Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 14, Number 10 (2018), pp. 1347–1362

http://www.ripublication.com/gjpam.htm

Numerical Reconstruction and Remediation of Soil Acidity on A one Dimensional Flow Domain with Constant and Linear Temporally Dependent Flow Parameters

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Abstract

A one dimensional mass transport equation whose solution is ill-posed is considered to model flow of solutes in porous medium. The diffusion coefficient and advection velocity in the governing partial differential equation (PDE) are first taken constant and secondly linearly time dependent and not proportional to each other. Flow domain is assumed semi infinitely deep and homogeneous and it is subdivided into small units called control volumes of uniform dimension. Finite volume and Finite difference methods are used to discretize space and time respectively in the governing PDE. Discretized equations are inverted to obtain the concentrations at various nodes of the control volumes by using mathematical codes developed in Mat-lab and the results presented using graphs at different soil depths and time to determine the parameters that can help detect the contamination levels before disastrous levels are reached and with ease. It is observed that the concentration levels of ions with depth and time can easily be detected when diffusion coefficient and advection velocities are linearly depended on time.

AMS subject classification:
Keywords: Ill-posed, Finite Volume Method, Advection, Diffusion, Reconstruction, Remediation.

1. Introduction

Movement of pollutants from a ground surface of soil through plant root zone to the groundwater is a major pollution to the hydrological environment in the subsurface. This phenomenon has negative impact on human life, livestock who depend heavily on groundwater in addition to degradation of flora and fauna on the terrestrial and aquatic environment. The uncontrolled and excess use of chemical fertilizers are known to be major cause of this pollution. This is because chemicals under investigation in this study which include high nitrogen synthetic fertilizers, pesticides, salts and minerals that percolate in soil over time are becoming responsible for soil acidification.

Some of the studies conducted on soil acidification include [4] who started of by defining soil acidification as the decrease in acid neutralization capacity of the soil. It is one of the factors limiting crop production in many parts of the world. Crop production in the high rainfall areas like in Kenya is constrained by soil acidity and soil fertility depletion as suggested by [16]. Although soil acidification is a natural process, it has recently been accelerated by human practices on the farm lands which causes gradual accumulation of hydrogen ions in the soil. These practices include addition of agricultural synthetic fertilizers and pesticides, inorganic matter and minerals that break down in the soil over time. Some of the industrial effluent causes great concern because they hardly break down, are carcinogenic and their extraction is extremely expensive. In addition and to large extent, it has been documented that chemical fertilizer on excess percolation into the soil contribute immensely to acidification when they stay and break down over time.

These practices have caused great concern to environmentalists, hydrologists, civil engineers as well as mathematicians. In this paper,we intend to develop a mathematical understanding of the initial root causes and levels of acidification in priori, by solving mathematical backward problem which translates to inverse problem as opposed to solving a forward problem, whose solution is ill-posed in such a way that the infinitesimal error always magnifies un-proportionally in final solution hence requiring regularization schemes. This is what is being referred to as **reconstruction of acidity**. **Remediation** in this context is the reversibility of intensively acidified arable land to traditional health and fertile land. This should be a priority for land conservation.

To model this processes mathematically, we invoke a mathematical thinking by developing mathematical models from Navier-Stokes Equations to simulate advection and diffusion process of solute transport in homogeneous soil structures. Homogeneous soils are an exceptionally rare case of soil structure as much as the plant root zone can be considered to be almost homogeneous. This is expected in a farmland where the soil columns are often disturbed during land preparation and planting which lead to mixing of different soil layers leaving the transport behavior to be uniform all through. Homogeneous soils are not only ideal for pure studies but also for developing models that can predict the transport of both organic and inorganic materials when the soil is weakly heterogeneous.

In this work we have solved an inverse problem modelled from the advection diffusion equation, numerically by adopting a hybrid of Finite volume method and Finite difference schemes for spatial and temporal discretization respectively with some fundamental assumptions utilized.

The process of acidification is complex indeed expressible in terms of non linear PDEs. Thus determination of analytic solution involves a lot of assumptions thus making the results unrealistic. Hence numerical experiment is a cost effective avenue for obtaining better and reliable results for the PDEs and more so methods based on control volumes.

Flow of contaminated fluids from the soil surface in to the ground water has been studied by many researchers in the past all taking different view point. [19] quoted that water flow in the unsaturated zone is complicated due to the fact that the soil permeability to water depends on its water saturation. [25] in their paper cited that fertilizers, pesticides and industrial waste may be small in quantity but highly toxic and can be transported to ground water to remain there for hundreds of years.

