Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 9 (2017), pp. 5611–5623

http://www.ripublication.com/gjpam.htm

Operations on Accurate Edge Domination Number in Graphs

Venkatesh S.H

Research Scholar, Gouthama Research Centre,
(Affiliated to Kuvempu University),
Sri Siddhartha Institute of Technology,
Tumkur, Karnataka, India.

Venkanagouda M. Goudar

Department of Mathematics, Gouthama Research Centre,
(Affiliated to Kuvempu University),
Sri Siddhartha Institute of Technology,
Tumkur, karnataka, India.

Venkatesha

Department of Mathematics, Kuvempu University,
Shankarghatta, Shimoga, Karnataka, India.

Abstract

A edge dominating set F of a graph $G = (V, E)$ is an accurate edge dominating set, if $< E - F >$ has no edge dominating set of cardinality $|F|$. The accurate edge domination number $\gamma_{ae}(G)$ is the minimum cardinality of an accurate edge dominating set. We study the the relation between cartesian product, corona, composition, join and strong product of a accurate edge domination number of a graph.

AMS subject classification: 05C, 05C05, 05C70.
Keywords: Edge dominating set, Accurate edge dominating set.

1. Introduction

Let G be a finite, simple, non-trivial, undirected and connected (p, q) graph with vertex set $V(G)$ and edge set $E(G)$. A nonseparable graph is connected, nontrivial and has no cut points. A block of a graph is a maximal nonseparable subgraph. As usual P_p, C_p and K_p are respectively the path, cycle and complete graph.

The greatest distance between any two vertices of a connected graph G is called the diameter of G and is denoted by $diam(G)$. For any real number x, $\lceil x \rceil$ denotes the smallest integer not less than x and $\lfloor x \rfloor$ denotes the greatest integer not greater than x. In general $< X >$ to denote the subgraph induced by the set of vertices X.

A set $F \subseteq E(G)$ is said to be an edge dominating set if every edge in $< E(G) - F >$ is adjacent to some edges in F. The Edge domination number of G is the cardinality of smallest edge dominating set of G and is denoted by $\gamma'(G)$. This concept was introduced by Mitchell and Hedetniemi [6].

A dominating set D of a graph G is an accurate dominating set, if $< V - D >$ has no dominating set of cardinality $|D|$. The accurate domination $\gamma_a(G)$ is the minimum cardinality of an accurate dominating set. This concept was introduced by Kulli and Kattimani [11]. For further results on accurate edge domination in graphs see in [13].

In this paper we follow the notations of [3].

2. Preliminary Notes

We need the following results to prove further results.

Theorem 2.1. **[12]** If G is a (p, q) graph without isolated vertex then $\dfrac{q}{\Delta(G) + 1} \leq \gamma^{'}(G)$.

In the next section we discuss some operation on Accurate edge domination number of a graph.

3. Accurate edge domination number of a graph

A edge dominating set F of a graph $G = (V, E)$ is an accurate edge dominating set, if $< E - F >$ has no edge dominating set of cardinality $|F|$. The accurate edge domination number $\gamma_{ae}(G)$ is the minimum cardinality of an accurate edge dominating set [10].

In this paper we study the the relation between cartesian product, corona, compositions, join and strong product of a accurate edge domination number of a graph. For example, we consider the corona of cycle C_3 and path P_2 of a graph in the figure 3.1. The Accurate edge dominating set of $C_3 \circ P_2$ is $A = \{d, i, g, k\}$. Therefore $\gamma_{ae}(G) = |A| = 4$.

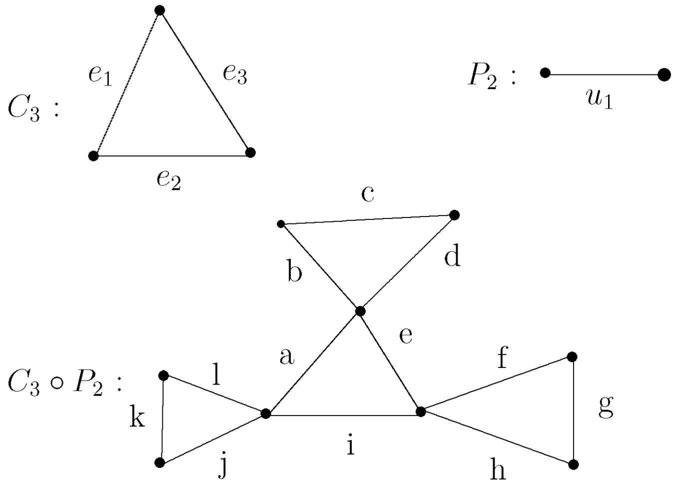

Figure 3.1:

4. Accurate edge domination number on join and composition of Graphs

Here we discuss the results on accurate edge domination number of join and composition of two graphs. The join of two graphs G_1 and G_2 is the graph $G_1 + G_2$ with the vertex set $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and the edge set $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup E_{G_1}^{G_2}$, where $E_{G_1}^{G_2} = \{uv/u \in V(G_1), v \in V(G_2)\}$.

