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Fuzzy Soft Proximity Structures – I

V.M.Vijayalakshmi [1], A.Kalaichelvi [2]

[1] *Assistant professor, Department of Science and Humanities, Faculty of Engineering, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, India.*

[2] *Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, India*

Abstract

In this paper the concept of fuzzy soft proximity is introduced in two different ways by referring the fuzzy proximity introduced by Artico, G. and Moresco, R., [1, 2]. It is shown that every fuzzy soft proximity $\tilde{\rho}$ induces a fuzzy soft topology $\tilde{\tau}(\tilde{\rho})$. It is proved that the association $\tilde{\rho} \to \tilde{\tau}(\tilde{\rho})$ is functorial.

Keywords: AM1 – Fuzzy proximity, AM2 – Fuzzy proximity, AM1 – Fuzzy Soft Proximity, AM2 – Fuzzy Soft Proximity, Fuzzy Soft Mapping, AM1– Fuzzy Soft Proximity Mapping.

I. INTRODUCTION

There are many mathematical tools for dealing with uncertainties. The most appropriate theory for dealing with uncertainty is fuzzy set theory introduced by Zadeh in 1965 [11]. In 1968, Chang [3] defined fuzzy topology and later in 1976, Lowen [4] redefined fuzzy topology in a different way. In 1999, Molodstov [6] introduced the concept of soft set and in 2011, Shabir and Naz [9] defined the concept of soft topology. In 2001, Maji et al. [5] introduced the concept of fuzzy soft set and in 2011, Tanay and Kandemir [10] introduced the concept of fuzzy soft topology. In 1984, Artico and Moresco [1, 2] introduced the first definition of fuzzy proximity in two versions.

In this paper the concept of fuzzy soft proximity is introduced in two different ways by following the definition of fuzzy proximity introduced by Artico and Moresco.

In section II of this paper, the preliminary definitions regarding fuzzy soft sets, fuzzy soft topological spaces and fuzzy proximity spaces are given.

In section III of this paper the concept of fuzzy soft proximity is introduced following the sense of Artico and Moresco. It is shown that every fuzzy soft proximity $\tilde{\rho}$ induces a fuzzy soft topology $\tilde{\tau}(\tilde{\rho})$. It is proved that the association $\tilde{\rho} \to \tilde{\tau}(\tilde{\rho})$ is functorial.

II. PRELIMINARY DEFINITIONS

Throughout this paper, X denotes initial universe and E denotes the set of parameters for the universe X.

Definition 2.1[3]

A **fuzzy set** in X is a map f: X $\to$ [0, 1] = I. The family of fuzzy sets in X is denoted by I^X. Following are some basic operations on fuzzy sets. For the fuzzy sets f and g in X,

(1) $f = g \Leftrightarrow f(x) = g(x)$ for all $x \in X$.

(2) $f \le g \Leftrightarrow f(x) \le g(x)$ for all $x \in X$.

(3) $(f \vee g)(x) = \max\{f(x), g(x)\}$ for all $x \in X$.

(4) $(f \wedge g)(x) = \min\{f(x), g(x)\}$ for all $x \in X$.

(5) $f^C(x) = 1 - f(x)$ for all $x \in X$. Here f^c denotes the complement of f.

(6) For a family $\{f_\lambda / \lambda \in \Lambda\}$ of fuzzy sets defined on a set X.

   (i) $(V_{\lambda\in\Lambda} f_\lambda)(x) = V_{\lambda\in\Lambda} f_\lambda(x)$

   (ii) $(\Lambda_{\lambda\in\Lambda} f_\lambda)(x) = \Lambda_{\lambda\in\Lambda} f_\lambda(x)$

(7) For any $\alpha \in$ I, the constant fuzzy set $\boldsymbol{\alpha}$ in X is a fuzzy set in X defined by $\boldsymbol{\alpha}(x) = \alpha$ for all $x \in X$. **0** denotes null fuzzy set in X and **1** denotes universal fuzzy set in X.

