\lambda_9\}(1,x) + \{2\omega^2\lambda_1-\omega^2\lambda_2-\omega^2\lambda_3+2\omega\lambda_4-\omega\lambda_5-\omega\lambda_6+2\lambda_7-\lambda_8-\lambda_9\}(1,x^2) + \{-\lambda_1+2\lambda_2-\lambda_3-\omega^2\lambda_4+2\omega^2\lambda_5-\omega^2\lambda_6-\omega\lambda_7+2\omega\lambda_8-\omega\lambda_9\}(a,1) + \{-\omega\lambda_1+2\omega\lambda_2-\omega\lambda_3-\lambda_4+2\lambda_5-\lambda_6-\omega^2\lambda_7+2\omega^2\lambda_8-\omega^2\lambda_9\}(a,x) + \{-\omega^2\lambda_1+2\omega^2\lambda_2-\omega^2\lambda_3-\omega\lambda_4+2\omega\lambda_5-\omega\lambda_6-\lambda_7+2\lambda_8-\lambda_9\}(a,x^2) + \{-\lambda_1-\lambda_2+2\lambda_3-\omega^2\lambda_4-\omega^2\lambda_5+2\omega^2\lambda_6-\omega\lambda_7-\omega\lambda_8+2\omega\lambda_9\}(a^2,1) + \{-\omega\lambda_1-\omega\lambda_2+2\omega\lambda_3-\lambda_4-\lambda_5+2\lambda_6-\omega^2\lambda_7-\omega^2\lambda_8+2\omega^2\lambda_9\}(a^2, x) + \{-\omega^2\lambda_1-\omega^2\lambda_2+2\omega^2\lambda_3-\omega\lambda_4-\omega\lambda_5+2\omega\lambda_6-\lambda_7-\lambda_8+2\lambda_9\}(a^2, x^2) + \{2\lambda_{10}-\lambda_{11}-\lambda_{12}+2\omega^2\lambda_{13}-\omega^2(\lambda_{14}+\lambda_{15})+2\omega\lambda_{16}-\omega(\lambda_{17}+\lambda_{18})\}(b,1) + \{2\omega\lambda_{10}-\omega\lambda_{11}-\omega\lambda_{12}+2\lambda_{13}-\lambda_{14}-\lambda_{15}+2\omega^2\lambda_{16}-\omega^2\lambda_{17}-\omega^2\lambda_{18}\}(b,x)$ +

$\{2\omega^2\lambda_{10}-\omega^2\lambda_{11}-\omega^2\lambda_{12}+2\omega\lambda_{13}-\omega\lambda_{14}-\omega\lambda_{15}+2\lambda_{16}-\lambda_{17}-\lambda_{18}\}(b,x^2)$ +

$\{-\lambda_{10}+2\lambda_{11}-\lambda_{12}-\omega^2\lambda_{13}+2\omega^2\lambda_{14}-\omega^2\lambda_{15}-\omega\lambda_{16}+2\omega\lambda_{17}-\omega\lambda_{18}\}(ab,1)$ +

$\{-\omega\lambda_{10}+2\omega\lambda_{11}-\omega\lambda_{12}-\lambda_{13}+2\lambda_{14}-\lambda_{15}-\omega^2\lambda_{16}+2\omega^2\lambda_{17}-\omega^2\lambda_{18}\}(ab, x)$ +

$\{-\omega^2\lambda_{10}+2\omega^2\lambda_{11}-\omega^2\lambda_{12}-\omega\lambda_{13}+2\omega\lambda_{14}-\omega\lambda_{15}-\lambda_{16}+2\lambda_{17}-\lambda_{18}\}(ab,x^2)$ +

$\{-\lambda_{10}-\lambda_{11}+2\lambda_{12}-\omega^2\lambda_{13}-\omega^2\lambda_{14}+2\omega^2\lambda_{15}-\omega\lambda_{16}-\omega\lambda_{17}+2\omega\lambda_{18}\}(a^2b,1)$ +

$\{-\omega\lambda_{10}-\omega\lambda_{11}+2\omega\lambda_{12}-\lambda_{13}-\lambda_{14}+2\lambda_{15}-\omega^2\lambda_{16}-\omega^2\lambda_{17}+2\omega^2\lambda_{18}\}(a^2b, x)$ +

