Stolarsky-3 Mean Labeling of Some Special Graphs

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Abstract

Let G = (V, E) be a graph with p vertices and q edges. G is said to be Stolarsky-3 Mean graph if each vertex x ∈ V is assigned distinct labels f(x) from 1,2,...,q+1 and each edge e=uv is assigned the distinct labels f(e=uv) = \[ \left\lfloor \frac{f(u)^2 + f(u)f(v) + f(v)^2}{3} \right\rfloor \] (or) \[ \left\lceil \frac{f(u)^2 + f(u)f(v) + f(v)^2}{3} \right\rceil \] then the resulting edge labels are distinct. In this case f is called a Stolarsky-3 Mean labeling of G and G is called a Stolarsky-3 Mean graph. In this paper we investigate the Stolarsky-3 Mean labeling of some special graphs.

Keywords: Graph Labeling, Mean Labeling, Stolarsky-3 Mean Labeling, Slanting Ladder, Triangular Ladder, H-graph, Twig graph, Middle graph, Total graph.
1. INTRODUCTION

The graphs $G = (V,E)$ considered in this paper are finite, undirected and without loops or multiple edges. We follow Gallian\[1\] for all detailed survey of graph labeling and we refer Harary\[2\] for all other standard terminologies and notations. The concept of “Mean Labeling of graphs” has been introduced S. Somasundaram, R. Ponraj and S.S. Sandhya in 2004\[3\] and S. Somasundaram and S.S. Sandhya introduced the concept of “Harmonic Mean Labeling of graphs” in\[4\]. “Stolarsky-3 Mean Labeling of graphs” was introduced by S.S. Sandhya, E. Ebin Raja Merely and S. Kavitha \[7\].

The following definitions are necessary for the present study.

**Definition 1.1:** A graph $G$ with $p$ vertices and $q$ edges is said to be Stolarsky-3 Mean graph if each vertex $x \in V$ is assigned distinct labels $f(x)$ from $1, 2, \ldots, q+1$ and each edge $e=uv$ is assigned the distinct labels $f(e=uv) = \left\lceil \sqrt{\frac{f(u)^2 + f(u)f(v) + f(v)^2}{3}} \right\rceil$ (or) $\left\lfloor \sqrt{\frac{f(u)^2 + f(u)f(v) + f(v)^2}{3}} \right\rfloor$ then the resulting edge labels are distinct. In this case $f$ is called a Stolarsky-3 Mean labeling of $G$.

**Definition 1.2:** The Slanting ladder $SL_n$ is a graph obtained from two points $u_1, u_2, \ldots, u_n$ & $v_1, v_2, \ldots, v_n$ by joining each $u_i$ with $v_{i+1}$ $1 \leq i \leq n - 1$.

**Definition 1.3:** A Triangular ladder is a graph obtained from $L_n$ by adding the edges $u_i v_{i+1}$, $1 \leq i \leq n - 1$, where $u_i$ and $v_i$ $1 \leq i \leq n$ are the vertices of $L_n$ such that $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ are two paths of length $n$ in the graph $L_n$.

**Definition 1.4:** The H-graph of a path $P_n$ is the graph obtained from two copies of $P_n$ with vertices $v_1, v_2, v_3, \ldots, v_n$ & $u_1, u_2, \ldots, u_n$ by joining the vertices $v_{n+1}$ & $u_{n+1}$ if $n$ is odd and the vertices $v_{\frac{n+1}{2}}$ & $u_{\frac{n+1}{2}}$ if $n$ is even.

**Definition 1.5:** The Middle graph $M(G)$ of a graph $G$ is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of $G$ or one is a vertex of $G$ and the other is an edge incident on it.
**Definition 1.6:** The Total graph \( T(G) \) of graph \( G \) is the graph whose vertex set is \( V(G) \cup E(G) \) and two vertices are adjacent whenever they are either adjacent or incident in \( G \).

**Definition 1.7:** A graph \( (V,E) \) obtained from a path by attaching exactly two pendant edges to each interval vertices of the path is called a Twig graph.

### 2. MAIN RESULTS

**Theorem 2.1:** Slanting Ladder \( SL_n \) is Stolarsky-3 Mean graph.

**Proof:** Let \( G \) be the slanting ladder graph with the vertices \( u_1, u_2, ..., u_n \) and \( v_1, v_2, ..., v_n \).

