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Decomposition of (μ, λ)-Continuity

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Abstract

In this paper, we introduce and study the notion of weak$^*_{(\mu,\lambda)}$-continuity in generalized topological spaces. Also, we prove that $f : (X, \mu) \to (Y, \lambda)$ is (μ, λ)-continuous if and only if it is weakly (μ, λ)-continuous and weak$^*_{(\mu,\lambda)}$-continuous.

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1. Introduction and Preliminaries

In 2002, Csaszar [2] introduced the notions of generalized topology and generalized continuity. Let X be a nonempty set and μ be a collection of subsets of X. Then μ is called a *generalized topology* (briefly GT) on X iff $\emptyset \in \mu$ and the union of an arbitrary class of elements of μ always belong to μ. We call the pair (X, μ) be a *generalized topological space* (briefly GTS) on X. Let μ be a GT in X. The elements of μ are said to be *μ-open*, their complements are *μ-closed*. We consider the largest μ-open subset of $A \subset X$ and denote it by $i_\mu(A)$ and the smallest μ-closed superset of A and denoted it by $c_\mu(A)$. A function $f : (X, \mu) \to (Y, \lambda)$ is said to be (μ, λ)-continuous [2], iff $U \in \lambda$ implies that $f^{-1}(U)$ is μ-open A function $f : (X, \mu) \to (Y, \lambda)$ is said to be *weakly* (μ, λ)*-continuous* [6], if for each $x \in X$ and each λ-open neighbourhood V of $f(x)$, there exist a μ-open neighbourhood U of x such that $f(U) \subseteq c_\lambda(V)$.

Definition 1.1. [2] A function $f : (X, \mu) \to (Y, \lambda)$ is said to be $\theta(\mu, \lambda)$-continuous at x if for each λ-open neighbourhood V of $f(x)$, there is a μ-open neighbourhood U of x such that $f(c_\mu(U)) \subseteq c_\lambda(V)$.

2. Weak$^*_{(\mu,\lambda)}$-continuous functions

Definition 2.1. A function $f : (X, \mu) \to (Y, \lambda)$ is said to be weak$^*_{(\mu,\lambda)}$-continuous (briefly $w^*_{(\mu,\lambda)}$-continuity), if for each λ-open set V in (Y, λ), $f^{-1}(f_r(V))$ is μ-closed in (X, μ), where $f_r(V) = c_\lambda(V) - i_\lambda(V)$. For every $V \subset Y$, $f_r(V)$ is always λ-closed in (Y, λ).

Lemma 2.2. Every (μ, λ)-continuity is weak$^*_{(\mu,\lambda)}$-continuity but not conversely.

Example 2.3. Let $X = Y = \{a, b, c\}$, $\mu = \{\emptyset, \{a\}, \{a, b\}\}$ and $\lambda = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Then the identity function$f : (X, \mu) \to (Y, \lambda)$ is weak$^*_{(\mu,\lambda)}$-continuity but not (μ, λ)-continuity.

(i) Let $a \in X$ and $V = \{\{a\}, \{a, b\}\} \in \lambda$ such that $f(a) \in V$. Then $f_r(V) = \{c\}$ and $f^{-1}(f_r(V)) = \{c\}$ is μ-closed in (X, μ).

(ii) Let $b \in X$ and $V = \{\{b\}, \{a, b\}\} \in \lambda$ such that $f(b) \in V$. Then $f_r(V) = \{c\}$ and $f^{-1}(f_r(V)) = \{c\}$ is μ-closed in (X, μ).

By (i), (ii), f is weak$^*_{(\mu,\lambda)}$-continuity. On the other hand $\{b\}$ be a λ-open set in (Y, λ) and $f^{-1}(U) = \{b\}$ is not μ-open in (X, μ). Hence f is not (μ, λ)-continuity.

Remark 2.4. The notions of $\theta(\mu, \lambda)$-continuity and weak$^*_{(\mu,\lambda)}$-continuity are independent.

The following examples shows that the notion of $\theta(\mu, \lambda)$-continuity and weak$^*_{(\mu,\lambda)}$-continuity are independent.

Example 2.5. Let $X = Y = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a, b\}, \{c, d\}, X\}$ and $\lambda = \{\emptyset,$ $\{a, c, d\}\{b, c, d\}, Y\}$. Then the identity function $f : (X, \mu) \to (Y, \lambda)$ is $\theta(\mu, \lambda)$-continuous but not weak$^*_{(\mu,\lambda)}$-continuous.

(i) Let $a \in X$ and $V = \{\{a, c, d\}, Y\} \in \lambda$ such that $f(a) \in V$. Then, there exist μ-open sets $U = \{\{a, b\}, X\}$ and such that $a \in U$ and $f(c_\mu(U)) = U \subset c_\lambda(V) = Y$.

(ii) Let $b \in X$ and $V = \{\{b, c, d\}, Y\} \in \lambda$ such that $f(b) \in V$. Then, there exist μ-open sets $U = \{\{a, b\}, X\}$ and such that $b \in U$ and $f(c_\mu(U)) = U \subset c_\lambda(V) = Y$.