A chemical becomes a pollutant if its concentration exceed some prescribed water quality standard or soil attains an un-allowable PH after chemicals have been applied. This impairment of beneficial water and soil use has been known to be induced by natural processes and human activities. Specifically, when fertilizers are applied on a wet ground they dissolve easily to form a solute because of their characteristic nature of been highly miscible with water, volatile and hygroscopic. Thereafter the solute will be transported through advection also referred to as convection, deep into the soil due to the bulk fluid motion after an irrigation or even a heavy downfall. However when advection slows down due to soil saturation, the level of wetness attained will vary from the surface soil downwards. As this infiltration process occurs, the solute simply disperses away from the source in a diffusive manner and thus the flow of the chemicals can be described using the Advection- diffusive equation (ADE).

The classical Advection and Dispersion equation has commonly been used to characterize the transport of non-reactive solutes through homogeneous porous media consisting of impermeable grains, [8] and [2]. Other processes that can control the movement of the solute are sorption, volatilization, sorption, hydrolis, biotransformation and radioactive decay. For reactive solutes, the Advection Dispersion equation has been modified to incorporate the effects of adsorption and desorption ([14] and [17]) and hysteresis ([27]).

Various approaches have previously been employed to solve the Advection Dispersion equations applied to the transport of chemicals through saturated and unsaturated porous media. Analytical solutions have been reported by a number of researchers when assumptions made allow for simplification of the Navier stokes equation to represent a linear problem, see [21], [22], [13], [26]. Other researchers who employed numerical studies included [7], [23], [18], [9], [10], [11] among others.

We need to solve the unsteady advection diffusion equation in which the coefficients are unknown. Consequently the problem need to be regularized before one can give it a full numerical analysis using computer algorithm or other computational methods, representing ill posed problems which are treated as inverse problems. Ill posed problems are those that do not meet the three Hadamard criteria for being well posed. For the regularization, one needs to bring in new assumptions to fully define the problem and narrow it down. Identification of the unknown diffusion coefficient in a linear parabolic equation via semi group approach was performed by [6]. Identification of coefficients for a parabolic equation where the unknown coefficient depends on an over specified datum is presented by [24]. Identification of a Robin Coefficient on a non accessible part of the boundary from available data on the other part is reported by [3]. Coefficients problems are used to estimate values of parameters in a governing equation.

Techniques for remediation of polluted soil and groundwater previously applied include pump- and -treat, using a combination of the optimization methods and simulation models as proposed by [12], Hot water flushing ([15], [20]), air sparging [1], Cosolvent flushing [15], the use of surfactants [5], In situ bio-remediation [28]. The effectiveness of the remediation may be substantially improved if the location and extent of the contaminants source are known.

In this paper we intend to determine which, between constant and linearly time dependent diffusion coefficient and advection velocities for various flow conditions can help detect contamination levels early before percolation of chemicals penetrate to inaccessible levels into the ground.

2. One Dimensional Advection Diffusion Model

Let the domain of flow $\Omega = [0, z]$ represent the semi infinite flow domain given that $0 \leq z \leq \infty$ and t varies from 0 to final time T. The general non-linear form of one dimensional advection diffusion equation describing solute flow in Cartesian system given by equation (1)

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\left[D_z\left(z,t\right) \frac{\partial C}{\partial z} - w\left(z,t\right) C \right] + S_0 \tag{1}$$

where $C\left(z,t\right)$ is the function representing concentration the substance to be transported at depth z of the domain at time t taking z axis as the direction of flow, $D_z\left(z,t\right)$ is the diffusion coefficient which can represent molecular diffusion while $w\left(z,t\right)$ is the average pore water velocity.

The last term on the right hand side is taken to be the source or sink term for production or loss of solutes within the system. Since we are concerned with solutes flow in agricultural land S_0 is the source term taken to represent fertilizer application and other human related activities that can lead to inequilibrium in soil PH. It is assumed that in this paper, soil is of semi infinite depth and the soil properties like the permeability and porosity are uniform along the z axis. We need to analyze the situation where the source term is zero and when the source term is present is left out for further research. In the present case, we need to reconstruct the initial condition $C(z,t) = f(z)\ t = t_0$ and the flow parameters $D_z(z,t)$and $w(z,t)$. We shall determine a suitable function $f(t)$ for boundary condition $C(0,t)$on one side of the domain taking $C(z,t) = 0, z = z_\infty$on the other side of the domain.

3. Well-Posedness of the Problem

Problems expressible in terms of PDE given by equation (1) subject to relevant boundary or initial condition(s) is well posed if a solution exists, the solution is unique and it continuously depends on the data given. We consider the continuous problem above for $0 \le z \le 1$, $D_z \ge 0$. The problem is strongly well-posed if the solution is bounded in terms of all the data i.e. the terms are known explicitly. However we can demonstrate the well-posedness by considering the source term and the boundaries on either sides of the domain to be zero using the Energy method.