Definition 2.2[3]

A **fuzzy topological space** is a pair (X, τ) where X is a nonempty set and τ is a family of fuzzy sets on X satisfying the following properties :

(1) the constant fuzzy sets **0** and **1** belong to τ.

(2) $f, g \in \tau$ implies $f \wedge g \in \tau$

(3) $f_\lambda \in \tau$ for each $\lambda \in \Lambda$ implies $V_{\lambda\in\Lambda} f_\lambda \in \tau$.

Then τ is called a **fuzzy topology** on X. Every member of τ is called fuzzy open. g is called fuzzy closed in

(X, τ) if $g^C \in \tau$.

Definition: 2.3 [1]

A function $\rho: I^X \times I^X \to \{0,1\}$ is called an **AM1-fuzzy proximity** if ρ satisfies the following axioms:

For any f,g,h $\in I^X$,

(AMFP 1) $\rho(0, 1) = 0$

(AMFP 2) $\rho(f, g) = \rho(g, f)$.

(AMFP 3) $\rho(f, g) \vee \rho(h, g) = \rho(f \vee h, g)$.

(AMFP 4) If $\rho(f, g) = 0$, then there exists $h \in I^X$ such that $\rho(f, h) = 0$, $\rho(g, h^C) = 0$.

(AMFP 5) $\rho(f, g) = 0 \Rightarrow f \leq g^C$.

The pair (X, ρ) is called a **AM1-fuzzy proximity space**.

Definition : 2.4 [2]

A function $\rho : I^X \times I^X \to \{0,1\}$ is called an **AM2-fuzzy proximity** if ρ satisfies the following axioms :

For any f, g, h $\in I^X$,

(*AMFP 1) $\rho(0,1) = 0$

(*AMFP 2) $\rho(f, g) = \rho(g, f)$.

(*AMFP 3) $\rho(f, g) \vee \rho(h, g) = \rho(f \vee h, g)$.

(*AMFP 4) If $\rho(f, g) = a$, for every $\varepsilon > 0$, there exists $C \subseteq X$ such that

$$\rho(f, \chi_C) < a + \varepsilon \text{ and } \rho(\chi_{X-C}, g) < a + \varepsilon.$$

(*AMFP 5) $\rho(f, g) > (f \wedge g)(x)$, for every $x \in X$.

(*AMFP 6) If $|g_1 - g_2| \leq \varepsilon$, for $\varepsilon \in I$, then $|\rho(f, g_1) - \rho(f, g_2)| \leq \varepsilon$, for every $f \in I^X$).

$$(\text{ Here } |g_1 - g_2| = \vee_{x \in X} |g_1(x) - g_2(x)|)$$

The pair (X, ρ) is called a **AM2-fuzzy proximity space**.

Definition : 2.5 [1]

Let (X, ρ_1) and (Y, ρ_2) be two AM1 – fuzzy proximity spaces. A function $\theta : X \to Y$ is said to be an **AM1 – fuzzy proximity mapping** if any one of the following equivalent conditions hold:

(1) For every $h, k \in I^X$

$$\rho_2(h,k) = 0 \Rightarrow \rho_1\left(\theta^{-1}(h), \theta^{-1}(k)\right) = 0$$

(2) For every $f, g \in I^X$

$$\rho_1(f,g) = 1 \Rightarrow \rho_2\left(\theta(f), \theta(g)\right) = 1$$

Definition : 2.6 [1]

Let (X, ρ) be an AM1-fuzzy proximity space. For any $f \in I^X$, define $int(f) = \vee \{g \in I^X / \rho(g, f^c) = 0\}$. Then the function $f \to int\ f$ is an interior operator on I^X. The collection $\delta(\rho) = \{f \in I^X / int\ f = f\}$ is a fuzzy topology on X and it is called the fuzzy topology induced by ρ.

Result : 2.7 [1]

Let (X, ρ_1) and (Y, ρ_2) be two AM1-fuzzy proximity spaces. If θ: X → Y is an AM1-fuzzy proximity map, then θ is fuzzy continuous with respect to the corresponding fuzzy topologies $\tau(\rho_1)$ and $\tau(\rho_2)$.