Theorem 4.1. Let P_3 be a path of length two and P_{p_1} be any path with $p_1 \geq 3$ then $\gamma_{ae}(P_3 + P_{p_1}) = \lceil \frac{p_1}{2} \rceil + 2$.

Proof. Let $G_1 = P_3$ and $G_2 = P_{p_1}$ be the two paths labeled in order as $u_1 e_1 u_2 e_2 u_3$ and $v_1 e_1^{'} v_2 e_2^{'} ... v_{p_1 - 1} e_{p_1 - 1}^{'} v_{p_1}$ respectively. Let $E(G_1 + G_2) = \{e_1^{''}, e_2^{''} ... e_{4p_1+1}^{''}\}$ be the edge set of $G_1 + G_2$. Let $F \subseteq E(G_1 + G_2)$ be the minimum edge dominating set of $G_1 + G_2$. Let $F_1 = F \cap E(G_1)$, $F_2 = F \cap E(G_2)$ and $F_3 = F \cap E_{G_1}^{G_2}$. Suppose $F_1 = F_2 = F_3 = \emptyset$ or $F_1 \neq \emptyset$, $F_2 = F_3 = \emptyset$ or $F_2 \neq \emptyset$, $F_1 = F_3 = \emptyset$ then the set $F = F_1 \cup F_2 \cup F_3$ does not forms edge dominating set of $G_1 + G_2$. Which is contradiction. So except above cases $F = F_1 \cup F_2 \cup F_3$ forms the minimum edge dominating set of $G_1 + G_2$. If the induced subgraph $< E(G_1 + G_2) - F >$ has no edge dominating set of cardinality $|F|$ then $A = F$ itself forms minimum accurate edge dominating set of $G_1 + G_2$. Otherwise, consider $\{e_r^{''}/1 \leq r \leq 4p_1 + 1\} \subseteq < E(G_1 + G_2) - F >$ and $A = F \cup \{e_r^{''}/1 \leq r \leq 4p_1 + 1\}$ be the minimum accurate edge dominating set of $G_1 + G_2$ such that $< E(G_1 + G_2) - A >$ has no edge dominating set of cardinality $|A|$.

Thus, $|A| \leq \frac{|V(G_2)|}{2} + 2$,

$$\gamma_{ae}(P_3 + P_{p_1}) \leq \lceil \frac{p_1}{2} \rceil + 2.$$

Equality is obvious. Hence the proof. ■

Now we define the composition of two graphs G_1 and G_2 is the graph $G_1[G_2]$ with vertex set $V(G_1[G_2]) = V(G_1) \times V(G_2)$ and edge set $E(G_1[G_2])$ satisfying the following condition : $(x_1, y_1)(x_2, y_2) \in E(G_1[G_2])$ if and only if either $x_1x_2 \in E(G_1)$ or $x_1 = x_2$ and $y_1y_2 \in E(G_2)$.

Theorem 4.2. Let P_{p_1} and K_2 be the graphs of order $p_1 \geq 3$ and two respectively then, $\gamma_{ae}(P_{p_1}[K_2]) = p_1$.

Proof. Let $G = P_{p_1}[K_2]$ be the graph formed from p_1 copies of G_1 and G_2 of K_2. Let

$$X = \{x_1, x_2, ...x_{p_1}\} \in V(G_1), Y = \{y_1, y_2\} \in V(G_2)$$

and

$$V(G) = \{((x_1, y_1), (x_1, y_2)), ((x_2, y_1), (x_2, y_2))...((x_{p_1}, y_1), (x_{p_1}, y_2))\}$$

be the vertex set of G. Let $E(G) = \{(x_i, y_j)(x_{i+1}, y_{j+1})/1 \leq i \leq p_1, 1 \leq j \leq 2\}$ be the edge set satisfies the condition that either $(x_ix_{i+1}) \in E(G_1)$ or $x_i = x_{i+1}$ and $y_iy_j \in E(G_2)$. Let $I(G) = \{(x_{i+1}, y_j)/1 \leq i \leq p_1, 1 \leq j \leq 2\}$ be the internal vertex set of G and degree of each internal vertex is five. Let $F \subseteq E(G)$ be the minimum edge dominating set of G and every edges in F are adjacent to at least one vertex of $I(G)$. If the induced subgraph $< E(G) - F >$ has no edge dominating set of cardinality $|F|$ then $A = F$ itself forms minimum accurate edge dominating set of G. Otherwise, consider $K \subseteq (E(G) - F)$ and $A = F \cup K$ be the minimum accurate edge dominating set of G such that $< E(G) - A >$ has no edge dominating set of cardinality $|A|$. Thus, $|A| = |V(G_1)|$. It implies that, $\gamma_{ae}(G) = p_1$. Therefore $\gamma_{ae}(P_{p_1}[K_2]) = p_1$. Hence the proof. ■