Take a one dimensional A-D model in equation (1), initial condition $C(z,t_0) = f(z)$ and boundary conditions

$$C(0,t) = f(t)$$

and

$$C(1,t) = 0$$

Multiply the differential equation by $2C$ for constant $D(z,t)$, $W(z,t)$ and $S_0 = 0$ to get

$$2C\frac{\partial C}{\partial t} = 2CD\frac{\partial^2 C}{\partial z^2} - 2CW\frac{\partial C}{\partial z} \tag{2}$$

Integrating equation (2) over the spatial domain $0 \le z \le 1$,

$$\int_0^1 2C\frac{\partial C}{\partial t}dz = \int_0^1 2CD\frac{\partial^2 C}{\partial z^2}dz - \int_0^1 2CW\frac{\partial C}{\partial z}dz \tag{3}$$

$$\frac{d}{dt}\|C\|^2 = 2D\{-C(0,t)\,C_z(0,z) + C(1,t)\,C_z(1,t)\}+WC(0,t)^2-WC(1,t)^2-\|C_z\|^2$$

where

$$\|C\|^2 = \int_0^1 C^2 dz$$

Given $D = 0$, the differential equation is hyperbolic and we need only one boundary condition. If $W > 0$, $C(0,t) = f(t)$ has to be given; if $W < 0$, $C(1,z) = 0$ has to be given instead. If $W > 0$ and $f(t) = 0$ then $\frac{d}{dt}\|C\|^2 = -WC(1,t)^2$.

Assuming a parabolic case where $D \neq 0$ and that we have to give data at both boundaries, inserting the zero boundary data yields,

$$\frac{d}{dt}\|C\|^2 = -2D\|C_z\|^2.$$

Time integration of the above two results gives that the original differential equation is well posed in the classical sense assuming that the correct number of boundary condition is used.

4. Time Variation of Advection Velocity $W(z,t)$ and Diffusion Coefficient $D(z,t)$ in the Absence of Source Term

When the diffusivity $D_z(z,t) = D(t)$ and the flow velocity $w(z,t) = w(\mathbf{t})$, we obtain a particular case to the problem in the equation (1) given by equation (4)

$$\frac{\partial C}{\partial t} = D_z(t)\frac{\partial^2 C}{\partial z^2} - w(t)\frac{\partial \mathbf{C}}{\partial \mathbf{z}}, \tag{4}$$

for $\Omega \times t\epsilon\,(0,T]$.

In the current problem we shall consider varying the parameter values of $w(z,t)$ as:

(i) constant advection velocity$w(z,t) = w_0$

(ii) advection velocity is a linear function of time $w(z,t) = w_0(at+b)$ where a is the rate at which the flow velocity is varying with time and b is the initial velocity at time $t = 0$.

Similarly we shall also consider varying the diffusion coefficient $D(z,t)$ as

(iii) Constant Diffusion coefficient $D(z,t) = D_0$

(iv) Diffusion coefficient is a linear function of time $D(z,t) = D_0(at+b)$ where a is the rate at which solutes diffusion is varying with time and b is the initial solutes diffusion at time $t = 0$ and W_0 and D_0 are constant values.

5. Discretization of the Given Space and Temporal Domain

Finite volume method developed by Pantanker and Spalding in 1972 involves subdivision of the flow domain into infinitesimal volumes called control volumes and representation of the differential equations in integral form. The integral form of each conservation law is written separately for each control volume. Discretization process of time is then carried out for each control volume by finite difference scheme. Higher order terms are reduced into weak form which are then solved numerically by inversion the components of the discretised equation. Discrete values are estimated at the centre of

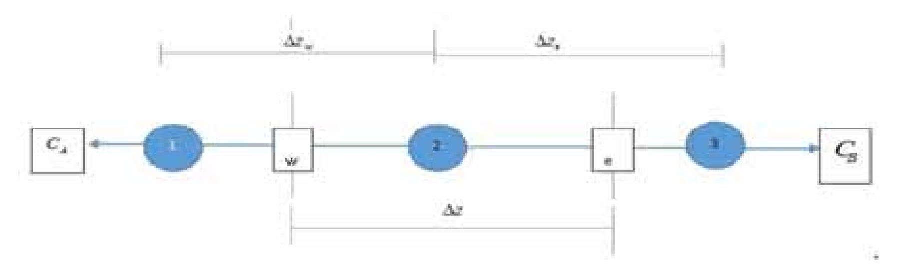

Figure 1: Discretized one dimensional domain into control volumes of width $\triangle z$

the control volume of the domain after implementing the prescribed initial and boundary conditions.

