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Non-Smooth Multi objective Programs with Generalized $(\phi-\nu-\rho)$-Invexity

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Abstract

In this paper, we introduce a new class of functions called locally as Lipschitz (ϕ, ν, ρ) invex functions. Some properties are studied. In this, various optimality and duality results were derived for such a class of non smooth multi objective programming problem.
Key words: Non smooth multi objective programme, optimality, duality, (ϕ, ν, ρ) – invexity.

1. INTRODUCTION

In the recent past, several attempts were made by different authors to define various classes of differentiable and non-differentiable non-convex functions to study their optimality and duality results [1, 29]. First Hanson et al. [10] generalized the concept of convexity by introducing invexity for differentiable mathematical programming problem. Later, Craven [29] named it as invex. Many generalizations of invexity concept have been given in the literature [1,2,7,10] for both smooth and non-smooth optimization problems. Recently, many authors (eg.[7,8,12,13]) studied various optimality and duality results for non-smooth locally Lipschitz optimization problems using Clarke's sub differentials. Further, Hanson's invex function was also generalized for non smooth problem. In [7] Caristi et.al., generalized invexity notion to the non-differentiability case by defining invexity for Lipschitz function. Also, he used principal analytic tool as a generalized gradient in the Clarke's sense.

In [24, 25], Reddy and Mukherjee generalized invexity notion to the non-differentiable case by defining invexity for Lipschitz function.

Also the definition of non smooth invex function was weakened function by different authors (eg.[1,18,19,25]) to establish different optimality and duality results. Consequently other generalized convex functions introduced like ρ convexity Vail et. Al.[30] F- convexity by Hanson and Mond [31], (F,ρ)-convexity by Preda [23], $(\phi-\rho)$- invexity by Caristi et al [7], and generalized (ϕ,ρ)-invexity by Antczak et al. [1].

The main aim of this paper is to introduce (ϕ,v,ρ)-invexity for locally Lipschitz (ϕ,v,ρ)invex function to establish optimality conditions and Mond-Weir duality results for a new class of function using non smooth programs. Further, Kuk, Lee and Kim [12] discussed $v-\rho$-invexity for vector valued functions of non smooth multi objective programs.

2. PRELIMINARIES

Let R^n be the n-dimensional Euclidean space and R^n_+ be its non-negative part of R^n. Throughout this chapter, the following convections for vectors in R^n will be used $x > y$ if and only if $x_i > y_i, i = 1,2,...,n$ $x \geq y$ if and only if $x_i \geq y_i$ for $i = 1,2,...,n$ and x not less than y is the negative of $x > y$.

Definition 2.1. A real –valued function $f : R^n - R$ is said to be locally Lipschitz if for $z \in R^n$ there exists a positive constant K and a neighborhood N of z, such that, for $x, y \in N$, we have $|f(x) - f(y)| \leq K\|x - y\|$, where || || denotes a norm in R^n.

Definition 2.2. The Clarke generalized directional derivative of a locally Lipschitz function $f : R^n \to R$ at $x \in R^n$ in the direction $d \in R^n$ denoted by $f^0(x;d)$

$$f^0(x;d) = \lim sub_{\substack{y \to r \\ t \downarrow 0}} \frac{f(y+td) - f(y)}{t}$$

Definition 2.3. The Clarke generalized sub gradient of $f : R^n \to R$ at $x \in R^n$ denoted $\partial f(x)$ is defined as:

$$\partial f(x) = \left\{ \xi \in R^n : f^0(x; d) \geq \xi^T d, \forall d \in R^n \right\}$$

Definition 2.4. Let $f : R^n \to R$ be a locally Lipschitz function on R^n. A point $\bar{x} \in R^n$ is said to be a stationary point of f, if $0 \in \partial f(\bar{x})$.

In this chapter, we consider the following non smooth multi objective composite program (MCP).

(MCP) Minimize $(f_1(F_1(x)), f_2(F_2(x), ..., f_n(F_n(x))$

Subject to $g_j(G_j(x)) \leq 0, j = 1, 2, ..., m$

And $x \in X = \left\{ x \in R^n : g_j(G_j(x,)) \leq 0, j = 1, 2, ..., m \right\}$

Where $f_i(F_i) : R^n \to R, i = 1, 2, ..., n$

$g_j(G_j) : R^n \to R, j = 1, 2, ..., m$ are locally Lipschitz functions.

