Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 14, Number 3 (2018), pp. 347–355

http://www.ripublication.com/gjpam.htm

Connected Edge-Vertex Mixed Domination on S-Valued Graphs

S. Kiruthiga Deepa
*Bharath Niketan College of Engineering,
Aundipatti - 625536, Tamilnadu, INDIA.*

M. Chandramouleeswaran
*Saiva Bhanu Kshatriya College,
Aruppukottai - 626101, Tamilnadu, INDIA.*

Abstract

In this paper we discuss the notion of connected $ev-$weight $m-$domination set on the $S-$valued graph, G^S.

AMS subject classification: 05C25, 16Y60.
Keywords: S-valued graphs, ev-weight m-domination number, connected ev-weight m-domination set..

1. Introduction

Corresponding to a given graph $G = (V, E)$, Chandramouleeswaran et al., [5] introduced the concept of S-valued graph $G^S = (V, E, \sigma, \psi)$, where σ is a mapping from $V \to S$ and ψ is a mapping from $E \to S$, called respectively the σ-set and ψ-set. In [2] we have discussed the notion of edge-vertex mixed domination on G^S. In [1] the authors discussed the notion of connected weight domination vertex set and in [4] the authors discussed on connected weight domination edge set of G^S. Motivated by this, in this paper we discuss the notion of connected ev-weight m-domination set on G^S.

2. Preliminaries

In this section, we recall some basic definitions that are needed for our work.

Definition 2.1. A semiring $(S, +, \cdot)$ is an algebraic system with a non-empty set S together with two binary operations $+$ and $\cdot$ such that

1. $(S, +, 0)$ is a monoid.
2. $(S, \cdot)$ is a semigroup.
3. For all $a, b, c \in S$, $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$.
4. $0 \cdot x = x \cdot 0 = 0 \; \forall \, x \in S$.

Definition 2.2. Let $(S, +, \cdot)$ be a semiring. A Canonical Pre-order $\preceq$ in S defined as follows: for $a, b \in S$, $a \preceq b$ if and only if, there exists an element $c \in S$ such that $a + c = b$.

Definition 2.3. [5] Let $G = (V, E \subset V \times V)$ be a given graph with $V, E \neq \phi$. For any semiring $(S, +, \cdot)$, a semiring-valued graph (or a S-valued graph), G^S, is defined to be the graph $G^S = (V, E, \sigma, \psi)$ where $\sigma : V \to S$ and $\psi : E \to S$ is defined to be

$$\psi(x, y) = \begin{cases} min\,\{\sigma(x), \sigma(y)\} & if\ \sigma(x) \preceq \sigma(y)\ or\ \sigma(y) \preceq \sigma(x) \\ 0 & otherwise \end{cases}$$

for every unordered pair (x, y) of $E \subset V \times V$. We call σ, a S-vertex set and ψ, a S-edge set of G^S.

Definition 2.4. [5] Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. The length of the path $P^S(uv)$, denoted by $l(P^S(uv))$, is defined to be the number of edges along the path $P^S(uv)$.

Definition 2.5. [2] Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. Let $T \subseteq E$. If every vertex of G^S is weight m-dominated by any edge in T, then T is said to be a ev-weight m-dominating set.

Definition 2.6. [3] Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. The edge-vertex mixed domination number of G^S denoted by $\gamma^S_{EV}(G^S)$ is defined by $\gamma^S_{EV}(G^S) = (|T|_S, |T|)$, where T is the minimal ev-weight m-dominating set.

3. Connected Vertex-Edge Mixed Domination on S-valued Graphs

In this section, we introduce the notion of connected edge-vertex mixed domination in S-valued graph, analogous to the notion in crisp graph theory, and prove some simple results.

Definition 3.1. Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. A ev-weight m-dominating set $T \subseteq E$ of G^S is said to be a connected ev-weight m-dominating set if $\langle T \rangle^S$ is connected in G^S.

Example 3.2. Let $(S = \{0, a, b, c\}, +, \cdot)$ be a semiring with the following Cayley Tables:

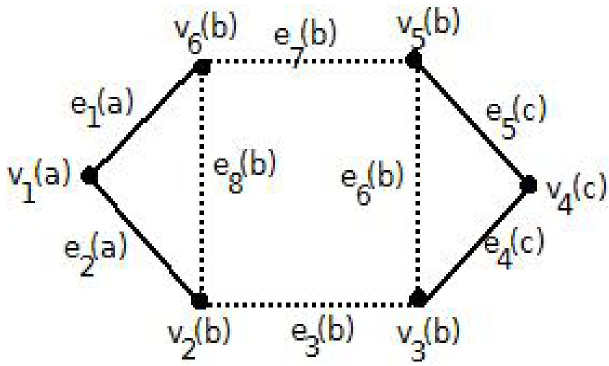
$+$	0	a	b	c
0	0	a	b	c
a	a	a	b	c
b	b	b	b	b
c	c	c	b	c