Taking the discretized flow domain illustrated in figure 1, in which node 2 serves as the centre node of the control volumes $\Delta x \Delta y \Delta z$ with unit thickness and nodes 1 and 3 and are the centres of the neighbouring control volumes, w and e are the western and eastern boundaries of the control volume respectively. Since the control volume is taken to be one dimension, the thickness $\Delta x = \Delta y = 1$ thus the control volume reduces to Δz. C_A and C_B are the conditions at the western and eastern boundaries of the control volume respectively that can be assumed to be known or unknown and thus need to be determined. When C_A and C_B are known, the problem becomes a forward problem and it can easily be solved using the standard techniques available. However whenever they aren't known the problem is ill posed and thus calls for the techniques of solving inverse problems to be employed. Specifically C_A is condition prevailing at the surface of the soil and C_B is representing the condition deep down in the flow domain.This study will test the validity of chosen functions C_A and C_B numerically.

With no loss of generality we focus on flow of fertilizer represented by smooth function $C\,(z,t)$ in porous medium assumed to have uniform structure in the solution domain.

6. Discretisation of Governing Equation when Flow Parameters $W\,(z,t) = W_0$ and $D\,(z,t) = D_0$ are Constant

The conservation law applies to each domain and equation (4) integrated over the i^{th} control volume over the time interval from t_{j-1} to t_j and assuming the dimensions of the control volume $\triangle x$ and $\triangle y$ are unity, the following procedure is observed

$$\int_{cv} \left\{ \int_{t_{j-1}}^{t_j} \frac{\partial C}{\partial t} dt \right\} dz = \int_{t_{j-1}}^{t_j} \left\{ \int_{cv} \left[D_z \frac{\partial^2 C}{\partial z^2} - w \frac{\partial C}{\partial z} \right] dz \right\} dt \tag{5}$$

The velocity field must satisfy the mass conservation law and the continuity equation becomes

$$\frac{dw}{dz} = 0 \tag{6}$$

Equation (5) becomes

$$\Delta z \int_{t_{j-1}}^{t_j} \frac{dC}{dt} dt = \left\{ \left[D_z \frac{dC}{dz} \right]_e - \left[D_z \frac{dC}{dz} \right]_w - (wC)_e + (wC)_w \right\} dt \tag{7}$$

From this result we have the first order and zeroth order derivatives for the diffusion and advection terms respectively in the conservation law. This reduction of the order of the derivative is important in dealing with situations which change so rapidly in space that the spatial derivative does not exist. The diffusion coefficient and advection velocity are taken to be uniform on either sides of the control volume thus equation 5 reduces to

$$\frac{\Delta z}{dt} \left[C\left(z_P, t_j\right) - C\left(z_P, t_{j-1}\right) \right] = D_z \left[\frac{dC}{dz}_e - \frac{dC}{dz}_w \right] - w\left(C_e - C_w\right) \tag{8}$$

For control volume 1, C_w is assumed to be known from the boundary conditions and RHS of equation (8) can now be reduced to

$$D_z \left(\frac{dC}{dz}_e - \frac{dC}{dz}_w \right) - w\left(C_e - C_w\right) = D_z \left\{ \frac{C_E - C_P}{\delta z_e} - \frac{C_P - C_w}{\frac{\delta z_w}{2}} \right\} - w \left\{ \frac{C_E + C_P}{2} - C_w \right\} \tag{9}$$

Taking $\delta z_e = \delta z_w = \delta z$, then the above equation (9) reduces to

$$\frac{D_z}{\delta z} \left\{ C_E - 3C_P + 2C_w \right\} - \frac{w}{2} \left\{ C_E + C_P - 2C_w \right\}$$

Simplifying by grouping like terms, we obtain

$$\left(2\frac{D_z}{\delta z} + w \right) C_w + \left(-3\frac{D_z}{\delta z} - \frac{w}{2} \right) C_P + \left(\frac{D_z}{\delta z} - \frac{w}{2} \right) C_E$$

If we use the notation $C_{i,j}$ for $C\left(z_P, t_j\right)$ and $C_{i,j-1}$ for $C\left(z_P, t_{j-1}\right)$ equation (8) and (9) can be written as

$$\frac{\Delta z}{dt} \left[C_{i,j} - C_{i,j-1} \right] = \left(2\frac{D_z}{\delta z} + w \right) C_w + \left(-3\frac{D_z}{\delta z} - \frac{w}{2} \right) C_P + \left(\frac{D_z}{\delta z} - \frac{w}{2} \right) C_E$$

In the control volume 1,we take $C_w = C_{i-\frac{1}{2},j}$, $C_P = C_{i,j}$ $C_E = C_{i+1,j}$, this equation now becomes

$$\frac{\Delta z}{dt} \left[C_{i,j} - C_{i,j-1} \right] = \left(2\frac{D_z}{\delta z} + w \right) C_{i-\frac{1}{2},j} + \left(-3\frac{D_z}{\delta z} - \frac{w}{2} \right) C_{i,j} + \left(\frac{D_z}{\delta z} - \frac{w}{2} \right) C_{i+1,j}$$