$\{-\omega^2\lambda_{10}-\omega^2\lambda_{11}+2\omega^2\lambda_{12}-\omega\lambda_{13}-\omega\lambda_{14}+2\omega\lambda_{15}-\lambda_{16}-\lambda_{17}+2\lambda_{18}\}(a^2b, x^2)]$ (3.21)

Example 3.4.Idempotents in $F(D_4 \times C_2)$

We define some notations:

$\overline{C_1} = (1,1) + (a^2,1)$

$\overline{C_2} = (1,-1) + (a^2,-1)$

$\overline{C_3} = (a,1) + (a^3,1)$

$\overline{C_4} = (a,-1) + (a^3,-1)$

$\overline{C_5} = (b,1) + (a^2b,1)$

$\overline{C_6} = (b,-1) + (a^2b,-1)$

$\overline{C_7} = (ab,1) + (a^3b,1)$

$\overline{C_8} = (ab,-1) + (a^3b,-1)$

Idempotents are given by

$e_1 = \frac{1}{16}\sum_{g\in D_4\times C_2} g$

$e_2 = \frac{1}{16}[\overline{C_1} - \overline{C_2} + \overline{C_3} - \overline{C_4} + \overline{C_5} - \overline{C_6} + \overline{C_7} - \overline{C_8}]$

$e_3 = \frac{1}{16}[\overline{C_1} + \overline{C_2} + \overline{C_3} + \overline{C_4} - \overline{C_5} - \overline{C_6} - \overline{C_7} - \overline{C_8}]$

$e_4 = \frac{1}{16}[\overline{C_1} - \overline{C_2} + \overline{C_3} - \overline{C_4} - \overline{C_5} + \overline{C_6} - \overline{C_7} + \overline{C_8}]$

$e_5 = \frac{1}{16}[\overline{C_1} + \overline{C_2} - \overline{C_3} - \overline{C_4} + \overline{C_5} + \overline{C_6} - \overline{C_7} - \overline{C_8}]$

$e_6 = \frac{1}{16}[\overline{C_1} - \overline{C_2} - \overline{C_3} + \overline{C_4} + \overline{C_5} - \overline{C_6} - \overline{C_7} - \overline{C_8}]$

$e_7 = \frac{1}{16}[\overline{C_1} + \overline{C_2} - \overline{C_3} - \overline{C_4} - \overline{C_5} - \overline{C_6} + \overline{C_7} + \overline{C_8}]$

$e_8 = \frac{1}{16}[\overline{C_1} - \overline{C_2} - \overline{C_3} + \overline{C_4} - \overline{C_5} + \overline{C_6} + \overline{C_7} - \overline{C_8}]$

$e_9 = \frac{1}{4}[(1,1) + (1,-1) - (a^2,1) - (a^2,-1)]$

$e_{10} = \frac{1}{4}[(1,1) - (1,-1) - (a^2,1) + (a^2,-1)]$

3.5. Minimum distance and dimension of Codes over $F(D_4\times C_2)$

(i) $I_{\{e_i\}}$ is (16,15,2) group code for i =1,2,…8

(ii) $I_{\{e_i\}}$ is (16,12,2) group code for i =9,10

(iii) $d(I_{\{e_i,e_j\}}) = 2$ for $1\le i, j \le 10$, $i\neq j$

(iv) $d(I_{\{e_i,e_j,e_k\}}) = 2$ for $1\le i, j,k \le 8$, $i\neq j\neq k$

(v) $d(I_{\{e_1,e_2,e_3,e_4\}}) = 2$ and $dim(I_{\{e_1,e_2,e_3,e_4\}}) = 12$

(vi) $d(I_{\{e_5,e_6,e_7,e_8\}}) = 2$ and $dim(I_{\{e_5,e_6,e_7,e_8\}}) = 12$.