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	0	a	0
b	0	a	b	c
c	0	0	c	c

Let $\preceq$ be a canonical pre-order in S, given by

$$0 \preceq 0,\ 0 \preceq a,\ 0 \preceq b,\ 0 \preceq c,\ a \preceq a,\ a \preceq b,\ a \preceq c,\ b \preceq b,\ c \preceq b,\ c \preceq c$$

Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$.



Define $\sigma : V \to S$ by

$$\sigma(v_1) = a, \sigma(v_2) = \sigma(v_3) = \sigma(v_5) = \sigma(v_6) = b, \sigma(v_4) = c$$

and $\psi : E \to S$ by

$$\psi(e_1) = \psi(e_2) = a, \psi(e_3) = \psi(e_6) = \psi(e_7) = \psi(e_8) = b, \psi(e_4) = \psi(e_5) = c$$

Consider the set $T_1 = \{e_3, e_6\}$.
Here the vertices $v_1, v_2, v_3, v_4, v_5$and$v_6 \in \langle N_S[e_3] \rangle$,
also $\sigma(v_1) = a \preceq b = \psi(e_3)$
$\sigma(v_2) = b \preceq b = \psi(e_3)$
$\sigma(v_3) = b \preceq b = \psi(e_3)$
$\sigma(v_4) = c \preceq b = \psi(e_3)$
$\sigma(v_5) = b \preceq b = \psi(e_3)$
$\sigma(v_6) = b \preceq b = \psi(e_3)$

$\therefore$ The edge e_3 is a ev-weight m-dominating edge of the vertices v_1, v_2, v_3, v_4, v_5 and v_6.
And the vertices v_2, v_3, v_4, v_5 and $v_6 \in \langle N_S[e_6]\rangle$,
also $\sigma(v_2) = b \preceq b = \psi(e_6)$
$\sigma(v_3) = b \preceq b = \psi(e_6)$
$\sigma(v_4) = c \preceq b = \psi(e_6)$
$\sigma(v_5) = b \preceq b = \psi(e_6)$
$\sigma(v_6) = b \preceq b = \psi(e_6)$
$\therefore$ The edge e_6 is a ev-weight m-dominating edge of the vertices v_2, v_3, v_4, v_5 and v_6.
Hence $T_1 = \{e_3, e_6\}$ is a ev-weight m-dominating set. Also $\langle T_1\rangle^S$ is connected.
Therefore T_1 is a connected ev-weight m-dominating set.
Similarly $T_2 = \{e_6, e_7\}, T_3 = \{e_7, e_8\}, T_4 = \{e_3, e_8\}, T_5 = \{e_3, e_6, e_7\}, T_6 = \{e_6, e_7, e_8\}, T_7 = \{e_3, e_7, e_8\}, T_8 = \{e_3, e_6, e_8\}, T_9 = \{e_3, e_6, e_7, e_8\}$ are all connected ev-weight m-dominating sets.

Remark 3.3. From the definition, we have, every connected ev-weight m-dominating set is a ev-weight m-dominating set. But the converse need not be true as shown in the following example.

In example 3.2, clearly $T_{10} = \{e_3, e_7\}$ and$T_{11} = \{e_6, e_8\}$ are ev-weight m-dominating sets, and also $\langle T_{10}\rangle^S$, $\langle T_{11}\rangle^S$ are not connected. Hence T_{10}, T_{11} are not connected ev-weight m-dominating sets.

Definition 3.4. Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$.A subset $T \subseteq E$ is said to be a minimal connected ev-weight m-dominating set, if

1. T is a connected ev-weight m-dominating set.
2. No proper subset of T is a connected ev-weight m-dominating set.

In example 3.2, clearly T_1, T_2, T_3, T_4 are minimal connected ev-weight m-dominating sets.

Definition 3.5. Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. A subset $T \subseteq E$ is said to be a maximal connected ev-weight m-dominating set, if

1. T is a connected ev-weight m-dominating set.
2. there is no connected ev-weight m-dominating set $T' \subset E$ such that $T \subset T' \subset E$.

In example 3.2, clearly T_9 is a maximal connected ev-weight m-dominating set.

Definition 3.6. Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. The connected edge-vertex mixed domination number of G^S is defined by $\gamma^S_{CEV}(G^S) = (|T|_S, |T|)$, where T is the minimal connected ev-weight m-dominating set.