Define a function \( f : V(G) \to \{ 1, 2, ..., q+1 \} \) by

\[
\begin{align*}
f(u_i) &= 3i, \quad 1 \leq i \leq n - 1. \\
f(u_n) &= 3n - 2. \\
f(v_1) &= 1. \\
f(v_i) &= 3i - 4, \quad 2 \leq i \leq n.
\end{align*}
\]

Then the edges are labeled with

\[
\begin{align*}
f(u_i u_{i+1}) &= 3i + 1, \quad 1 \leq i \leq n - 1. \\
f(u_i v_{i+1}) &= 3i - 1, \quad 1 \leq i \leq n - 1. \\
f(v_1 v_2) &= 1. \\
f(v_i v_{i+1}) &= 3(i - 1), \quad 2 \leq i \leq n - 2.
\end{align*}
\]

Then the edge labels are distinct.

Hence \( SL_n \) is Stolarsky-3 Mean graph.

**Example 2.2:** The Stolarsky-3 Mean labeling of \( SL_6 \) is given below.

![Figure 1](image-url)
**Theorem 2.3:** Triangular Ladder $TL_n$ is Stolarsky-3 Mean graph.

**Proof:** Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be two paths of length $n$. Join $u_i$ and $v_i$, $1 \leq i \leq n$, and join $u_i$ and $v_{i+1}$, $1 \leq i \leq n - 1$. The resulting graph is $TL_n$.

Define a function $f : V(TL_n) \to \{1, 2, \ldots, q+1\}$ by

- $f(u_i) = 4i - 2$, $1 \leq i \leq n$.
- $f(v_1) = 1$.
- $f(v_i) = 4(i-1)$, $2 \leq i \leq n$.

Then the edges are labeled with

- $f(u_iu_{i+1}) = 4i$, $1 \leq i \leq n - 1$.
- $f(u_iv_i) = 4i - 3$, $1 \leq i \leq n$.
- $f(v_iv_{i+1}) = 4i - 2$, $1 \leq i \leq n - 1$.
- $f(u_iv_{i+1}) = 4i - 1$, $1 \leq i \leq n - 1$.

Then the edge labels are distinct. Hence $TL_n$ is Stolarsky-3 Mean graph.

**Example 2.4:** The Stolarsky-3 Mean labeling of $TL_6$ is given below.

![Figure 2](image.png)

**Theorem 2.5:** $H$ graph is Stolarsky-3 Mean graph for all $n$ if $n$ is even and $n \leq 11$ if $n$ is odd.

**Proof:** Let $G$ be the graph with the vertices $v_1, v_2, \ldots, v_n$ & $u_1, u_2, \ldots, u_n$.

Define a function $f : V(G) \to \{1, 2, \ldots, q+1\}$ by
\[ f(v_i) = i, \quad 1 \leq i \leq n. \]
\[ f(u_i) = n + i, \quad 1 \leq i \leq n. \]

Then the edges are labeled as
\[ f(v_i v_{i+1}) = i, \quad 1 \leq i \leq n - 1. \]
\[ f(u_i u_{i+1}) = n + i, \quad 1 \leq i \leq n - 1. \]
\[ f(\frac{v_{n+1} u_{n+1}}{2}) = n \quad \text{if } n \text{ is odd.} \]
\[ f(\frac{v_{n/2+1} u_{n/2}}{2}) = n \quad \text{if } n \text{ is even.} \]

Then we get distinct edge labels.

Hence \( f \) is Stolarsky-3 Mean labeling.

**Example 2.6:** The labeling pattern of H graph is given below.

When \( n=5 \)
Theorem 2.7: Twig graph $T_m$ is Stolarsky-3 Mean graph.

Proof: Let $G$ be the twig graph.

Let $u_1, u_2, ..., u_n$ be the vertices of the path $P_n$ and $v_1, v_2, ..., v_{n-2} \& w_1, w_2, ..., w_{n-2}$ be two pendant vertices attached to $u_i$.

