Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 14, Number 11 (2018), pp. 1533-1537

http://www.ripublication.com/gjpam.htm

A Discovery of the taxi –cab number 1729

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ABSTRACT

In this paper we prove the taxi-cab number Theorem. The theorem states that'' we can separate a positive integer as a sum of two different cubes in two different ways.

The paper also demonstrates the answers to the following questions.

1- Why,1729 is the smallest taxi –cab number ?
2- How can we point out the series of hardy- Ramanujan numbers ?
3- Either Fermate's statement is true or false ?

We connect the elimination method in the pair of third power equations to prove the taxi –cab number theorem and to point out answers to the above questions.

M.S.C : 14GO5 , 14H52 ,

Keywords: Elementry Number Theory, Hardy- Ramanujan numbers, Law's of Exponents, F.L.T, Pair of third power Equations, Elimination Method, Transponding.

1. INTRODUCTION

This is a famous story about one of India's great mathematical genius Shri Niwash Ramanujan. Once another famous mathematician .G.H. hardy came to visit him in a taxi whose number was 1729.while talking to Ramanujan, Hardy described this number "a dull number''

Ramanujan quickly pointed out that no Mr. Hardy, 1729 was indeed interesting .he said it is the smallest number that can be expressed as a sum of two cubes in two different ways.

2. RELATION TO OTHER CONJECTURES

2.1- Fermate's last theorem: we cannot separate a cube as a sum of two cubes.

2.2- Pythagoras theorem: we can separate a square as a sum of two squares.

3. THE PROOF

Taxi-Cab Number Theorem: we can separate a positive integer as a sum of two different cubes in two different ways

Proof:

We know that

$a^3+b^3+c^3-3abc =[a+b+c]\ [a^2+b^2+c^2-ab-bc-ca]$

$a^3+b^3+c^3= [a+b+c]\ [a^2+b^2+c^2-ab-bc-ca] +3abc$

If, $[a+b+c]\ [a^2+b^2+c^2-ab-bc-ca] +3abc=d^3$

Then, $a^3+b^3+c^3=d^3$____[1] here, a,b,c and d all are positive integers.

First three third power equations are given below to understand.

$3^3+4^3+5^3= 6^3$______[II]

$1^3+6^3+8^3= 9^3$_____[iii]

$3^3+10^3+18^3=19^3$——[iv]

Now let a pair of third power equations.

$a_1^3+b_1^3+c_1^3=d_1^3$______[v]

$a_2^3+b_2^3+c_2^3=d_2^3$______[vi]

Case1: If $a_1=a_2$ and $c_1=c_2$

Then we can re- write the pair of equations [v] and [vi] as

$a_1^3+b_1^3+c_1^3= d_1^3$____[v]

$a_1^3+b_2^3+c_1^3=d_2^3$______[vi]

Subtract equations [vi] from equations [v] to eliminate two terms in the L.H.S of the pair of third power equations [v] and [vi].

$b_1^3-b_2^3=d_1^3-d_2^3$

$b_1^3+d_2^3=d_1^3+b_2^3=x$________[vii] here x is a positive integer.

Case2: If $b_1=b_2$ and $d_1=d_2$ in the pair of equations [v] and [vi]

Then we can re-write the pair of equations [v] and [vi] as

$a_1^3+b_1^3+c_1^3+=d_1^3$ ______[v]

$a_2^3+b_1^3+c_2^3=\ d_1^3$ _______[vi]

Subtract equations [vi] from equation [v]to eliminate one term in the L.H.S and the final term in the R.H.S of the pair of third power equation [v] and [vi]

$a_1^3-a_2^3+c_1^3-c_2^3=0$

$a_1^3+c_1^3=a_2^3+c_2^3=y$ ______[viii] here y is a positive integer.

4. NUMERICALS

Now we have to pear into numerical examples to find out the answers for three questions mentioned in the abstract.

Let first three third power equations

$3^3+4^3+5^3=6^3$ ______[ii]

$1^3+6^3+8^3+=9^3$ ______[iii]

$3^3+10^3+18^3=19^3$— [iv]

Five examples are given below to understand

***Example 1*:** multiply equation [ii] by 2^3

$6^3+8^3+10^3=12^3$ ___[ix]

$6^3+8^3+1^3=9^3$ _______[lll]

Subtract equation [iii] from equation [ix]

$10^3-1^3=12^3-9^3$

$9^3+10^3=1^3+12^3=1729$------------[X]

***Example 2*:** multiply equation [ii] by 3^3 and equation [iii] by 2^3

$9^3+12^3+15^3=18^3$ ______[xi]

$2^3+12^3+16^3=18^3$ ____[xii]

Subtract equation [xii] from equation [xi]

$9^3-2^3+15^3-16^3=0$

$9^3+15^3=2^3+16^3= 4104$______[xiii] here 4104 is not a cube number

***Example 3*:** multiply equation [ii] by 4^3 and equation [iii] by 2^3

$12^3+16^3+20^3=24^3$______[xiv]

$12^3+16^3+2^3=18^3$_____[xv]

Subtract equation [xv]from equation [xiv]

$20^3-2^3=24^3-18^3$

$18^3+20^3=2^3+24^3=13832$———[xvi]

Here, 13832 is not a cube number

***Example 4*:** multiply equation [iii] by 3^3

$3^3+18^3+24^3=27^3$______[xvii]

$3^3+18^3+10^3=19^3$_____[iv]

Subtract equation [iv] from equation [xvii]

$24^3-10^3=27^3-19^3$

$19^3+24^3=10^3+27^3=20683$--------[xviii]

Here, 20683 is not a cube number

***Example 5*:** multiply equation [ii] by 6^3 and equation [iii] by 4^3

$18^3+24^3+30^3=36^3$____[xix]

$4^3+24^3+32^3=36^3$_____[xx]

Subtract equation [xx] from equation [xix]

$18^3-4^3+30^3-32^3=0$

$18^3+30^3=4^3+32^3=32832$———[xxi]

Here, 32832 is not a cube number

5. FINDINGS

5.1 According to the equation [1] we can separate a cube as a sum of three different cubes.

5.2 According to the equation [vii] and equation [viii] if two same terms of a pair of third power equations can be eliminated then, We observe that there are many positive integers those can be separate as a sum of two different cubes in two different ways . In this way the Taxi-Cab Number Theorem has been proved.

5.3 Equation [x] is the product of the connection of elimination method in the simplest pair of third power equations that satisfies the Taxi-Cab Number Theorem. Therefore 1729 is the smallest number that can be expressed as a sum of two different cubes in two different ways.

5.4 Equations [x] ,[xiii][xvi],[xviii],and [xxi] proves the series of first five Hardy – Ramanujan number .

5.5 Anyhow we could not find the sum of two cubes is equal to a cube. It proves the truth of Fermate's statement.

6. REMARK

All Pythagorean triplets may not be obtained using the form $2m, m^2-1, m^2+1$ for m is positive integer >1. For example another triplet 5,12,13

CONCLUSION

Hence, The Taxi-Cab Number Theorem has been proved.

The truth of the fermate's last Theorem and the series of first five Hardy – Ramanujan Numbers have been discovered.