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On nano $\pi^\star g$-closed sets

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Abstract

In this paper, new class of set is called nano $\pi^\star g$-closed in nano topological spaces is introduced and its properties.

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1. Introduction

Lellis Thivagar et al. [5] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space.

Bhuvaneswari et al. [4] introduced and investigated nano g-closed sets in nano topological spaces. In this paper, we define and study the properties of nano $\pi^\star g$-closed sets which is a weaker form of nano πg-closed sets but stronger than nano rwg-closed sets.

2. Preliminaries

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. [8] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.2. [5] If (U, R) is an approximation space and $X, Y \subseteq U$; then

1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3. [5] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, R(X) satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,

2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4. [5] If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5. [5]If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by $Nint(H)$.

That is, $Nint(H)$ is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$.

That is, $Ncl(H)$ is the smallest nano closed set containing H.

Definition 2.6. A subset H of a nano topological space $(U, \tau_R(X))$ is called

1. nano semi-open [5] if $H \subseteq Ncl(Nint(H))$.

2. nano regular-open [5] if $H = Nint(Ncl(H))$.

3. nano pre-open [5] if $H \subseteq Nint(Ncl(H))$.

4. nano α-open [7] if $H \subseteq N(int(Ncl(Nint(H))))$.

5. nano β-open [10] if $H \subseteq N(cl(Nint(Ncl(H))))$.

6. nano π-open [1] if the finite union of nano regular-open sets.

The complements of the above mentioned sets is called their respective closed sets.

Definition 2.7. A subset H of a nano topological space $(U, \tau_R(X))$ is called

1. nano g-closed [3] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.

2. nano sg-closed [2] if $Nscl(H)' \subseteq G$, whenever $H \subseteq G$ and G is nano semi-open.

3. nano wg-closed [6] if $Ncl(Nint(H)' \subseteq G$, whenever $H \subseteq G$ and G is nano open.

4. nano πg-closed [9] if $Ncl(H)' \subseteq G$, whenever $H \subseteq G$ and G is nano π-open.

3. On nano $\pi^{\star}g$-closed sets

Definition 3.1. A subset H of space $(U, \tau_R(X))$ is called nano $\pi^{\star}g$-closed if $Ncl(Nint(H)) \subseteq G$, whenever $H \subseteq G$ and G is nano π-open.

The complement of nano $\pi^{\star}g$-open if $H^c = U - H$ is nano $\pi^{\star}g$-closed.

Example 3.2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a, b\}, \{c\}, \{d\}\}$ and $X = \{a, d\}$. Then the nano topology $\tau_R(X) = \{\phi, \{d\}, \{a, b\}, \{a, b, d\}, U\}$.

1. then $\{b, c, d\}$ is nano $\pi^{\star}g$-closed.
2. then $\{a, c, d\}$ is nano $\pi^{\star}g$-open.

Theorem 3.3. In a space $(U, \tau_R(X))$, every nano closed is nano $\pi^{\star}g$-closed.

Proof. Obvious. ■

Remark 3.4. The converse of the Theorem 3.3 need not be true in general as shown in the following example.

Example 3.5. In Example 3.2, observe that $\{a\}$ is nano $\pi^{\star}g$-closed but not nano closed.

Theorem 3.6. In a space $(U, \tau_R(X))$, every nano g-closed is nano $\pi^{\star}g$-closed.

Proof. Suppose that H is nano g-closed in U. Let $H \subseteq G$ where G is nano π-open. Since H is nano g-closed, $Ncl(H) \subseteq G$ and $Ncl(Nint(H)) \subseteq Ncl(H)$, we get $Ncl(Nint(H)) \subseteq G$. Using the fact that every nano π-open is nano open, thus H is nano $\pi^{\star}g$-closed. ■

Remark 3.7. The converse of the Theorem 3.6 need not be true in general as shown in the following example.

Example 3.8. In Example 3.2, observe that $\{b\}$ is nano $\pi^{\star}g$-closed but not nano g-closed.

Theorem 3.9. In a space $(U, \tau_R(X))$, every nano α-closed is nano $\pi^{\star}g$-closed.

Proof. Let H be nano α-closed and $H \subseteq G$ where G is nano π-open. Since H is nano α-closed, $Ncl(Nint(Ncl(H))) \subseteq H \subseteq G$. Hence $Ncl(Nint(H)) \subseteq Ncl(Nint(Ncl(H))) \subseteq G$. Hence $Ncl(Nint(H)) \subseteq G$ and H is nano $\pi^{\star}g$-closed. ■

Remark 3.10. The converse of the Theorem 3.9 need not be true in general as shown in the following example.

Example 3.11. In Example 3.2, observe that $\{a, c\}$ is nano $\pi^{\star}g$-closed but not nano α-closed.

Theorem 3.12. In a space $(U, \tau_R(X))$, every nano πg-closed is nano $\pi^{\star}g$-closed.