Now, we define $(\phi - v - \rho)$-invexity for locally Lipschitz functions as follows.

Definition 2.5. Let $f_i(F_i) : R^n \to R$ and $g_j(G_j) : R^n \to R$ be locally Lipschitz functions for $i = 1, 2, ..., n$ and $j = 1, 2, ..., m$, respectively.

(i) $f(F) = (f_1(F_1), f_2(F_2), ..., f_n(F_n))$ is $(\phi - v - \rho)$-invex with respect to function η and θ: $R^n \times R^n \to R^n$, If there exists a function $\phi : R^n \times R^n \times R^{n+1} \to R$

Where $\phi(x,u,...)$ is convex on $R^{n+1} \phi(x, u, (0, a)) \geq 0$ for every $x \in R^n$, and any $a \in R_+$ such that $\alpha_i(x,u)[(f_i((F_i)(x_i) - f_i(F_i))(x) \geq \phi(x,u)(\xi, \eta) + \rho_i \left\| \phi(x,u) \right\|^2$

(ii) $g(G) = (g_1(G_1), g_2(G_2), ..., g_m(G_m))$ is said to be $(\phi - v - \sigma)$-invex with respect to function η and $\theta : R^n \times R^n \to R^n$, there exists $\beta_j : R^n \times R^n \to R_+ \setminus \{0\}$ and $\sigma_i \in R, j = 1, 2, ..., m$,and $\phi : R^n \times R^n \times R^{n+1} \to R$ Where $\phi(x,u.)$ is convex on R^{n+1}, $\phi = (x, u.(0, a)) \geq 0$ for every $x \in R^n$, and any $a \in R_+$ such that

$$\beta_j(x,u)[(g_j(G_j))(x) - (g_j(G_j))(u)] \geq \phi(x,u)\xi_j \eta(x,u) + \sigma_j \left\| (x,u) \right\|^2$$

Based on Sawaragi et.al [32], we have the following.

Definition 2.6. Let $u \in X \subset R^n$ is said to be a weak minimum of (MCP) if there exists no integers $x \in X \subset R^n$, such that $f_i(F_i(x)) \leq f_i(F_i(u)), i = 1, 2, ..., p.$

3. RESULTS

Theorem 3.1. (Generalized Fritz - John Necessary optimality conditions)

Let $\tilde{x} \in R^n$ be an optimal solution in problem (MCP) then, there exists $\tilde{\tau} \in R^n{}_+, \tilde{\lambda} \in R^m{}_+$ Such that

$$0 \in \tilde{\tau}\partial(f_i(F_i(x)) + \sum_{j=1}^{m} \tilde{\lambda}_j \partial g_j(G_j(\tilde{x})) \qquad (3.1)$$

$$\tilde{\lambda}_j(g_j(G_j(\tilde{x})) = 0, j \in \{1, 2, ..., m\} \qquad (3.2)$$

$$(\tilde{\tau}, \tilde{\lambda}) \neq (0, 0) \qquad (3.3)$$

Then, corresponding Karush-Khun Tucker conditions.

Theorem 3.2. Let $\tilde{x} \in R^n$ be an optimal solution in problem (MCP) and some suitable constraints qualification to be satisfied at $\tilde{x}$ then, there exists $\tilde{\lambda} \in R^m$ such that

$$0 \in \partial(f(F(\tilde{x})) + \sum_{j=1}^{m} \tilde{\lambda}_j \partial g_j(G_j(\tilde{x})) \qquad (3.4)$$

$$\tilde{\lambda}_j(g_j(G_j(\tilde{x})) = 0, j = \{1, 2, ..., m\} \qquad (3.5)$$

$$\tilde{\lambda}_j \in R_+, j \in I \qquad (3.6)$$

Definition 3.1. The point $(\tilde{x}, \tilde{\lambda}) \in R^n \times R^m{}_+$ is said to be a Karush-Kuhn-Tucker Point for the considered non smooth optimization problem (MCP), if there exits conditions (3.1), (3.2), (3.3, (3.4)), (3.5), (3.6) are satisfied at $\tilde{x}$ with the Lagrange multipliers $\tilde{\lambda} \in R^m{}_+$.