In example 3.2, the connected edge-vertex mixed domination number is $\gamma^S_{CEV}(G^S) = (|T_1|_S, |T_1|) = (|T_2|_S, |T_2|) = (|T_3|_S, |T_3|) = (|T_4|_S, |T_4|) = (b, 2)$.

Theorem 3.7. A S-valued graph G^S will have a connected ev-weight m-dominating set if and only if G^S is connected.

Proof. Let $C_i^S = (V_i, E_i, \sigma_i, \psi_i)$ be the connected components of G^S, $i = 1, 2, \ldots, m$ where $\sigma_i = \sigma \backslash v_i$, $\psi_i = \psi \backslash E_i$.
Let T_i be the ev-weight m-dominating set of C_i^S whose elements have maximal S-value. Since a ev-weight m-dominating set T of G^S will have an edge from every component of G^S, $T = \cup_{i=1}^m T_i$.
Now T is a connected ev-weight m-dominating set $\Leftrightarrow$ $\langle T \rangle^S$ is connected.
$\Leftrightarrow$ there is a common vertex between $e_i \in T_i$ and $e_j \in T_j$, $i \neq j, i, j = 1, 2, \ldots, m$
$\Leftrightarrow$ G^S is connected. ■

Theorem 3.8. For a S-valued graph $G^S = (V, E, \sigma, \psi)$,

$$\gamma_{EV}^S(G^S) \preceq \gamma_{CEV}^S(G^S) \preceq 3\gamma_{EV}^S(G^S) + 2(0, -1)$$

Proof. By definition, Every connected ev-weight m-dominating set is necessarily a ev-weight m-dominating set $\Rightarrow \gamma_{EV}^S(G^S) \preceq \gamma_{CEV}^S(G^S)$.
Let T be a ev-weight m-dominating set of G^S, such that $\gamma_{EV}^S(G^S) = (|T|_S, |T|)$, and let the induced subgraph $\langle T \rangle^S$ have m components, then $|T| \geqslant m$.

Claim: There exists two components C_i^S and C_j^S where $i \neq j$ of $\langle T^S \rangle$ such that the length of a shortest path between C_i^S and C_j^S is atmost 3 in G^S. Assume that there exists a shortest path between C_i^S and C_j^S of length atleast 4. Let P_S be the shortest of all shortest path between two distinct components of $\langle T \rangle^S$. Hence we can find a edge $(e, \psi(e))$ in the path P_S such that e is at a distance of atleast 2 from the end points of P_S. Since T is a ev-weight m-dominating set, then the edge e must be at a distance of atmost 1 from a component. Thus the edge e lies on a path P'_S between the two components C_i^S and C_j^S, $(i \neq j)$ such that $l(P'_S) \leqslant l(P_S)$. This contradicts the assumption that the length of the path P_S is atleast 4. This proves that, there exists two components C_i^S and C_j^S where $i \neq j$ of $\langle T \rangle^S$ such that the path between them has length atmost 3. Adding an edge in the path P to the ev-weight m-dominating set T, decreases the number of components of $\langle T \rangle^S$ by 1. Continuing this procedure, we obtain only one component in $\langle T \rangle^S$, proving that T is a ev-weight m-dominating set. Thus we can add atmost $2(m-1)$edges to the ev-weight m-dominating set T, so as to form a connected ev-weight m-dominating set.

Thus $\gamma_{CEV}^S(G^S) \preceq (|T|_S, |T|) + 2(\sum_{i=1}^{m-1} \psi(e_i), m - 1)$

$\preceq (|T|_S, |T|) + 2(\sum_{e \in T} \psi(e) + 0, (|T| - 1))$

$\preceq (|T|_S, |T|) + 2((|T|_S, |T|) + (0, -1))$

$= \gamma_{EV}^S(G^S) + 2(\gamma_{EV}^S(G^S) + (0, -1))$

$= 3\gamma_{EV}^S(G^S) + 2(0, -1)$.

Hence $\gamma^S_{EV}(G^S) \preceq \gamma^S_{CEV}(G^S) \preceq 3\gamma^S_{EV}(G^S) + 2(0, -1)$. ■

4. Some Discussion on Mixed Domination Numbers on S-Valued Graphs

In this section, we discuss and compare the mixed domination numbers. In particular we establish the following theorem.

Theorem 4.1. For any S-valued graph G^S, we have

$$\gamma^S_{EV}(G^S) \preceq \gamma^S_{CEV}(G^S) \preceq \gamma^S_{VE}(G^S) \preceq \gamma^S_{CVE}(G^S).$$

First we discuss some examples.