Let u be any element of $F(D_4\times C_2)$

$u = \sum_{i=1}^{4}\lambda_i\,(a^{i-1},1) + \sum_{i=5}^{8}\lambda_i\left(a^{i-5},-1\right) + \sum_{i=9}^{12}\lambda_i\,(a^{i-9}b,1) + \sum_{i=13}^{16}\lambda_i\,(a^{i-13},-1)$

then $ue_1 = (\sum_{i=1}^{16}\lambda_i)e_1$

$ue_2 = (\sum_{i=1}^{4}\lambda_i - \sum_{i=5}^{8}\lambda_i + \sum_{i=9}^{12}\lambda_i - \sum_{i=13}^{16}\lambda_i)e_2$

$ue_3 = (\sum_{i=1}^{8}\lambda_i - \sum_{i=9}^{16}\lambda_i)e_3$

$ue_4 = (\sum_{i=1}^{4}\lambda_i - \sum_{i=5}^{8}\lambda_i - \sum_{i=9}^{12}\lambda_i + \sum_{i=13}^{16}\lambda_i)e_4$

$ue_5 = \left(\sum_{i=1}^{16}\lambda_i\,(-1)^{i-1}\right)e_5$

$ue_6 = (\sum_{i=1}^{4}\lambda_i\,(-1)^{i-1} + \sum_{i=5}^{8}\lambda_i\,(-1)^{i} + \sum_{i=9}^{12}\lambda_i\,(-1)^{i-1} + \sum_{i=13}^{16}\lambda_i\,(-1)^{i})e_6$

$ue_7 = (\sum_{i=1}^{8}\lambda_i\,(-1)^{i-1} + \sum_{i=9}^{16}\lambda_i\,(-1)^{i})e_7$

$ue_8 = (\sum_{i=1}^{4}\lambda_i\,(-1)^{i-1} + \sum_{i=5}^{12}\lambda_i\,(-1)^{i} + \sum_{i=13}^{16}\lambda_i\,(-1)^{i-1})e_8$

$ue_9 = \frac{1}{4}[(\lambda_1-\lambda_3+\lambda_5-\lambda_7)\,\{(1,1)+(1,-1) - (a^2,1) - (a^2,-1)\} + (\lambda_2-\lambda_4+\lambda_6-\lambda_8)\,\{(a,1)+(a,-1) - (a^3,1) - (a^3,-1)\} + (\lambda_9-\lambda_{11}+\lambda_{13}-\lambda_{15})\,\{(b,1)+(b,-1) - (a^2b,1) -$

$(a^2b,-1)\}+(\lambda_{10}-\lambda_{12}+\lambda_{14}-\lambda_{16})$ $\{(ab,1)+(ab,-1)-(a^3b,1)-(a^3b,-1)\}]$

$ue_{10} = \frac{1}{4}[(\lambda_1-\lambda_3-\lambda_5+\lambda_7)\{(1,1)-(1,-1)-(a^2,1)+(a^2,-1)\}+(\lambda_2-\lambda_4-\lambda_6+\lambda_8)\{(a,1)-(a,-1)-(a^3,1)+(a^3,-1)\}+(\lambda_9-\lambda_{11}-\lambda_{13}-\lambda_{15})\{(b,1)-(b,-1)-(a^2b,1)+(a^2b,-1)\}+(\lambda_{10}-\lambda_{12}+\lambda_{14}-\lambda_{16})\{(b,1)-(b,-1)-(a^2b,1)+(a^2b,-1)\}]$

4. IDEMPOTENTS IN THE GROUP ALGEBRA $F(D_n \times D_n)$

Notations:

$C_{1,a}=\sum_{r=0}^{n-1}(1,a^r)$

$C_{a,1} = \sum_{r=1}^{n-1}(a^r,1)$

$$C_{a,a} = \sum_{i,j=1}^{n-1}(a^i,a^j)$$

$$C_{b,1} = \sum_{r=0}^{n-1}(a^r b,1)$$

$C_{1,b}=\sum_{r=0}^{n-1}(1,a^r b)$

$C_{a,b} = \sum_{r=1,j=0}^{n-1}(a^r,a^j b)$

$C_{b,a}=\sum_{i=0,j=1}^{n-1}(a^i b,a^j)$

$$C_{b,b} = \sum_{i,j=0}^{n-1}(a^i b,a^j b)$$

Idempotents are given by

$e_1 = \frac{1}{4n^2}\sum_{g\in D_n\times D_n} g$

$e_2 = \frac{1}{4n^2}[C_{1,a}+C_{a,1}+C_{a,a}+C_{1,b}+C_{a,b}-C_{b,1}-C_{b,a}-C_{b,b}]$