Definition 3.2. Let (x, τ, λ) be a Lagrange's function. Then, the Lagrange function associated with the constraint minimization problem (MCP) is the function $L : R^n \times R_+ \times R_+^m \to R$, defined as $L(x_0, \lambda_0, \tau_0) = \tau_0 f(F(x_0)) + \sum_{j=1}^{m} \lambda_{0j}(g_j(G_j))(x_0).$

3.1. Generalized Slater constraint qualification

Let $\tilde{x} \in R^n$ is said to satisfy the Slater constraint qualification for the considered non smooth multi objective composite program, if there exists a feasible solution $\tilde{x}$ such that $(g_j(G_j(\tilde{x}))<0, j\in I(\tilde{x})$. And more over $(g_j(G_j)), j\in I(\tilde{x})$ are locally Lipschitz $(\phi-\nu-\rho g_j(G_j))$ invex at $\tilde{x}$ on R^n.

Now, we state and prove optimality conditions for the considered non smooth multi objective composite program (MCP) under generalized $(\phi-\nu-p)$-invexity.

Theorem 3.3. Let $(\tilde{x},\tilde{\lambda},\tilde{\tau})\in R^n\times R_+\times R_+^m$ be a feasible solution in the consider problem (MCP) at which Generalized Fritz John necessary optimality conditions are satisfied. Further, assume that the constraint functions $(g_j(G_j)), j\in I(\tilde{x})$ Lipschitz $(\phi-\nu-\rho g_j(G_j))$ invex at $\tilde{x}$ on R^n and $\sum_{j\in I(x)}\tilde{\lambda}_j\rho g_j(G_j)\geq 0$. If the generalized Slater's constraint qualification is satisfied at $\tilde{x}$ than the Generalized Karush-Khun Tucker optimality conditions are satisfied.

Proof: Suppose $\tilde{x}$ be a feasible solution to the problem (MCP), at which the above stated Generalized Fritz-John optimality conditions (3.1), (3.2), (3.3) are satisfied with the corresponding Lagrange multipliers $\tilde{\lambda}\in R_+^m, \tilde{\tau}\in R_+$.

Again since the generalized Slater's constraint qualification is satisfied at $\tilde{x}$ then we need to prove that the Lagrange multiplies $\tilde{x}$ associated with the objective function is not equal to 0.

We prove this by contradiction.

Let us suppose that $\tilde{x}=0$

Then, by condition (3.3) we have $\sum_{j=1}^{m}\tilde{\tau}_j>0$

Let us denote $\alpha_j=\dfrac{\tilde{\lambda}_j}{\sum_{j=1}^{m}\tilde{\lambda}_j}, j\in I$

Hence $0\leq\alpha_j\leq 1, j\in I$ and $\sum_{j=1}^{m}\alpha_j=1$

Also, $\alpha_j(g_j(G_j))(\tilde{x}), j\in I$ (3.7)

But, the constraints functions $(g_jG_j(\tilde{x})), j\in I,(\tilde{x})$ are locally Lipschitz $(\phi-\nu-\rho_i)$ invex at $\tilde{x}$ and R^n then, by det, we have

$$(g_j(G_j))(x)-(g_j(G_j))(\tilde{x})\geq\phi\eta(x,\tilde{x})(\xi_j,\rho(g_j(G_j))+\rho\left\|\vartheta(x,\tilde{x})\right\|^2, j\in I(\tilde{x}).$$

Holds for any $\zeta_j \in \partial g_j(G_j(\tilde{x}), j \in I(\tilde{x})$ and for all $+\left(\dfrac{\partial \eta}{\partial t}\right)^T h_x(t, x, x^*, u^*)$.

Since $0 \le v_j \le 1, j \in I$ then, the above in equations yields,

$$\alpha_i(x,\tilde{x})[g_j(G_j)(x) - g_j(G_j)(\tilde{x})] \ge \alpha_j \phi\eta(x, \tilde{x}(\xi_j \rho g_j(G_j)) + \rho\|\theta(x,\tilde{x})\|^2, j \in I(\tilde{x})$$

Adding both sides of the above, we get

$$\sum_{j\in I(\tilde{x})} \alpha_j(g_j(G_j))(x) - \sum_{j\in I(\tilde{x})} \alpha_j(\rho g_j(G_j))(x) \ge \sum_{j\in I(\tilde{x})} \alpha_j \phi\eta(x, \tilde{x}, (\xi_j, \rho g_j(G_j)) \quad (3.8)$$

But, by definition, $\phi_i(x,\tilde{x})$ is a convex function on R^{n+1}.