Definition 2.8 [5]

Let $A \subseteq E$. A **soft set** f_A over X is a mapping from E to P(X) i.e., $f_A : E \to P(X)$ where P(X) is the power set of X.

Definition 2.9 [5]

Let $A \subseteq E$. A **fuzzy soft set** $\tilde{f}_A$ over X is a mapping from E to I^X i.e., $\tilde{f}_A : E \to I^X$ where $\tilde{f}_A(e) \neq \mathbf{0}$ if $e \in A \subseteq E$ and $\tilde{f}_A(e) = \mathbf{0}$ if $e \notin A$. The family of fuzzy soft sets over X is denoted by FS(X, E).

Definition 2.10 [5]

The fuzzy soft set $\tilde{f}_A \in FS(X, E)$ is called **null fuzzy soft set** denoted by $\tilde{0}$ if for all e $\in$ E, $\tilde{f}_A(e) = \mathbf{0}$.

Definition 2.11 [5]

The fuzzy soft set $\tilde{f}_A \in FS(X, E)$ is called **universal fuzzy soft set** denoted by $\tilde{1}$ if for all e $\in$ E, $\tilde{f}(e) = \mathbf{1}$.

Definition 2.12 [5]

Let $\tilde{f}_A, \tilde{g}_B \in FS(X, E)$. $\tilde{f}_A$ is called a **fuzzy soft subset** of $\tilde{g}_B$ if $\tilde{f}_A(e) \le \tilde{g}_B(e)$ for every e $\in$ E and we write
$\tilde{f}_A \tilde{\subseteq} \tilde{g}_B$.

Definition 2.13 [5]

Let $\tilde{f}_A, \tilde{g}_B \in FS(X, E)$. $\tilde{f}_A$ and $\tilde{g}_B$ are said to be **equal** denoted by $\tilde{f}_A \cong \tilde{g}_B$ if $\tilde{f}_A \tilde{\subseteq} \tilde{g}_B$ and $\tilde{g}_B \tilde{\subseteq} \tilde{f}_A$.

Definition 2.14 [5]

Let $\tilde{f}_A, \tilde{g}_B \in FS(X, E)$. The **union** of $\tilde{f}_A$ and $\tilde{g}_B$ is also a fuzzy soft set $\tilde{h}_C$ defined by $\tilde{h}_C(e) = \tilde{f}_A(e) \vee \tilde{g}_B(e)$ for all e $\in$ E where C = A $\cup$ B. Here we write $\tilde{h}_C = \tilde{f}_A \tilde{\cup} \tilde{g}_B$.

Definition 2.15 [5]

Let $\tilde{f}_A, \tilde{g}_B \in FS(X, E)$. The **intersection** of $\tilde{f}_A$ and $\tilde{g}_B$ is also a fuzzy soft set $\tilde{h}_C$ defined by $\tilde{h}_C(e) = \tilde{f}_A(e) \wedge \tilde{g}_B(e)$ for all e $\in$ E where C = A $\cap$ B. Here we write $\tilde{h}_C = \tilde{f}_A \tilde{\cap} \tilde{g}_B$.

Definition 2.16 [5]

Let Ω be an index set and $\{(f_A)_i : i \in \Omega\}$ be a family of fuzzy soft sets over X. Then their union $\tilde{\cup}_{i\in\Omega}(f_A)_i$ and intersection $\tilde{\cap}_{i\in\Omega}(f_A)_i$ are defined respectively as follows:

(a) $(\tilde{\cup}_{i\in\Omega}(f_A)_i)(e) = \vee_{i\in\Omega}((f_A)_i(e))$, for all $e \in E$.

(b) $(\tilde{\cap}_{i\in\Omega}(f_A)_i)(e) = \wedge_{i\in\Omega}((f_A)_i(e))$, for all $e \in E$.