$e_3 = \frac{1}{4n^2}[C_{1,a}+C_{a,1}+C_{a,a}-C_{1,b}-C_{a,b}+C_{b,1}+C_{b,a}-C_{b,b}]$

$e_4 = \frac{1}{4n^2}[C_{1,a}+C_{a,1}+C_{a,a}-C_{1,b}-C_{a,b}-C_{b,1}-C_{b,a}+C_{b,b}]$

$\eta_j = \frac{1}{2n^2}[2(1,1)+\sum_{r=1}^{n-1}(1,a^r)2\cos\frac{2\pi jr}{n}+2C_{a,1}+2C_{b,1}+\sum_{i=0}^{n-1}\sum_{r=1}^{n-1}(a^i b,a^r)2\cos\frac{2\pi jr}{n}+\sum_{r=1}^{\frac{n-1}{2}}\{(a^r,a^r)+(a^r,a^{-r})+(a^{-r},a^r)+(a^{-r},a^{-r})\}(2\cos\frac{2\pi jr}{n})]$

$\delta_j = \frac{1}{2n^2}[2(1,1)+\sum_{r=1}^{n-1}(1,a^r)2\cos\frac{2\pi jr}{n}+2C_{a,1}-2C_{b,1}-\sum_{i=0}^{n-1}\sum_{r=1}^{n-1}(a^i b,a^r)2\cos\frac{2\pi jr}{n}+\sum_{r=1}^{\frac{n-1}{2}}\{(a^r,a^r)+(a^r,a^{-r})+(a^{-r},a^r)+(a^{-r},a^{-r})\}(2\cos\frac{2\pi jr}{n})]$

$\zeta_j = \frac{1}{2n^2}[2(1,1)+2C_{1,a}+2C_{1,b}+\sum_{r=1}^{n-1}(a^r,1)2\cos\frac{2\pi jr}{n}+\sum_{i=0}^{n-1}\sum_{r=1}^{n-1}(a^r,a^i b)2\cos\frac{2\pi jr}{n}+$

$\sum_{r=1}^{\frac{n-1}{2}}\{(a^r,a^r)+(a^r,a^{-r})+(a^{-r},a^r)+(a^{-r},a^{-r})\}(2\cos\frac{2\pi jr}{n})]$

$\theta_j=\frac{1}{2n^2}[2(1,1)+2C_{1,a}-2C_{1,b}+\sum_{r=1}^{n-1}(a^r,1)2\cos\frac{2\pi jr}{n}-\sum_{i=0}^{n-1}\sum_{r=1}^{n-1}(a^r,a^ib)2\cos\frac{2\pi jr}{n}+$
$\sum_{r=1}^{\frac{n-1}{2}}\{(a^r,a^r)+(a^r,a^{-r})+(a^{-r},a^r)+(a^{-r},a^{-r})\}(2\cos\frac{2\pi jr}{n})]$

$\vartheta_j=\frac{1}{n^2}[4(1,1)+\sum_{r=1}^{n-1}(1,a^r)4\cos\frac{2\pi jr}{n}+\sum_{r=1}^{n-1}(a^r,1)4\cos\frac{2\pi jr}{n}+\sum_{r=1}^{\frac{n-1}{2}}\{(a^r,a^r)+(a^r,a^{-r})+$
$(a^{-r},a^r)+(a^{-r},a^{-r})\}(2\cos\frac{2\pi jr}{n})^2]$ where $1\le j\le\frac{n-1}{2}$.

4.2.Codes over F($D_n \times D_n$)

In this section we find the minimum distance and dimension of the codes.