Also, since $0 \le \alpha_j \le 1, j \in I$ and $\sum_{j=1}^{m} \alpha_j = 1$ then, definition of generalize invex,

we have

$$\sum_{j\in I(\tilde{x})} \alpha_j \phi\eta(x,\tilde{x})(\xi_j, \rho g_j(G_j)) \ge \phi\eta(x,\tilde{x})\left\lfloor \sum_{j\in I(\tilde{x})} \alpha_j \xi_j, \sum_{j\in I(\tilde{x})} \alpha_j \rho g_j(G_j) \right\rfloor \quad (3.9)$$

Now, combining (3.7) and (3.8), we obtain

$$\sum_{j\in I(\tilde{x})} \alpha_j(g_j(G_j))(x) - \sum_{j\in I(\tilde{x})} \alpha_j(g_j(G_j))(\tilde{x}) \ge \phi\eta(x,\tilde{x})[\sum_{j\in I(\tilde{x})} \alpha_j \xi_j, \sum_{j\in I(\tilde{x})} \alpha_j \rho g_j(G_j)] \quad (3.10)$$

But our assumption is $\tilde{\tau} = 0$. Therefore, by Generalized Fritz- John necessary optimality condition (3.3), we have $\sum_{j\in I(\tilde{x})} \tilde{\lambda}_j \xi_j = 0$ then by definition of α_j

we have $$\sum_{j\in I(\tilde{x})} \alpha_j \xi_j = 0 \quad (3.11)$$

From (3.10) and (3.11), one can obtain

$$\sum_{j\in I(\tilde{x})} \alpha_j(g_j(G_j))(x) - \sum_{j\in I(\tilde{x})} \alpha_j(g_j(G_j))(\tilde{x}) \ge \phi\eta(x, \tilde{x}, (0, \sum_{j\in I(\tilde{x})} \alpha_j \rho g_j(G_j))$$

But from defamation of convexity we have $\phi(x, \tilde{x}, (0,u)) \ge 0$ for every $x \in R^n$ any $a \in R_+$,

By assumption we have $\sum_{j\in I(\tilde{x})} \alpha_j(g_j(G_j))(x) - \sum_{j\in I(\tilde{x})} \alpha_j(g_j(G_j))(\tilde{x}) \ge 0 \ \ldots$

which holds for all $x \in R^n$.

According to assumption, generalized Slater's constraints qualification is satisfied then, there exists a feasible point $\tilde{x}$ such that $(g_j(G_j))(\tilde{x}) < 0, \quad j \in I(\tilde{x})$. Since the above inequality (5.3.6.7) is satisfied for all $x \in R^n$ then it is also satisfied for $x = \tilde{x}$

$$i.e. \quad \sum_{j \in I(\tilde{x})} \alpha_j (g_j(G_j))(\tilde{x}) = 0 \tag{3.14}$$

$$0 \le \alpha_j \le 1, j \in I(\tilde{x}).$$

$$\sum_{i=1}^{m} \alpha_j = 1, \text{ and } (g_j(G_j))(\tilde{x}) < 0, j \in I(\tilde{x})$$

$$\text{We get} \quad \sum_{j \in I(\tilde{x})} \alpha_j (g_j(G_j))(\tilde{x}) < 0 \tag{3.15}$$

This is a contradiction to inequality (3.15), hence $\tilde{\tau} \neq 0$.

This completes the proof of the theorem.

Theorem 3.4. If $(\tilde{x}, \tilde{\lambda}) \in R^n \times R_+^m$ is a Krause Kuhn Tenses point in the problem (MCP),

Let us assume that the objective $f_i(F_i)$ is locally Lipschitz $(\phi - v - \rho_{f(F)})$- invex at $\tilde{x} \in R^n$ also the constraint function $g_j(G_j)$, $j \in I(\tilde{x})$ are locally Lipschitz

$(f - v - \rho g_j(G_j))$-invex at x on R^n and $\rho f(F) + \sum_{j \in I(\tilde{x})} \lambda_j \rho g_j(G_j) \ge 0$. Then $\tilde{x}$ is an optimal for the problem (MCP).