Proof. Let H be a nano πg-closed and suppose that $H \subseteq G$ where G is nano π-open. Then $Ncl(Nint(H)) \subseteq Ncl(H) \subseteq G$ and hence $Ncl(Nint(H)) \subseteq G$. Thus H is nano $\pi^\star g$-closed. ■

Remark 3.13. The converse of the Theorem 3.12 need not be true in general as shown in the following example.

Example 3.14. In Example 3.2, observe that $\{b\}$ is nano $\pi^\star g$-closed but not nano πg-closed.

Theorem 3.15. In a space $(U, \tau_R(X))$, every nano wg-closed is nano $\pi^\star g$-closed.

Proof. Let H be a nano wg-closed in U. Let $H \subseteq G$ where G is nano π-open. By definition of nano wg-closed and the fact that every nano regular open is nano π-open, thus H is nano $\pi^\star g$-closed. ■

Remark 3.16. The converse of the Theorem 3.15 need not be true in general as shown in the following example.

Example 3.17. In Example 3.2, observe that $\{a, b, d\}$ is nano $\pi^\star g$-closed but not nano wg-closed.

Theorem 3.18. In a space $(U, \tau_R(X))$, every nano pre-closed is nano $\pi^\star g$-closed.

Proof. Obvious. ■

Remark 3.19. The converse of the Theorem 3.18 need not be true in general as shown in the following example.

Example 3.20. In Example 3.2, observe that $\{a, b, d\}$ is nano $\pi^\star g$-closed but not nano pre-closed.

Remark 3.21. In a space $(U, \tau_R(X))$, the Union and Intersection of two nano $\pi^\star g$-closed but not nano $\pi^\star g$-closed.

Example 3.22. In Example 3.2, then

1. We have $S = \{a\}$ and $T = \{b\}$. Clearly $S \cup T = \{a, b\}$ is not nano $\pi^\star g$-closed.

2. We have $S = \{a, b, c\}$ and $T = \{a, b, d\}$. Clearly $S \cap T = \{a, b\}$ is not nano $\pi^\star g$-closed.

Remark 3.23. The following example prove that nano $\pi^\star g$-closed and nano semi-closed are independent of each other.

Example 3.24. In Example 3.2, then

1. We have $\{b\}$ is nano $\pi^\star g$-closed but not nano semi-closed.

2. We have $\{a, b\}$ is nano semi-closed but not nano $\pi^{\star}g$-closed.

Remark 3.25. The following example prove that nano $\pi^{\star}g$-closed and nano sg-closed are independent of each other.

Example 3.26. In Example 3.2, then

1. We have $\{c\}$ is nano $\pi^{\star}g$-closed but not nano sg-closed.
2. We have $\{d\}$ is nano sg-closed but not nano $\pi^{\star}g$-closed.

Remark 3.27. The following example prove that nano $\pi^{\star}g$-closed and nano β-closed are independent of each other.

Example 3.28. In Example 3.2, then

1. We have $\{a, b, d\}$ is nano $\pi^{\star}g$-closed but not nano β-closed.
2. We have $\{a, b\}$ is nano gs-closed but not nano $\pi^{\star}g$-closed.

Theorem 3.29. In a space $(U, \tau_R(X))$, if H is nano $\pi^{\star}g$-closed and $H \subseteq K \subseteq Ncl(Nint(H))$ then K is also nano $\pi^{\star}g$-closed.

Proof. Let $K \subseteq G$ where G is nano π-open. Then $H \subseteq K$ implies $H \subseteq G$ and G is nano π-open. Since H is nano $\pi^{\star}g$-closed, $Ncl(Nint(H)) \subseteq G$. Using hypothesis, $Ncl(Nint(K)) \subseteq G$. Thus K is nano $\pi^{\star}g$-closed. ■

Theorem 3.30. In a space $(U, \tau_R(X))$, if H is both nano regular open and nano $\pi^{\star}g$-closed then it is nano clopen.

Proof. Since H is nano regular open, H is open and $H = Nint(H)$. H is nano $\pi^{\star}g$-closed implies $Ncl(Nint(H)) \subseteq H$. $Ncl(H) = Ncl(Nint(H)) \subseteq H$ implies $Ncl(H) = H$. Hence H is nano clopen. ■

Lemma 3.31. The following properties are equivalent for a subset H of U.

1. H is nano clopen.
2. H is nano regular open and nano $\pi^{\star}g$-closed.
3. H is nano π-open and nano $\pi^{\star}g$-closed.

Proof. Follows from Theorem 3.30 and the fact that every nano regular open is nano π-open. ■

Theorem 3.32. In a space $(U, \tau_R(X))$, if H is nano $\pi^{\star}g$-closed then $Ncl(Nint(H)) - H$ contains no non-empty nano π-closed set.