Rearranging this, we have

$$\left(-3\frac{D_z}{\delta z}-\frac{w}{2}-\frac{\delta z}{dt}\right)C_{i,j}+\left(\frac{D_z}{\delta z}-\frac{w}{2}\right)C_{i+1,j}=-\left(2\frac{D_z}{\delta z}+w\right)C_{i-\frac{1}{2},j}-\frac{\Delta z}{dt}C_{i,j-1} \tag{10}$$

For control volumes 2 to $N-1$, we have the following discretised equation

$$\left(\frac{D_z}{\Delta z}+\frac{w}{2}\right)C_{i-1,j}+\left(-2\frac{D_z}{\Delta z}-\frac{\Delta z}{\Delta t}\right)C_{i,j}+\left(\frac{D_z}{\Delta z}-\frac{w}{2}\right)C_{i+1,j}=-\frac{\Delta z}{dt}C_{i,j-1} \tag{11}$$

Lastly the N^{th} control volume give the discretised equation of the form

$$\left(\frac{D_z}{\Delta z}+\frac{w}{2}\right)C_{N-1,j}+\left(-3\frac{D_z}{\Delta z}+\frac{w}{2}-\frac{\Delta z}{\Delta t}\right)C_{N,j}=-\left(\frac{2D_z}{\Delta z}-w\right)C_{N+\frac{1}{2},j}-\frac{\Delta z}{\Delta t}C_{N,j-1} \tag{12}$$

Using equations (10) to (12), we set

$$A=-3\frac{D_z}{\Delta z}-\frac{w}{2}, B=\frac{D_z}{\Delta z}-\frac{w}{2},$$

$$E=\frac{D_z}{\Delta z}+\frac{w}{2}$$

$F=-3\frac{D_z}{\Delta z}+\frac{w}{2}$, $G=\frac{D_z}{\Delta z}$, $H=\Delta z$, $HH=\frac{\Delta z}{\Delta t}$ and the three equations above make the system of equations (13,14) and (15) respectively for $i=1$,

$$(A-HH)\,C_{1,j}+BC_{1+1,j}=-2EC_{1+\frac{1}{2},j}-HHC_{1,j-1} \tag{13}$$

$i=2,......N-1$,

$$EC_{i-1,j}+(-2G-HH)\,C_{i,j}+BC_{i+1,j}=-HHC_{i,j-1} \tag{14}$$

$i=N$,

$$EC_{N-1,j}+(F-HH)\,C_{N,j}=-2BC_{N+\frac{1}{2},j}-HHC_{N,j-1} \tag{15}$$

In order to guarantee that the numerical scheme is stable, we have to make sure that the matrix is symmetric, diagonally dominant and real, with non negative diagonal entries then the matrix is positive definite.

7. Linear Variation in $w\left(z,t\right)=w_0\left(at+b\right)$ and $D\left(z,t\right)=D_0\left(at+b\right)$ with Time

Here we consider a case where the diffusivity and advection velocity are a linear functions of time as $w\left(z,t\right)=w_0(at+b)$ and $D\left(z,t\right)=D_0(at+b)$, a and b are constants. The a in the linear function above is used to denote the rate at which the advection velocity and solutes diffusion are varying with time where as b defines the initial velocity and diffusion coefficient at time $t=0$.

Integrating over the control volume and over the time interval from $t_{j-1}$to t_j, we see that equation (4) will give

$$\triangle z\left[C_{i,j}-C_{i,j-1}\right]=\left[\frac{D_0}{2}a\left(t_j^2-t_{j-1}^2\right)+D_0b\left(t_j-t_{j-1}\right)\right]\left(\frac{dC}{dz}_e-\frac{dC}{dw}_w\right)$$
$$-\left[\frac{W_0}{2}a\left(t_j^2-t_{j-1}^2\right)+W_0b\left(t_j-t_{j-1}\right)\right](C_e-C_w)$$

This reduces to

$$\triangle z\left[C_{i,j}-C_{i,j-1}\right]=\left[\frac{D_0}{2}a\triangle t\left(t_j+t_{j-1}\right)+D_0b\triangle t\right]\left(\frac{dC}{dz}_e-\frac{dC}{dw}_w\right)$$
$$-\left[\frac{W_0}{2}a\triangle t\left(t_j+t_{j-1}\right)+W_0b\triangle t\right](C_e-C_w) \quad (16)$$

which represents the general discretised equation in this case. At the $1st$ control volume, the discretised equation becomes

$$\left\{\left[\frac{3D_0\triangle t}{\triangle z}+\frac{W_0\triangle t}{2}\right]\left[\frac{a}{2}\left(t_j+t_{j-1}\right)+b\right]+\triangle z\right\}C_{i,j}$$
$$-\left[\frac{D_0\triangle t}{\triangle z}-\frac{W_0\triangle t}{2}\right]\left[\frac{a}{2}\left(t_j+t_{j-1}\right)+b\right]C_{i+1,j}$$
$$=\left[2\frac{D_0\triangle t}{\triangle z}+W_0\triangle t\right]\left[\frac{a}{2}\left(t_j+t_{j-1}\right)+b\right]C_{i-\frac{1}{2},j}+\triangle zC_{i,j-1} \quad (17)$$