Definition 2.17 [7]

Let $\tilde{f}_A \in FS(X, E)$. The **complement** of $\tilde{f}_A$ denoted by $\tilde{f}_A^C$ is a fuzzy soft set defined by

$\tilde{f}_A^C(e) = \mathbf{1} - \tilde{f}_A(e)$ for every $e \in E$. Clearly $(\tilde{f}_A^C)^C = \tilde{f}_A$, $\tilde{1}^C = \tilde{0}$ and $\tilde{0}^C = \tilde{1}$.

Definition 2.18 [10]

A **fuzzy soft topological space** is a triple $(X, E, \tilde{\tau})$ where X is a nonempty set, E is a parameter set and $\tilde{\tau}$ is a family of fuzzy soft sets over X satisfying the following properties :

(1) $\tilde{0}$, $\tilde{1} \in \tilde{\tau}$

(2) $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}$ then $\tilde{f}_A \tilde{\cap} \tilde{g}_B \in \tilde{\tau}$

(3) If $\tilde{f}_{A_i} \in \tilde{\tau}\ \forall\ i \in \Lambda$ then $\tilde{\cup}_{\lambda\in\Lambda} \tilde{f}_{A_i} \in \tilde{\tau}$

Then $\tilde{\tau}$ is called a **fuzzy soft topology** over X. Every member of $\tilde{\tau}$ is called fuzzy soft open. $\tilde{g}_B$ is called fuzzy soft closed in $(X, E, \tilde{\tau})$ if $\tilde{g}_B^C \in \tilde{\tau}$.

Definition 2.19 [8]

Let FS(X, E) and FS(Y, K) be the families of all fuzzy soft sets over (X, E) and (Y, K) respectively. Let

$u : X \to Y$ and $p : E \to K$ be two functions. Then $\tilde{f}_{up}$ is called a fuzzy **soft mapping** from X to Y and denoted by $\tilde{f}_{up} : FS(X, E) \to FS(Y, K)$.

(1) Let $\tilde{f}_A \in FS(X, E)$, then the image of $\tilde{f}_A$ under the fuzzy soft mapping $\tilde{f}_{up}$ is the fuzzy soft set over Y defined by $\tilde{f}_{up}(\tilde{f}_A)$, where

$$\tilde{f}_{up}(\tilde{f}_A)(k)(y) = \vee_{x\in u^{-1}(y)}\left(\vee_{e\in p^{-1}(k)\cap A}\tilde{f}_A(e)\right)(x) \quad \text{if} \quad u^{-1}(y) \neq \phi,\ p^{-1}(k) \cap A \neq \phi$$

$$= 0, \quad \text{otherwise.}$$

(2) Let $\tilde{g}_B \in FS(Y, K)$, then the preimage of $\tilde{g}_B$ under the fuzzy soft mapping $\tilde{f}_{up}$ is the fuzzy soft set over X defined by $(\tilde{f}_{up})^{-1}(\tilde{g}_B)$ where

$$\left(\tilde{f}_{up}\right)^{-1}\left(\tilde{g}_B\right)(e)\,(x) = \begin{cases} \tilde{g}_B(p(e))(u(x)), & \text{for } p(e) \in B \\ 0, & \text{otherwise} \end{cases}$$

Theorem 2.20 [8]

Let $\tilde{f}_A \in FS(X, E)$, $\left(\tilde{f}_{A_i}\right)_{i\in\wedge} \in FS(X, E)$ and $\tilde{g}_B \in FS(Y, K)$, $\left(\tilde{g}_{B_i}\right)_{i\in\wedge} \in FS(Y, K)$, where $\wedge$ is an index set.