(iii) Let $c \in X$ and $V = \{\{a, c, d\}, \{b, c, d\}, Y\} \in \lambda$ such that $f(c) \in V$. Then, there exist μ-open sets $U = \{\{c, d\}, X\}$ and such that $c \in U$ and $f(c_\mu(U)) = U \subset c_\lambda(V) = Y$.

(iv) Let $d \in X$ and $V = \{\{a, c, d\}, \{b, c, d\}, Y\} \in \lambda$ such that $f(d) \in V$. Then, there exist μ-open sets $U = \{\{c, d\}, X\}$ and such that $d \in U$ and $f(c_\mu(U)) = U \subset c_\lambda(V) = Y$.

By (i), (ii), (iii), and (iv), f is weakly(μ, λ)-continuity. On the other hand, for $V = \{\{a, c, d\}, Y\} \in \lambda$, $c_\lambda(\{a, c, d\}) = Y$, $i_\lambda(\{a, c, d\}) = \{a, c, d\}$, $f_r(\{a, c, d\}) = \{b\}$ and $f^{-1}(\{b\}) = \{b\}$ is not μ-closed in (X, μ), f is not weak$^*_{(\mu,\lambda)}$-continuity.

Example 2.6. Let $X = Y = \{a, b, c\}$, $\mu = \{\emptyset, \{a\}, \{a, b\}\}$ and $\lambda = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Then the identity function $f : (X, \mu) \to (Y, \lambda)$ is weak$^*_{(\mu,\lambda)}$-continuity but not $\theta(\mu, \lambda)$-continuity.

(i) Let $a \in X$ and $V = \{\{a\}, \{a, b\}\} \in \lambda$ such that $f(a) \in V$. Then $f_r(V) = \{c\}$ and $f^{-1}(f_r(V)) = \{c\}$ is μ-closed in (X, μ).

(ii) Let $b \in X$ and $V = \{\{b\}, \{a, b\}\} \in \lambda$ such that $f(b) \in V$. Then $f_r(V) = \{c\}$ and $f^{-1}(f_r(V)) = \{c\}$ is μ-closed in (X, μ).

By (i), (ii), f is weak$^*_{(\mu,\lambda)}$-continuity. On the other hand, for $V = \{\{a\}, \{a, b\}\} \in \lambda$, $c_\lambda(\{a\}) = \{a, c\}$. Then, there exists a μ-open $U = \{\{a, b\}\}$ such that $a \in U$, $f(c_\mu(U)) = X \nsubseteq c_\lambda(V)$, f is not $\theta(\mu, \lambda)$-continuity.

Definition 2.7. A GTS (X, μ) is R_μ-space, if for each $x \in X$ and each μ-open neighbourhood V of x, there exists a μ-open neighbourhood U of x such that $x \in U \subset c_\mu(U) \subset V$.

Theorem 2.8. Let (Y, λ) be a R_μ-space. Then $f : (X, \mu) \to (Y, \lambda)$ is weakly$_{(\mu,\lambda)}$-continuous if and only if f is (μ, λ)-continuous.

Proof. Let $x \in X$ and V be λ-open set of Y containing $f(x)$. Since Y is R_μ-space, there exists a λ-open set U of Y such that $f(x) \in U \subset c_\lambda(U) \subset V$. Since f is weakly$_{(\mu,\lambda)}$-continuous there exists a μ-open set W such that $x \in W$ and $f(W) \subset c_\lambda(U) \subset V$ thus f is (μ, λ)-continuity. Conversely, let $x \in X$ and V be any λ-open set of Y containing $f(x)$. Since f is (μ, λ)-continuous, there exists a μ-open set U containing x such that $f(U) \subseteq c_\lambda(V)$. Hence f is weakly$_{(\mu,\lambda)}$-continuity. ■

Theorem 2.9. A function $f : (X, \mu) \to (Y, \lambda)$ is (μ, λ)-continuous if and only if it is weakly$_{(\mu,\lambda)}$-continuous and weak$^*_{(\mu,\lambda)}$-continuous.

Proof. Let $x \in X$ and V be any λ-open set of Y containing $f(x)$. Since f is (μ, λ)-continuity, there exists a μ-open set U containing x such that $f(U) \subseteq c_\lambda(V)$ and

$f^{-1}(f_r(V))$ is μ-closed in (X,μ). Hence f is weakly$_{(\mu,\lambda)}$-continuous and weak$^*_{(\mu,\lambda)}$-continuity. Conversely, let $x \in X$ and V be any λ-open set containing $f(x)$ in (Y,λ). Since f is weakly$_{(\mu,\lambda)}$-continuous, there exists a μ-open set U containing x such that $f(U) \subseteq c_\lambda(V)$. Now $f_r(V) = c_\lambda(V) - i_\lambda(V)$ and thus $f(x) \notin f_r(V)$. Hence $x \notin f^{-1}(f_r(V))$ and $U - f^{-1}(f_r(V))$ is an μ-open set containing x since f is weak$^*_{(\mu,\lambda)}$-continuous. The proof will be complete when we show $f(U - f^{-1}(f_r(V))) \subset V$. Let $y \in U - f^{-1}(f_r(V))$, then $y \in U$ and hence, $f(y) \in c_\lambda(V)$. But $y \notin f^{-1}(f_r(V))$ and thus $f(y) \notin f_r(V) = c_\lambda(V) - V$, which implies $f(y) \in V$, $f(U) \subseteq V$. Hence f is (μ,λ)-continuous. ■

The following examples shows that the notion of weakly$_{(\mu,\lambda)}$-continuity and weak$^*_{(\mu,\lambda)}$-continuity are independent.