Taking $E=i+1,j\ W=i-1,j\ P=i,j,e=i+\frac{1}{2},j\ w=i-\frac{1}{2},j$
For the $2nd$ to $(N-1)th$ control volumes, the discretised equations become

$$\left[\frac{D_0\triangle t}{\triangle z}-\frac{W_0\triangle t}{2}\right]\left[\frac{a}{2}\left(t_j+t_{j-1}\right)+b\right]C_{i-1,j}-\left[2\frac{D_0\triangle t}{\triangle z}\left(\frac{a}{2}\left(t_j+t_{j-1}\right)+b\right)+\triangle z\right]C_{i,j}$$
$$+\left[\frac{D_0\triangle t}{\triangle z}-\frac{W_0\triangle t}{2}\right]\left[\frac{a}{2}\left(t_j+t_{j-1}\right)+b\right]C_{i+1,j}=-\triangle zC_{i,j-1} \quad (18)$$

Lastly the N^{th} control volume results to the following discretised equation

$$\left[\frac{D_0\triangle t}{\triangle z}+\frac{W_0\triangle t}{2}\right]\left[\frac{a}{2}\left(t_j+t_{j-1}\right)+b\right]C_{i-1,j}$$
$$-\left[\left(3\frac{D_0\triangle t}{\triangle z}-\frac{W_0\triangle t}{2}\right)\left(\frac{a}{2}\left(t_j+t_{j-1}\right)+b\right)+\triangle z\right]C_{i,j}$$
$$=-2\left(\frac{D_0\triangle t}{\triangle z}-\frac{W_0\triangle t}{2}\right)\left[\frac{a}{2}\left(t_j+t_{j-1}\right)+b\right]C_{i+\frac{1}{2},j}-\triangle zC_{i,j-1} \qquad (19)$$

8. Results and Discussions

Both linear and non linear mass transport equation are used to determine flow characteristics of pollutants in soils. A one dimensional ADE is considered in which the coefficients were first taken constant. A variation was also made whereby both parameters are time dependent. The argument here is that as time increases then the $D_z(z,t)$ and $W(z,t)$ are also changing at varied depths in the soil until a point of saturation is reached.

Advection and diffusion processes are playing a key role in determination of the concentration at different levels in soils at different times. Here we have considered advection effect higher than diffusion. This is because fluids percolate deep into soil due to their bulk motion after irrigation or a downfall and slows down due to soil saturation. The level of wetness varies from surface soil downwards. Now when solutes are applied in form of fertilizers, upon dissolving, they move away from the point of application to points of low concentration in a diffusive manner. To the contrary when $D(z,t)$ dominates the flow, it means that the solutes are being applied to already water logged soils and advection velocity is negligible. We are referring to nitrogenous fertilizers highly responsible for soil acidification and are applied to growing plants thus diffusion here is taking place where advection is present.

The results are presented in form of graphs and discussions are made here under.

In the Figure 2 below flow parameters $D_z\,(z,t)$ and $W\,(z,t)$ are taken constant from time to time. This means even as time or depth changes, the two parameters remain unchanged. With advection is playing a significant role in the transport of solutes as opposed to diffusion which takes a less value, we notice Concentration is. The curve for $z=0.25$ in Figure 2 and 3 represents the first level in the flow domain. It is steep at the beginning then starts leveling near the concentration levels of 0.35. It means that this level is closer to the surface where application of fertilizers and other human activities are taking place. This level recieves solutions containing pollutants first and attains saturation first and faster as opposed to other levels in the domain. It takes longer to attain saturation level for Figure 3 compared to Figure 2. Here $D\,(z,t)=D_0\,(at+b)$ and $W\,(z,t)=W_0\,(at+b)$ are increasing at a constant rate though less than when$D\,(z,t)=D_0$ and $W\,(z,t)=W_0$.

The zone $z=0.5$ midway the depth of the semi infinite flow domain. As advection continues to take place, less pollutants reach this zone and consequently takes longer to reach saturation level. Less pollutants reach the level $z = 0.75$. There is minimal pollution at the level $z = 1.0$ because the flow domain is assumed to be semi-infinite, thus this is the zone of semi infinite depth it may take longer than the considered time.