(1) If $\tilde{f}_{A_1} \tilde{\subseteq} \tilde{f}_{A_2}$ then $\tilde{f}_{up}\left(\tilde{f}_{A_1}\right) \tilde{\subseteq} \tilde{f}_{up}\left(\tilde{f}_{A_2}\right)$

(2) If $\tilde{g}_{B_1} \tilde{\subseteq} \tilde{g}_{B_2}$ then $(\tilde{f}_{up})^{-1}\left(\tilde{g}_{B_1}\right) \tilde{\subseteq} (\tilde{f}_{up})^{-1}\left(\tilde{g}_{B_2}\right)$

(3) $\tilde{f}_{up}(\tilde{\bigcup}_{i\in\wedge} \tilde{f}_{A_i}) = \tilde{\bigcup}_{i\in\wedge} \tilde{f}_{up}(\tilde{f}_{A_i})$

(4) $\tilde{f}_{up}(\tilde{\bigcap}_{i\in\wedge} \tilde{f}_{A_i}) = \tilde{\bigcap}_{i\in\wedge} \tilde{f}_{up}(\tilde{f}_{A_i})$

(5) $(\tilde{f}_{up})^{-1}(\tilde{\bigcup}_{i\in\wedge} \tilde{g}_{B_i}) = \tilde{\bigcup}_{i\in\wedge} (\tilde{f}_{up})^{-1}(\tilde{g}_{B_i})$

(6) $(\tilde{f}_{up})^{-1}(\tilde{\bigcap}_{i\in\wedge} \tilde{g}_{B_i}) = \tilde{\bigcap}_{i\in\wedge} (\tilde{f}_{up})^{-1}(\tilde{g}_{B_i})$

(7) $(\tilde{f}_{up})^{-1}(\tilde{1}) = \tilde{1}$, $(\tilde{f}_{up})^{-1}(\tilde{0}) = \tilde{0}$

(8) $\tilde{f}_{up}(\tilde{0}) = \tilde{0}$, $\tilde{f}_{up}(\tilde{1}) = \tilde{1}$

Definition 2.21 [8]

Let $(X, E, \tilde{\tau}_1)$ and $(Y, K, \tilde{\tau}_2)$ be two fuzzy soft topological spaces. A fuzzy soft mapping

$\tilde{f}_{up} : (X, E, \tilde{\tau}_1) \to (Y, K, \tilde{\tau}_2)$ is called **fuzzy soft continuous** if $(\tilde{f}_{up})^{-1}(\tilde{g}_B) \in \tilde{\tau}_1$ for all $\tilde{g}_B \in \tilde{\tau}_2$.

III. FUZZY SOFT PROXIMITY STRUCTURES

Definition: 3.1

A function $\tilde{\rho} : FS(X, E) \times FS(X, E) \to \{0,1\}$ is said to be a **AM1 – fuzzy soft proximity** over X if the following conditions are satisfied:

For any $\tilde{f}_A, \tilde{g}_B, \tilde{h}_D \in FS(X, E)$, (AMFSP1) $\tilde{\rho}(\tilde{0},\tilde{1}) = 0$

(AMFSP2) $\tilde{\rho}(\tilde{f}_A,\tilde{g}_B) = \tilde{\rho}(\tilde{g}_B,\tilde{f}_A)$

(AMFSP3) $\tilde{\rho}(\tilde{f}_A,\tilde{g}_B) \vee \tilde{\rho}(\tilde{h}_D,\tilde{g}_B) = \tilde{\rho}(\tilde{f}_A \cup \tilde{h}_D, \tilde{g}_B)$

(AMFSP4) $\tilde{\rho}(\tilde{f}_A,\tilde{g}_B) = 0$, there exists a $\tilde{h}_D \in FS(X,E)$ such that $\tilde{\rho}(\tilde{f}_A,\tilde{h}_D)=0$ and $\tilde{\rho}(\tilde{g}_B,(\tilde{h}_D)^C)=0$

(AMFSP5) $\tilde{\rho}(\tilde{f}_A,\tilde{g}_B) = 0 \Longrightarrow \tilde{f}_A \tilde{\subseteq} (\tilde{g}_B)^C$

The triple (X, E, $\tilde{\rho}$) is said to be a **AM1 – fuzzy soft proximity space**.