Proof. Suppose that F is a non-empty nano π-closed subset of $Ncl(Nint(H))-H$. Now $F \subseteq Ncl(Nint(H))-H$ implies $F \subseteq Ncl(Nint(H)) \cap H^c$. Thus $F \subseteq Ncl(Nint(H))$. $F \subseteq H^c \Rightarrow H \subseteq F^c$. Since F^c is nano π-open and H is nano $\pi^{\star}g$-closed. We have $Ncl(Nint(H)) \subseteq F^c$ and $F \subseteq (Ncl(Nint(H))^c$. Therefore $F \subseteq Ncl(Nint(H)) \cap Ncl(Nint(H))^c = \phi$ (ie) $F = \phi$. This implies that $Ncl(Nint(H)) - H$ contains no non-empty nano π-closed. ■

Theorem 3.33. Suppose that $K \subseteq H \subseteq U$, K is nano $\pi^{\star}g$-closed relative to H and H is both nano regular open and nano $\pi^{\star}g$-closed subset of U, then K is nano $\pi^{\star}g$-closed relative to U.

Proof. Let $K \subseteq G$ and G be nano π-open in U. Given $K \subseteq H \subseteq U$. This $\Rightarrow$ $K \subseteq H \cap G$. Since K is nano $\pi^{\star}g$-closed relative to H, $Ncl(Nint_H(K)) \subseteq H \cap G$. $H \cap Ncl(Nint(K) \subseteq H \cap G$. Consequently $H \cap Ncl(Nint(K) \subseteq G$. Since H is nano regular open and nano $\pi^{\star}g$-closed. We have $H = Ncl(H)$. $Ncl(Nint(K)) \subseteq Ncl(K) \subseteq Ncl(H) = H$. Thus $Ncl(Nint(K)) \cap H = Ncl(Nint(K))$ and $Ncl(Nint(K)) \subseteq G$. Hence K is nano $\pi^{\star}g$-closed relative to U. ■

Corollary 3.34. Let H be both nano regular open and nano $\pi^{\star}g$-closed in U and suppose that F is nano-closed , then $(H \cap F)$ is nano $\pi^{\star}g$-closed.

Proof. We show that $Ncl(Nint(H \cap F)) \subseteq U$ whenever $(H \cap F) \subseteq G$ and G is nano π-open. Since F is nano closed, $(H \cap F)$ is nano closed in H and hence nano $\pi^{\star}g$-closed in H. Hence $(H \cap F)$ is nano $\pi^{\star}g$-closed in U. ■

Theorem 3.35. Let $H \subseteq T \subseteq S$. Suppose that H is nano $\pi^{\star}g$-closed in S and T is nano open in U then H is nano $\pi^{\star}g$-closed relative to T.

Proof. Given $H \subseteq T \subseteq S$ and H is nano $\pi^{\star}g$-closed in U. Let $H \subseteq (T \cap G)$ where G is nano π-open in U. Since H is nano $\pi^{\star}g$-closed in U, $H \subseteq G$ implies that $Ncl(Nint(H)) \subseteq G$. $T \cap Ncl(Nint(H)) \subseteq T \cap G$. Thus H is nano $\pi^{\star}g$-closed relative to T. ■

Theorem 3.36. A subset H of a nano topological space is nano $\pi^{\star}g$-open $\iff$ $F \subseteq Nint(Ncl(H))$ whenever F is nano π-closed and $F \subseteq H$.

Proof. Assume H is nano $\pi^{\star}g$-open then H^c is nano $\pi^{\star}g$-closed. Let F be a nano π-closed in U contained in H. Then F^c is a nano π-open set in U containing H^c. Since H^c is nano $\pi^{\star}g$-closed, $Ncl(Nint(H^c)) \subseteq F^c$. Consequently $F \subseteq Nint(Ncl(H))$.

Conversely, let $F \subseteq Nint(Ncl(H))$ whenever $F \subseteq H$ and F is nano π-closed in U. Let G be nano π-open containing H^c then $G^c \subseteq Nint(Ncl(H))$. Thus $Ncl(Nint(H^c)) \subseteq G$ implies H is nano $\pi^{\star}g$-open. ■

Remark 3.37. In a space $(U, \tau_R(X))$, both Union and Intersection of two nano $\pi^{\star}g$-open need not be nano $\pi^{\star}g$-open as seen in the following example.

Example 3.38. In Example 3.2, then

1. We have $S = \{c\}$ and $T = \{d\}$. Clearly $S \cup T = \{c, d\}$ is not nano $\pi^\star g$-open.
2. We have $S = \{a, c, d\}$ and $T = \{b, c, d\}$. Clearly $S \cap T = \{c, d\}$ is not nano $\pi^\star g$-open.

Remark 3.39. We obtain Definitions, Theorems, Results and Examples follows from the implications.

nano closed	**nano semi-closed**	**nano pre-closed**
$\searrow$	$\downarrow$	$\swarrow$
nano α-closed $\rightarrow$	**nano $\pi^\star g$-closed**	$\leftarrow$ **nano β-closed**
$\nearrow$	$\uparrow$	$\nwarrow$
nano g-closed	**nano πg-closed**	**nano wg-closed**

None the above implications are reversible.

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