Example 4.2. Let $(S = \{0, a, b, c\}, +, \cdot)$ be a semiring with the following Cayley Tables:

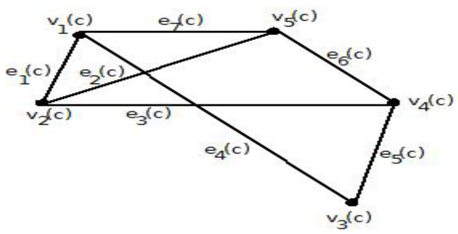
$+$	0	a	b	c
0	0	a	b	c
a	a	b	c	c
b	b	c	c	c
c	c	c	c	c

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	b	c	c
c	0	c	c	c

Let $\preceq$ be a canonical pre-order in S, given by

$$0 \preceq 0,\ 0 \preceq a,\ 0 \preceq b,\ 0 \preceq c,\ a \preceq a,\ a \preceq b,\ a \preceq c,\ b \preceq b,\ b \preceq c,\ c \preceq c$$

Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$



Clearly $D = \{v_1, v_2, v_3\}$ is the minimal ve-weight m-dominating set and $\gamma^S_{VE}(G^S) = (c, 3)$. Also $\langle D\rangle^S$ is connected. Hence D is a minimal connected ve-weight m-dominating set and $\gamma^S_{CVE}(G^S) = (c, 3)$. Clearly $T = \{e_1\}$ is the minimal ev-weight m-dominating set and $\gamma^S_{EV}(G^S) = (c, 1)$. Also $\langle T\rangle^S$ is connected. Hence T is a minimal connected ev-weight m-dominating set and $\gamma^S_{CEV}(G^S) = (c, 1)$. Hence we conclude that, if G^S is a vertex S-regular and an edge S-regular graph, then

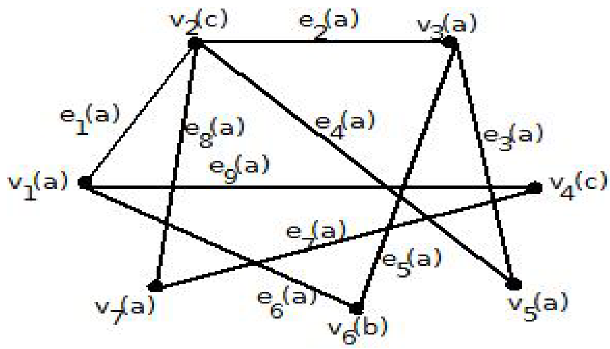
$$\gamma^S_{EV}(G^S) \preceq \gamma^S_{CEV}(G^S) \preceq \gamma^S_{VE}(G^S) \preceq \gamma^S_{CVE}(G^S).$$

Remark 4.3. If G^S is not a vertex regular but an edge regular graph, then

$$\gamma^S_{EV}(G^S) \preceq \gamma^S_{CEV}(G^S) \preceq \gamma^S_{VE}(G^S) \preceq \gamma^S_{CVE}(G^S).$$

For an edge regular graph G^S, which is not a vertex S-regular, we have $\gamma^S_{EV}(G^S) = \gamma^S_{CEV}(G^S) = (\psi(e), 0)$ where $\psi(e) \in S$ is smaller, for some $e \in E$. This is illustrated by the following example.

Example 4.4. Let $(S = \{0, a, b, c\}, +, \cdot)$ be a semiring with the canonical preorder given in example 4.2. Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$



Clearly $D = \{v_1, v_2, v_3, v_4\}$ is the minimal ve-weight m-dominating set and $\gamma^S_{VE}(G^S) = (c, 4)$. Also $\langle D\rangle^S$ is connected. Hence D is a minimal connected ve-weight m-dominating set and $\gamma^S_{CVE}(G^S) = (c, 4)$. Clearly G^S is an edge regular graph with minimal S-value a. So any edge of G^S can not weight m-dominate some vertex of G^S, since it is not a vertex S-regular graph. Therfore there will be no ev-weight m-dominating set. Hence $\gamma^S_{EV}(G^S) = \gamma^S_{CEV}(G^S) = (a, 0)$. Hence we conclude that

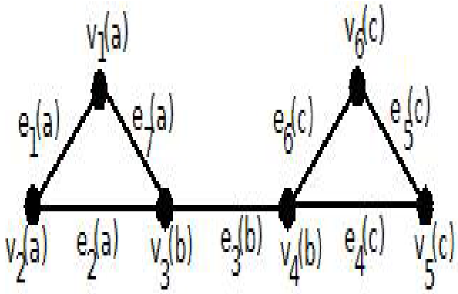
$$\gamma^S_{EV}(G^S) \preceq \gamma^S_{CEV}(G^S) \preceq \gamma^S_{VE}(G^S) \preceq \gamma^S_{CVE}(G^S).$$

Remark 4.5. If G^S is not a vertex regular and not an edge regular graph, then

$$\gamma^S_{EV}(G^S) \preceq \gamma^S_{CEV}(G^S) \preceq \gamma^S_{VE}(G^S) \preceq \gamma^S_{CVE}(G^S).$$

We illustrate this in the following example.