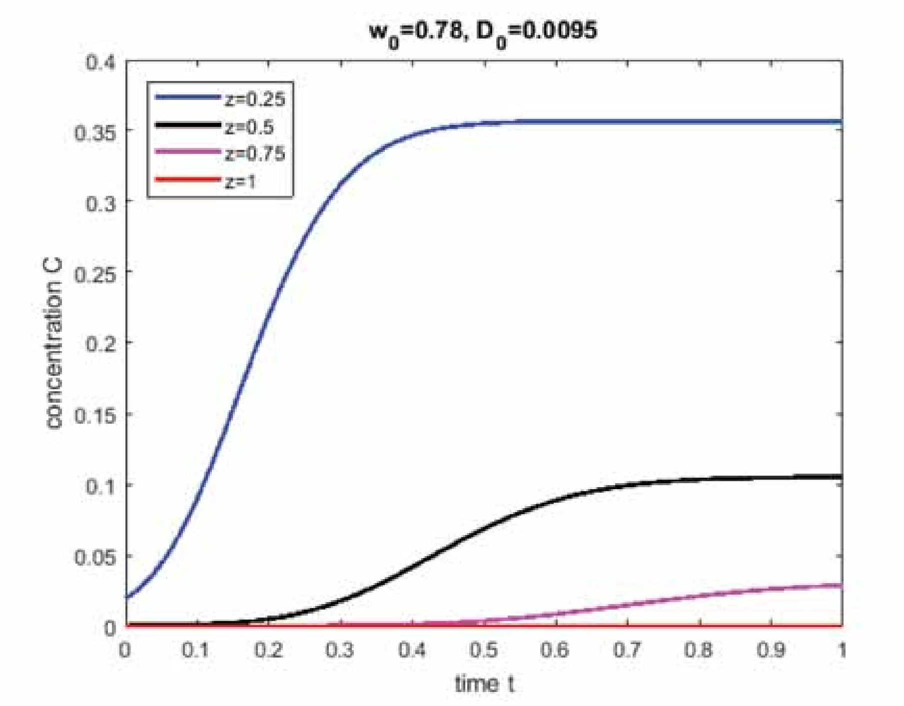

Figure 2: Non dimensional concentration C against time$t$ at different depths when $D\,(z, t)$ and velocity $W\,(z, t)$ are Constants

In Figure 4 and 5, concentration is taken to be a function of space/ depth. At time $t = 0$, the concentration is taking a maximum value of 1. As time increases by one step, pollution downwards decreases. It reduced to a non dimensional depth of 0.4 in Figure 5 and 0.5 in figure 4. Diffusion and advection are higher in Figure 4 than in Figure 5. Figure 4 can be linked to soils with bigger pore spaces than those demonstrated by Figure 5. A similar behavior is noted in the other time levels where higher levels concentration variation with depth are notiable in Figure 4 than in Figure 5. This shows that early control of pollution can easily be carried out before it sinks deep in to unreachable levels in situations where the flow parameters are linearly depended on time.

9. Conclusion and Recommentations

In this paper we considered a mathematical transport model in a homogeneous soil structure where reaction was negligible. The model helped to predict the flow characteristics of pollutants in soils. The model was anchored on the classical mass transport equation with appropriate initial and boundary conditions which were numerically tested for their

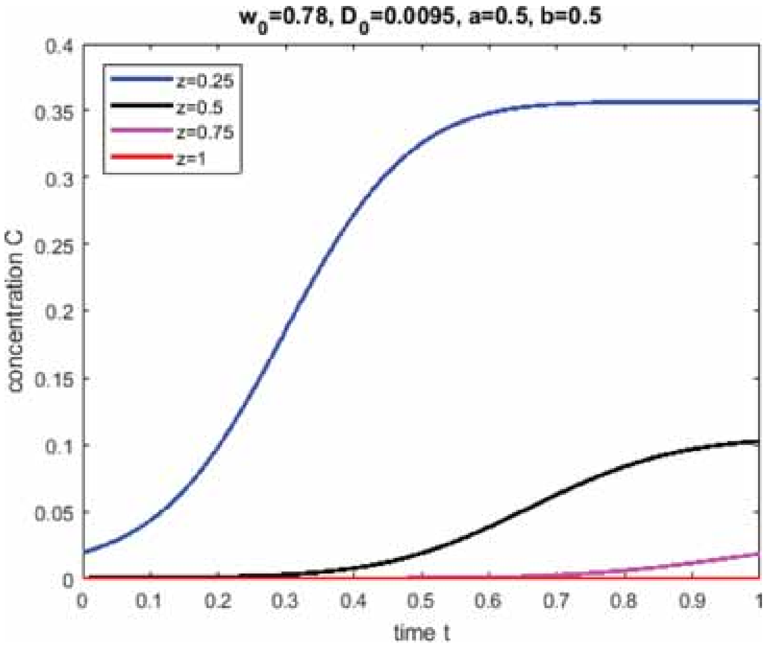

Figure 3: Non dimensional concentration C against time$t$ at varied depths z when both $D\ (z,t)$ and $W\ (z,t)$ are linear functions of time