Example 2.10. Let $X = Y = \{a,b,c,d\}$, $\mu = \{\emptyset, \{a,b\}, \{c,d\}, X\}$ and $\lambda = \{\emptyset,$ $\{a,c,d\}\{b,c,d\}, Y\}$. Then the identity function $f : (X,\mu) \to (Y,\lambda)$ is weakly$_{(\mu,\lambda)}$-continuous but not weak$^*_{(\mu,\lambda)}$-continuous.

(i) Let $a \in X$ and $V = \{\{a,c,d\}, Y\} \in \lambda$ such that $f(a) \in V$. Then, there exist μ-open sets $U = \{\{a,b\}, X\}$ and such that $a \in U$ and $f(U) = U \subset c_\lambda(V) = Y$.

(ii) Let $b \in X$ and $V = \{\{b,c,d\}, Y\} \in \lambda$ such that $f(b) \in V$. Then, there exist μ-open sets $U = \{\{a,b\}, X\}$ and such that $b \in U$ and $f(U) = U \subset c_\lambda(V) = Y$.

(iii) Let $c \in X$ and $V = \{\{a,c,d\}, \{b,c,d\}, Y\} \in \lambda$ such that $f(c) \in V$. Then, there exist μ-open sets $U = \{\{c,d\}, X\}$ and such that $c \in U$ and $f(U) = U \subset c_\lambda(V) = Y$.

(iv) Let $d \in X$ and $V = \{\{a,c,d\}, \{b,c,d\}, Y\} \in \lambda$ such that $f(d) \in V$. Then, there exist μ-open sets $U = \{\{c,d\}, X\}$ and such that $d \in U$ and $f(U) = U \subset c_\lambda(V) = Y$.

By (i), (ii), (iii), and (iv), f is weakly(μ,λ)-continuity. On the other hand, for $V = \{\{a,c,d\}, Y\} \in \lambda$, $c_\lambda(\{a,c,d\}) = Y$, $i_\lambda(\{a,c,d\}) = \{a,c,d\}$, $f_r(\{a,c,d\}) = \{b\}$ and $f^{-1}(\{b\}) = \{b\}$ is not μ-closed in (X,μ), f is not weak$^*_{(\mu,\lambda)}$-continuity.

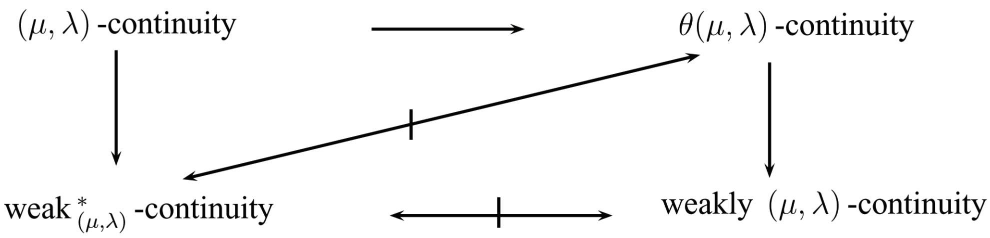
Example 2.11. Let $X = Y = \{a,b,c\}$, $\mu = \{\emptyset, \{a\}, \{a,b\}\}$ and $\lambda = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$. Then the identity function $f : (X,\mu) \to (Y,\lambda)$ is weak$^*_{(\mu,\lambda)}$-continuity but not weakly$_{(\mu,\lambda)}$-continuity.

(i) Let $a \in X$ and $V = \{\{a\}, \{a,b\}\} \in \lambda$ such that $f(a) \in V$. Then $f_r(V) = \{c\}$ and $f^{-1}(f_r(V)) = \{c\}$ is μ-closed in (X,μ).

(ii) Let $b \in X$ and $V = \{\{b\}, \{a,b\}\} \in \lambda$ such that $f(b) \in V$. Then $f_r(V) = \{c\}$ and $f^{-1}(f_r(V)) = \{c\}$ is μ-closed in (X,μ).

By (i), (ii), f is weak$^*_{(\mu,\lambda)}$-continuity. On the other hand, for $V = \{\{a\},\{a,b\}\} \in \lambda$, $c_\lambda(\{a\}) = \{a,c\}$. Then, there exists a μ-open $U = \{\{a,b\}\}$ such that $a \in U$, $f(U) = U \nsubseteq c_\lambda(V)$, f is not weakly$_{(\mu,\lambda)}$-continuity.

Remark 2.12. From the definitions we have the following implication:



The converse of the above relations need not be true from the above examples.

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