Definition: 3.2

A function $\tilde{\rho}$: FS(X, E) x FS(X, E)$\rightarrow \{0,1\}$ is said to be a **AM2 – fuzzy soft proximity** over X if the following conditions are satisfied:

For any $\tilde{f}_A,\tilde{g}_B,\tilde{h}_D \in FS(X,E)$,

(*AMFSP1) $\tilde{\rho}(\tilde{0},\tilde{1}) = 0$

(*AMFSP2) $\tilde{\rho}(\tilde{f}_A,\tilde{g}_B) = \tilde{\rho}(\tilde{g}_B,\tilde{f}_A)$

(*AMFSP3) $\tilde{\rho}(\tilde{f}_A,\tilde{g}_B) \vee \tilde{\rho}(\tilde{h}_D,\tilde{g}_B) = \tilde{\rho}(\tilde{f}_A \cup \tilde{h}_D, \tilde{g}_B)$

(*AMFSP4) $\tilde{\rho}(\tilde{f}_A,\tilde{g}_B) = 0$, there exists $\tilde{h}_D \in FS(X,E)$ such that $\tilde{\rho}(\tilde{f}_A,\tilde{h}_D) < a + \varepsilon$ and $\tilde{\rho}((\tilde{h}_D)^C,\tilde{g}_B) < a + \varepsilon$

(*AMFSP5) $\tilde{\rho}(\tilde{f}_A,\tilde{g}_B) \geq (\tilde{f}_A \tilde{\cap} \tilde{g}_B)(e)(x)$ for every $e \in E$ and for every $x \in X$.

(*AMFSP6) If $|\tilde{g}_{B_1} - \tilde{g}_{B_2}| \leq \varepsilon$, for $\varepsilon \in I$, $|\tilde{\rho}(\tilde{f}_A,\tilde{g}_{B_1}) - \tilde{\rho}(\tilde{f}_A,\tilde{g}_{B_{21}})| \leq \varepsilon$, for every $\tilde{f}_A \in FS(X,E)$

(Here $|\tilde{g}_{B_1} - \tilde{g}_{B_2}| = \vee \{|\tilde{g}_{B_1} - \tilde{g}_{B_2}|$ such that $e \in E$, $x \in X\}$)

The triple (X, E, $\tilde{\rho}$) is said to be an **AM2 – fuzzy soft proximity space**.

Definition: 3.3

Let (X, E, $\tilde{\rho}$) be a AM1 – fuzzy soft proximity space. For $\tilde{f}_A \in FS(X,E)$, define

$\operatorname{int}\tilde{f}_A = \tilde{\cup}\{\tilde{g}_B \in FS(X,E) / \tilde{\rho}(\tilde{g}_B,(\tilde{f}_A)^C = 0\}$.

Proposition 3.4

The function $\text{int}: FS(X,E) \to FS(X,E)$ is a fuzzy soft interior operator.

Proof:

(i) $\text{int}\,\tilde{1} = \tilde{1}$ follows from the axiom (AMFSP1)

(ii) $\text{int}\tilde{f}_A \,\tilde{\subseteq}\, \tilde{f}_A$ follows from the axiom (AMFSP5)

(iii) Trivially $\text{int}(\text{int}\tilde{f}_A) \,\tilde{\subseteq}\, \text{int}\ \tilde{f}_A$

Take $\tilde{g}_B \in FS(X,E)$ such that $\tilde{\rho}(\tilde{g}_B\ (\tilde{f}_A)^C) = 0$

$\therefore$ There exists

$\tilde{h}_D \in FS(X,E)$ such that $\tilde{\rho}\ (\tilde{g}_B, (\tilde{h}_D)^C) = 0$ and $\tilde{\rho}\ (\tilde{h}_D, (f_A)^C) = 0$

$\therefore \tilde{g}_B \,\tilde{\subseteq}\, \text{int}\,\tilde{h}_D$ and $\tilde{h}_D \,\tilde{\subseteq}\, \text{int}\,\tilde{f}_A$

$\therefore \tilde{g}_B \,\tilde{\subseteq}\, \text{int}(\text{int}\,\tilde{f}_A)$, for every $\tilde{g}_B \in FS(X,E)$ for which $\tilde{\rho}(\tilde{g}_B, (f_A)^C) = 0$