Define a function $f: V(G) \to \{1,2,..., q+1\}$ by

$f(u_1) = 1.$

$f(u_i) =3i-4$, $2 \leq i \leq n.$

$f(v_i) = 3i$, $1 \leq i \leq n - 2.$

$f(w_i) = 3i+1$, $1 \leq i \leq n - 2.$

Then the edges are labeled with

$f(u_iu_{i+1}) =3i -2$, $1 \leq i \leq n - 1.$

$f(v_{i} u_{i}) =3i -1$, $1 \leq i \leq n - 2.$

$f(w_{i} u_{i}) =3i$, $1 \leq i \leq n - 2.$
Then the edge labels are distinct.

Hence \( f \) is Stolarsky-3 Mean labeling.

**Example 2.8:** The Stolarsky-3 Mean labeling of Twig graph \( T_3 \) is given below.

![Figure 4](image)

**Theorem 2.9:** Middle graph \( M(P_n) \) is Stolarsky-3 Mean graph.

**Proof:** Let \( u_1, u_2, \ldots, u_n \) & \( v_1, v_2, \ldots, v_{n-1} \) be the vertices of the middle graph \( G=M(P_n) \).

By definition of middle graph \( V(M(P_n)) = V(P_n) \cup E(P_n) \) and whose edge set is

\[
E(M(P_n)) = \begin{cases} 
  u_i v_i, & 1 \leq i \leq n - 1 \\
  u_i v_{i-1}, & 2 \leq i \leq n \\
  v_i v_{i+1}, & 1 \leq i \leq n - 2 
\end{cases}
\]

Here \( |V(G)| = 2n-1 \) and \( |E(G)| = 3n-4 \).

We define \( f: V(G) \to \{1,2,3,\ldots,q+1\} \) by

\[
f(u_i) = 1,
\]

\[
f(v_i) = 3i - 1, \quad 1 \leq i \leq n - 1.
\]

Then the edges are labeled with

\[
f(u_i v_i) = 3i - 2, \quad 1 \leq i \leq n - 1.
\]

\[
f(u_i v_{i-1}) = 3i - 1, \quad 2 \leq i \leq n - 1.
\]

\[
f(v_i v_{i+1}) = 3i, \quad 1 \leq i \leq n - 2.
\]

Then the edge labels are distinct.

Hence Middle graph \( M(P_n) \) is stolarsky-3 Mean graph.
Example 2.10: The Stolarsky-3 Mean labeling of $M(P_6)$ is given below.

![Figure: 5](image)

Theorem 2.11: Total graph $T(P_n)$ is Stolarsky-3 Mean graph.

Proof: Let $u_1, u_2, \ldots, u_n$ & $v_1, v_2, \ldots, v_{n-1}$ be the vertices of the Total graph $T(P_n)$.

By definition of Total graph $V(T(P_n)) = V(P_n) \cup E(P_n)$ and

$$E(T(P_n)) = \begin{cases} 
    u_iu_{i+1}, & 1 \leq i \leq n-1. \\
    u_iv_i, & 1 \leq i \leq n-1. \\
    u_iv_{i-1}, & 2 \leq i \leq n. \\
    v_iv_{i+1}, & 1 \leq i \leq n-2. 
\end{cases}$$

Here $|V(G)| = 2n-1$ and $|E(G)| = 4n-5$.

Define $f: V(G) \to \{1, 2, \ldots, q+1\}$ as follows.

- $f(u_1) = 2$.
- $f(u_i) = 4i-1, \quad 2 \leq i \leq n.$
- $f(v_i) = 4i-3, \quad 1 \leq i \leq n - 1.$

Then the edges are labeled with

- $f(u_iu_{i+1}) = 4i - 2, \quad 1 \leq i \leq n - 1.$
- $f(u_iv_i) = 4i - 3, \quad 1 \leq i \leq n - 1.$
- $f(u_iv_{i-1}) = 4i - 2, \quad 2 \leq i \leq n.$
- $f(v_iv_{i+1}) = 4i, \quad 1 \leq i \leq n - 2.$

Then the edge labels are distinct.

Hence $T(P_n)$ is Stolarsky-3 Mean graph.
Example 2.12: The Stolarsky-3 Mean labeling of $T(P_6)$ is given below.

![Graph Image]

Figure: 6

REFERENCES