Proof: Proof is straight forward [25].

The following theorem illustration sufficient optimality conditions

Theorem 3.5. If $\tilde{x}$ is a feasible solution in problem (MCP), in which Generalized Krause- Kuhn- Tucker necessary optimality conditions are satisfied. Further, assume that the Lagrangian $L(x, \tau, \bar{\lambda}_0)$ locally Lipschitz $(\phi - v - \rho)$-invex for $\tilde{x} \in R^n$ with $\rho \ge 0$ then $\tilde{x}$ is optimal solution to the problem (MCP).

Proof: Proof is similar to [25]

4. DUALITY

Here, we consider the Mond- Weir type duality for the problem (MCP)

(MDP) maximize $f(F(y))$

Subject to $0 \in \partial(\tilde{\tau} f(F)) + \sum_{j=1}^{m} \tilde{\lambda}_j \partial(g_j(G_j))y$

$$\sum_{j=1}^{m} \lambda_j g_j(G_j(y)) \geq 0, \tilde{\tau} \in R_+$$

$$\tilde{\lambda}_j \in R_+ \ (\tilde{\tau}, \tilde{\lambda}_j) \geq 0$$

Now, the set of feasible solutions are denoted by

$$D = \left\{ (y, \tilde{\tau}, \tilde{\lambda}_0) \in R^n \times R \times R^m : 0 \in \partial(\tilde{\tau} f(F) + \sum_{j-1}^{m} \tilde{\lambda}_j g_j(G_j))(y) - \sum_{j=1}^{m} \tilde{\lambda}_j (g_j(G_j)(y) \geq 0, \tilde{\tau} > 0, \tilde{\lambda}_0 \geq 0 \right\}$$

Also $E = \left\{ Y \in R^n : (y, \tilde{\tau}, \tilde{\lambda}_0) \in D \right\}$. The following is used in the sequel according Antczak [1].

Lemma 4.1. If $(y, \tilde{\tau}, \tilde{\lambda}_0)$ is a certain feasible solution for dual (MDP) Further, assume that $g_j(G_j)$ $j \in I(y)$ is locally Lipschitz (ϕ, v, ρ) - invex not necessary with respect to the same ρ and η.

Theorem 4.1. (Weak Duality). If $\tilde{x}$ and $(\tilde{y}, \tilde{\tau}, \tilde{\lambda})$ are feasible solutions for primal problem (MCP) and corresponding dual problem (MDP), respectively further assume that $(f(F))$ is locally Lipschitz $(\phi - v - \rho f(F))$ invex at $\tilde{y}$ on $R^n UE$.

if $\tilde{\tau} \rho f(F) + \sum_{j \in I(\tilde{y})} \tilde{\lambda}_j \rho g_j(G_j) \geq 0$ then $(f(F(x)) \geq (f(F(y))$

Proof: proof is similar to [25].

Theorem 4.2. (Strong Duality). If $\tilde{x}$ is an optimal solution for problem (MCP) and certain Slater's Constraint qualification is satisfied at $\tilde{x}$ then there exists $\tilde{\tau} \in R_+$ such that $\tilde{\lambda}_0 \in R_+^m$ such that $(\tilde{x}, \tilde{\tau}, \tilde{\lambda}_0)$ is feasible for (MDP) and the objective functions of primal (MCP) and dual (MDP) are equal.

Proof: proof is similar to [25].

Theorem: 4.3. (Converse Duality). Assume that $\tilde{x}$ and $(\tilde{y},\tilde{\tau},\tilde{\lambda})$ are efficient solution of (MCP) and (MCD) respectively, then there exists vectors $\tilde{\lambda}\mu$ such that the optimal values are equal.

Proof: proof is similar to [25].

5. CONCLUSION

In this paper, the authors discussed optimality conditions. As an application Mond-Weir type duality results were proved under generalized invexity. These are the dynamic generalizations of the earlier results existing in the literature as a special case.

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