$\therefore \text{int}\,\tilde{f}_A \,\tilde{\subseteq}\, \text{int}(\text{int}\,\tilde{f}_A)$

$\therefore \text{int}\,\tilde{f}_A = \text{int}(\text{int}\,\tilde{f}_A)$

(iv) Trivially $\text{int}(\tilde{f}_A \,\tilde{\cap}\, \tilde{g}_B) \,\tilde{\subseteq}\, (\text{int}\,\tilde{f}_A)\ \tilde{\cap}\,(\text{int}\,\tilde{g}_B)$

Suppose $\text{int}(\tilde{f}_A \,\tilde{\cap}\, \tilde{g}_B) \,\tilde{\subset}\, (\text{int}\,\tilde{f}_A)\ \tilde{\cap}\,(\text{int}\tilde{g}_B)$, then there exists $e \in E$, $x \in X$ such that

$t_1 = \ (\text{int}(\tilde{f}_A \,\tilde{\cap}\, \tilde{g}_B))(e)(x) < t < ((\text{int}\ \tilde{f}_A)\ \tilde{\cap}\,(\text{int}\ \tilde{g}_B))(e)(x) = t_2$

$\tilde{h}_{D_1}(e)(x) > t$, $\tilde{h}_{D_2}(e)(x) > t$

$\therefore\ \tilde{\rho}(\tilde{h}_{D_1}\ \tilde{\cap}\,\tilde{h}_{D_2},\ (\tilde{f}_A)^C\ \tilde{\cup}\ (\tilde{g}_B)^C) = 0$ and $(\tilde{h}_{D_1}\ \tilde{\cap}\,\tilde{h}_{D_2})(e)(x) > t$

$\therefore\ \tilde{\rho}(\tilde{h}_{D_1}\ \tilde{\cap}\,\tilde{h}_{D_2},\ (\tilde{f}_A \,\tilde{\cap}\, \tilde{g}_B)^C) = 0$ and $(\tilde{h}_{D_1}\ \tilde{\cap}\,\tilde{h}_{D_2})(e)(x) > t$

(1)

$t_1 = (\text{int}(\tilde{f}_A\ \tilde{\cap}\ \tilde{g}_B))(e)(x) < t$

$\Rightarrow$ for every $\tilde{h}_D \in FS(X,E)$, $\tilde{\rho}\ (\tilde{h}_D, (\tilde{f}_A\ \tilde{\cap}\ \tilde{g}_B)^C) = 0$ and $\tilde{h}_D(e)(x) < t$

(2)

$\therefore$ (1) and (2) are contradictory.

$\therefore\quad \text{int}(\tilde{f}_A \,\tilde{\cap}\, \tilde{g}_B) = (\text{int}\ \tilde{f}_A)\ \tilde{\cap}\ (\text{int}\,\tilde{g}_B)$

Therefore the function $\tilde{f}_A \to \text{int}\,\tilde{f}_A$ is an interior operator.

Definition:3.5

The fuzzy soft topology induced by the AM1-fuzzy soft proximity $\tilde{\rho}$ over X is denoted by $\tilde{\tau}(\tilde{\rho})$ and it consists of all the fuzzy soft sets $\tilde{f}_A \in FS(X,E)$ for which $\tilde{f}_A = \text{int}\ \tilde{f}_A$.

Definition: 3.6

Let (X, E, $\tilde{\rho}_1$) and (Y, K, $\tilde{\rho}_2$) be two AM1-fuzzy soft proximity spaces. Let $u : X \to Y$ and $p : E \to K$ be the mappings. A function $\tilde{f}_{up} : FS(X,E) \to FS(Y,K)$ is said to be a **AM1 – fuzzy soft proximity mapping** if any one of the following equivalent conditions hold:

(1) for every $\tilde{h}_D, \tilde{k}_M \in FS(Y,K)$, $\tilde{\rho}_2(\tilde{h}_D, \tilde{k}_M) = 0 \Rightarrow \tilde{\rho}_1((\tilde{f}_{up})^{-1}(\tilde{h}_D), (\tilde{f}_{up})^{-1}(\tilde{k}_M)) = 0$

(2) for every $\tilde{f}_A, \tilde{g}_B \in FS(X,E)$, $\tilde{\rho}_1(\tilde{f}_A, \tilde{g}_B) = 1 \Rightarrow \tilde{\rho}_2(\tilde{f}_{up}(\tilde{f}_A), \tilde{f}_{up}(\tilde{g}_B)) = 1.$

Theorem : 3.7

Let (X, E, $\tilde{\rho}_1$) and (Y, K, $\tilde{\rho}_2$) be two AM1-fuzzy soft proximity spaces. If $\tilde{f}_{up} : (X, E, \tilde{\rho}_1) \to (Y, K, \tilde{\rho}_2)$ is said to be an AM1– fuzzy soft proximity map, then $\tilde{f}_{up} : (X, E, \tilde{\tau}(\tilde{\rho}_1)) \to (Y, K, \tilde{\tau}(\tilde{\rho}_2))$ is fuzzy soft continuous.

Proof:

Let $\tilde{g}_B \in \tilde{\tau}(\tilde{\rho}_2)$

$\therefore \tilde{g}_B = \text{int}\ \tilde{g}_B = \tilde{\bigcup}\{\tilde{h}_D \in FS(Y,K) \ / \ \tilde{\rho}_2(\tilde{h}_D, (\tilde{g}_B)^C) = 0\}$

$\therefore (\tilde{f}_{up})^{-1}(\tilde{g}_B) = \tilde{\bigcup}\{(\tilde{f}_{up})^{-1}(\tilde{h}_D) \ / \ \tilde{\rho}_2(\tilde{h}_D, (\tilde{g}_B)^C) = 0\}.$

$\tilde{\rho}_2(\tilde{h}_D, (\tilde{g}_B)^C) = 0 \Rightarrow \tilde{\rho}_1((\tilde{f}_{up})^{-1}(\tilde{h}_D), (\tilde{f}_{up})^{-1}((\tilde{g}_B)^C)) = 0$

$\Rightarrow \tilde{\rho}_1((\tilde{f}_{up})^{-1}(\tilde{h}_D), ((\tilde{f}_{up})^{-1}(\tilde{g}_B))^C) = 0$

$\therefore (\tilde{f}_{up})^{-1}(\tilde{g}_B) \tilde{\subseteq} \tilde{\bigcup}\{(\tilde{f}_{up})^{-1}(\tilde{h}_D) \ / \ \tilde{\rho}_1((\tilde{f}_{up})^{-1}(\tilde{h}_D), ((\tilde{f}_{up})^{-1}(\tilde{g}_B))^C) = 0$

$\tilde{\subseteq} \tilde{\bigcup}\{\tilde{k}_M \in FS(X,E) / \tilde{\rho}_1(\tilde{k}_M, ((\tilde{f}_{up})^{-1}(\tilde{g}_B))^C) = 0$

$= \text{int}\ ((\tilde{f}_{up})^{-1}(\tilde{g}_B))$

$\therefore (\tilde{f}_{up})^{-1}(\tilde{g}_B) \tilde{\subseteq} \text{int}\ ((\tilde{f}_{up})^{-1}(\tilde{g}_B))$

Hence $(\tilde{f}_{up})^{-1}(\tilde{g}_B) = \text{int}\ ((\tilde{f}_{up})^{-1}(\tilde{g}_B))$

$\therefore (\tilde{f}_{up})^{-1}(\tilde{g}_B) \tilde{\subseteq} \tilde{\tau}(\tilde{\rho}_1)$

$\therefore \tilde{f}_{up}$ is fuzzy soft continuous .

CONCLUSION

In this paper the concept of fuzzy soft proximity is introduced by using the definition of Artico and Moresco fuzzy proximity. It is proved that every fuzzy soft proximity induces a fuzzy soft topology.

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