Example 4.6. Let $(S = \{0, a, b, c\}, +, \cdot)$ be a semiring with the canonical preorder given in example 3.2
Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$.



Clearly $D = \{v_3, v_4\}$ is the minimal ve-weight m-dominating set and $\gamma^S_{VE}(G^S) = (b, 2)$. Also $\langle D\rangle^S$ is connected. Hence D is a minimal connected ve-weight m-dominating set and $\gamma^S_{CVE}(G^S) = (b, 2)$. Clearly$T = \{e_3\}$ is the minimal ev-weight m-dominating set and $\gamma^S_{EV}(G^S) = (b, 1)$. Also $\langle T\rangle^S$ is connected. Hence T is a minimal connected ev-weight m-dominating set and $\gamma^S_{CEV}(G^S) = (b, 1)$. Hence we conclude that

$$\gamma^S_{EV}(G^S) \preceq \gamma^S_{CEV}(G^S) \preceq \gamma^S_{VE}(G^S) \preceq \gamma^S_{CVE}(G^S).$$

Now we give the proof of the theorem 4.1.

Proof of Theorem 4.1**:**
Since every connected ve-weight m-dominating set is, by definition, a ve-weight m-dominating set, we have $\gamma^S_{VE}(G^S) \preceq \gamma^S_{CVE}(G^S)$. Similarly every connected ev-weight m-dominating set is, by definition, a ev-weight m-dominating set, we have $\gamma^S_{EV}(G^S) \preceq \gamma^S_{CEV}(G^S)$. To prove the theorem, it is enough to prove that $\gamma^S_{CEV}(G^S) \preceq \gamma^S_{VE}(G^S)$. Let $D \subseteq V$ be a minimal ve-weight m-dominating set, so that

$$\gamma^S_{VE}(G^S) = (\sum_{v_i \in D} \sigma(v_i), |D|).$$

Let $T \subseteq E$ be a minimal connected ev-weight m-dominating set. For any $e \in T$, the end vertices will dominate the other edges. And T will have minimum number of edges of G^S that are connected whose end vertices will dominate other edges. In that way, the end vertices of the edges in T will be an element of D. However all these vertices need not dominate the edges not in T. For, let $f \in E$ such that $f \notin T$.If the end vertex of f dominate the edges that are in T, then there is nothing to prove. Otherwise, if the end vertex of f dominate the edges that are not in T, then those vertices will be in D. Hence we need more vertices which will dominate the edges that are not in T. Hence $|D| \geqslant |V(\langle T\rangle^S)|$. Therefore $\gamma^S_{VE}(G^S) \succeq \gamma^S_{CEV}(G^S)$. ■

References

[1] Jeyalakshmi. S and Chandramouleeswaran. M, *Connected weight dominating vertex set on S-valued Graphs*, International Journal of Pure and Applied Mathematics (2017), Vol No. 112, pp 39–45, ISSN: 1314-3395, doi: 10.12732/ijpam.v112i5.16.

[2] Kiruthiga Deepa. S and Chandramouleeswaran. M, *Edge - Vertex Mixed Domination on S-Valued Graphs*, Mathematical Sciences International Research Journal, (2017), Vol No.6, pp. 142–146 ISSN: 2278-8697

[3] Kiruthiga Deepa. S and Chandramouleeswaran. M, *Mixed Domination Numbers on S-Valued Graphs*, Mathematical Sciences International Research Journal, (2017), Vol No. 6, Spl Issue(2017) pp. 248–252, ISSN: 2278-8697

[4] Mangala Lavanya. S and Chandramouleeswaran. M, *Connected weight dominating edge set on S-valued Graphs*, World Journal of Engineering Research and Technology (2017), ISSN: 2454-695X, Vol. 3, Issue 5, pp. 284–293.

[5] Rajkumar. M., Jeyalakshmi. S and Chandramouleeswaran. M, *Semiring-valued Graphs*, International Journal of Math. Sci. and Engg. Appls., Vol. 9 (III), 2015, 141–152.