(i) $d(I_{\{e_i\}})=2$ and $dim(I_{\{e_i\}})=4n^2-1$ for i =1,2,3,4

(ii) $d(I_{\{e_i,e_j\}})=2$ and $dim(I_{\{e_i,e_j\}})=4n^2-2$ for $1\le i,j\le 4$, $i\ne j$

(iii) $d(I_{\{e_i,e_j,e_k\}})=2$ and $dim(I_{\{e_i,e_j,e_k\}})=4n^2-3$ for $1\le i,j,k\le 4$, $i\ne j\ne k$

(iv) $d(I_\beta)=2$ and $dim(I_\beta)=4n^2-4$ where $\beta=\{e_1,e_2,e_3,e_4\}$

Let u= $\lambda_1(1,1)+\ldots+\lambda_{n^2}(a^{n-1},a^{n-1})+\lambda_{n^2+1}(1,b)+\cdots+\lambda_{2n^2}(a^{n-1},a^{n-1}b)+\ldots$ $+\lambda_{3n^2}(a^{n-1}b,a^{n-1})+\ldots+\lambda_{4n^2}(a^{n-1}b,a^{n-1}b)$ be any element of F($D_n \times D_n$) then

$ue_1=(\sum_{i=1}^{4n^2}\lambda_i)e_1$ (4.1)

$ue_2=(\sum_{i=1}^{2n^2}\lambda_i-\sum_{i=2n^2+1}^{4n^2}\lambda_i)e_2$ (4.2)

$ue_3=(\sum_{i=1}^{n^2}\lambda_i-\sum_{i=n^2+1}^{2n^2}\lambda_i+\sum_{i=2n^2+1}^{3n^2}\lambda_i-\sum_{i=3n^2+1}^{4n^2}\lambda_i)e_3$ (4.3)

$ue_4=(\sum_{i=1}^{n^2}\lambda_i-\sum_{i=n^2+1}^{3n^2}\lambda_i+\sum_{i=3n^2+1}^{4n^2}\lambda_i)e_4$ (4.4)

Let u = $\lambda g\in$ F($D_n \times D_n$) and $\lambda\ne 0$ such that wt(u)=1. Then $ue_1=\lambda e_1\ne 0$.Hence we have u = $\lambda g\notin I_{\{e_1\}}$ and so d ($I_{\{e_1\}}$) ≥ 2. Let u = $\lambda_1 g+\lambda_2 h\in$ F($D_n \times D_n$) with wt(u)=2 then $ue_1=0$ iff $\lambda_1=-\lambda_2$. So $u\in I_{\{e_1\}}$ hence $d(I_{\{e_1\}})=2$. Similarly we can prove for other cases.

4.3.Idempotents in the group algebra of $D_3 \times D_3$

Consider the group $D_3=\{1,a,a^2,b,ab,a^2b : ab=ba^{-1}\}$

The idempotents are given by

$e_1=\frac{1}{36}\sum_{g\in D_6\times D_6} g$

$e_2=\frac{1}{36}[(1,1)+C_{1,a}+C_{a,1}+C_{a,a}+C_{1,b}+C_{a,b}-C_{b,1}-C_{b,a}-C_{b,b}]$

$e_3=\frac{1}{36}[(1,1)+C_{1,a}+C_{a,1}+C_{a,a}-C_{1,b}-C_{a,b}+C_{b,1}+C_{b,a}-C_{b,b}]$

e4 = $\frac{1}{36}[(1,1)+C_{1,a}+C_{a,1}+C_{a,a}-C_{1,b}-C_{a,b}-C_{b,1}-C_{b,a}+C_{b,b}]$

e5 = $\frac{1}{18}[2(1,1)+2C_{1,a}-C_{a,1}-C_{a,a}+2C_{1,b}-C_{a,b}]$

e6 = $\frac{1}{18}[2(1,1)+2C_{1,a}-C_{a,1}-C_{a,a}-2C_{1,b}+C_{a,b}]$

e7 = $\frac{1}{18}[2(1,1)-C_{1,a}+2C_{a,1}-C_{a,a}+2C_{b,1}-C_{b,a}]$

e8 = $\frac{1}{18}[2(1,1)-C_{1,a}+2C_{a,1}-C_{a,a}-2C_{b,1}+C_{b,a}]$

e9 = $\frac{1}{9}[4(1,1)-2C_{1,a}-2C_{a,1}+C_{a,a}]$

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