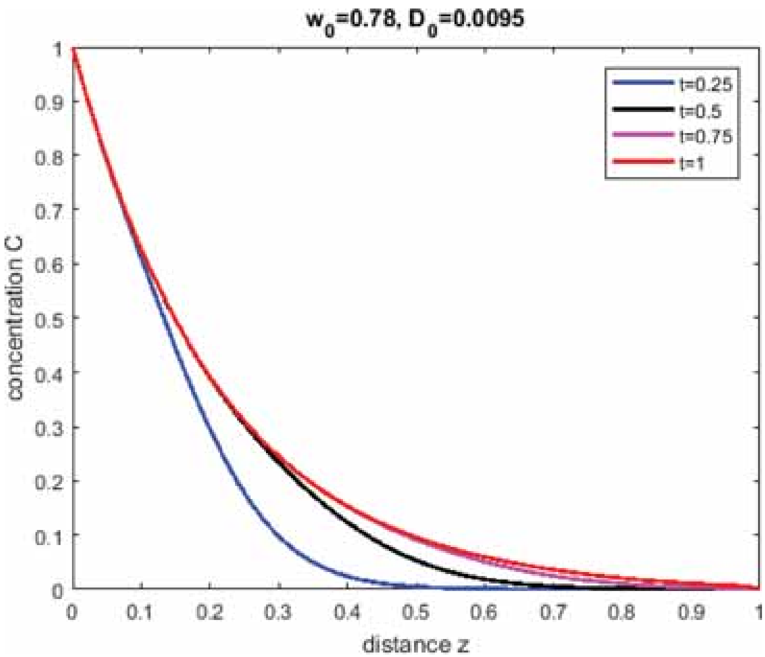

Figure 4: Non dimensional C against depth z when $D\ (z,t)$ and $W\ (z,t)$are constants

applicability. A one dimensional flow domain was considered where the flow parameters being investigated were analyzed constants and linear functions of time. A comparison

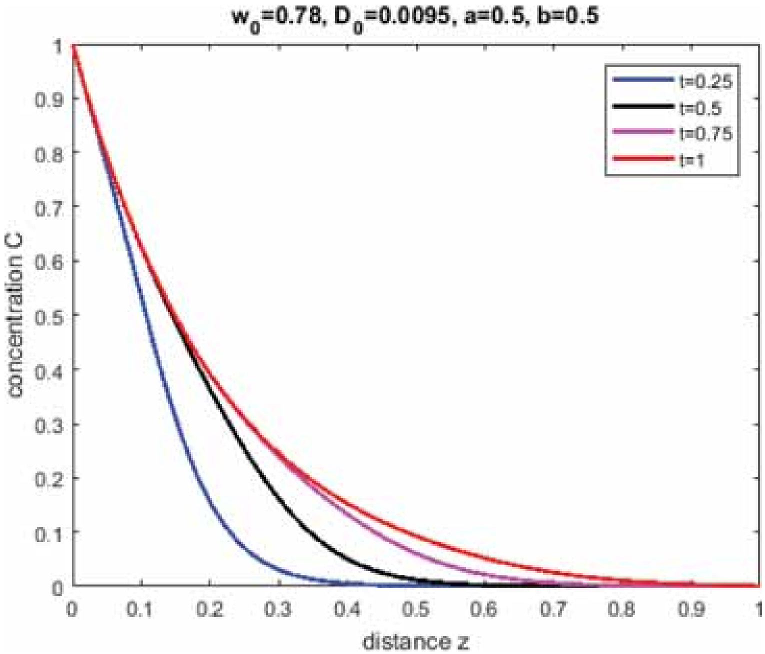

Figure 5: Non dimensional concentration C against depth z for varied time when $D\,(z,t)$ and $W\,(z,t)$ linear functions of time.

was also made for concentration with respect to depth and also time for varied diffusion coefficients and advection velocity in order to provide advice to all with interest on remediation strategies. Diffusion coefficient and advection velocity were varied with respect to time analysis performed with the help of graphs to determine how they will influence the transport of acids from the soil surface to unreachable levels in the ground. It was noted that for soils that allow pollutants to diffuse linearly with time take more time to reach saturation at the surface of the soil thus mitigation strategies can be employed to reduce on the rate of flow of more chemicals deep in to the soil. It is important to note that neutralization or extraction of pollutants can easily be performed in regions near the surface unlike when the pollutants have penetrated deep down to lower levels even though in small quantities. As a matter of policy, measures should be taken when fertilizers are been used in order to determine these two important flow parameters for the specific soil structures. This will help identify the best position to place the pollutants detectors as well as neutralizers. This can also help determine how to change the flow parameters for specific soils.

A lot more can be extended on the present work by considering the following

i) Analysis of the flow parameters which are exponentially depended on time

ii) Experimental determination of the flow parameters for one dimensional domain.

iii) Analysis of the flow parameters when the soil structure is heterogeneous.

It is our intention to carry our further research in one or more areas cited above though other researchers are encouraged to carry out investigations on the same.

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