5. Accurate edge domination number on Corona and cartesian product of Graphs

In this section we discuss the results on Accurate edge domination number of corona and cartesian product of two graphs. Let G_1 and G_2 be the graphs of order p_1 and p_2 respectively. The corona of two graphs G_1 and G_2 is the graph $G_1 \circ G_2$ obtained by taking one copy of G_1 and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex of the i^{th} copy of G_2.

Theorem 5.1. Let C_3 be a cycle and P_{p_1} be the path then

$$\gamma_{ae}(C_3 \circ P_{p_1}) \leq \begin{cases} 2p_1 + 1 & \text{if } p_1 \equiv 0(mod 2) \\ 2p_1 - 2 & \text{if } p_1 \equiv 1(mod 2) \end{cases}$$

Proof. Let $E_1 = \{e_1, e_2, e_3\}$ and $E_2 = \{e_j^{'}/1 \leq j \leq p_1\}$ be the edge set of C_3 and P_{p_1} respectively. Let $E = \{e_1^{''}, e_2^{''}...e_q^{''}\}$ be the edge set of $C_3 \circ P_{p_1}$.

Case 1. Let $p_1 \equiv 0(mod2)$.
Let $E = \{e_1^{''}, e_2^{''}...e_{6p_1}^{''}\}$ be the edge set of $C_3 \circ P_{p_1}$ and let $F \subseteq E(C_3 \circ P_{p_1})$ be the minimum edge dominating set of $C_3 \circ P_{p_1}$. We observe that $|F| = p_1 + 1$ and the induced subgraph $< E - F >$ has edge dominating set of cardinality $|F|$. Consider $F_1 \subseteq (E - F)$ and $A = F \cup F_1$ forms a accurate edge dominating set of $C_3 \circ P_{p_1}$ with minimum cardinality. Therefore $|A| \leq 2|F| - 1 = 2p_1 + 1$,
$\gamma_{ae}(C_3 \circ P_{p_1}) \leq 2p_1 + 1$.

Case 2. Let $p_1 \equiv 1(mod2)$.
Let $E = \{e_1^{''}, e_2^{''}...e_{6p_1}^{''}\}$ be the edge set of $C_3 \circ P_{p_1}$ and let $F \subseteq E(C_3 \circ P_{p_1})$ be the minimum edge dominating set of $C_3 \circ P_{p_1}$. If the induced subgraph $< E - F >$ has no edge dominating set of cardinality $|F|$ then $A = F$ forms a accurate edge dominating set of $C_3 \circ P_{p_1}$ with minimum cardinality. Otherwise, consider $F_1 \subseteq (E - F)$ and $A = F \cup F_1$ forms a accurate edge dominating set of $C_3 \circ P_{p_1}$ with minimum cardinality.
Thus, $|A| \leq 2|F|$,
$|A| \leq 2p_1 - 2$,

$$\gamma_{ae}(C_3 \circ P_{p_1}) \leq 2p_1 - 2.$$

Hence the proof. ■

Theorem 5.2. Let P_{p_1} and P_{p_2} be any two paths of order $p_1 \geq 3$ and $p_2 \geq 2$ respectively with $p_1 \geq p_2$. Then accurate edge domination number,

$$\gamma_{ae}(P_{p_1} \circ P_{p_2}) \leq \lceil \frac{m}{2} \rceil \lceil \frac{p_2}{2} \rceil.$$

Where m is the number of blocks of $P_{p_1} \circ P_{p_2}$.

Proof. Let $G = P_{p_1}$ and $H = P_{p_2}$ be any two paths of order p_1 and p_2 respectively and let $G \circ H$ be a (p, q) corona product graph. Let $E(G \circ H) = \{e_i/1 \leq i \leq q\}$ be the edge set of $G \circ H$. We have the following cases.

Case 1. Suppose $p_1 = p_2$.
Let $\{b_1, b_2, \ldots, b_{2p_1-1}\}$ be the number blocks in $G \circ H$ such that $|\{b_1, b_2, \ldots, b_{2p_1-1}\}| = m = 2p_1 - 1$. We observe that $\{b_1^{'}, b_2^{'}, \ldots, b_{p_1-1}^{'}\}$ be the number of blocks containing exactly one edge and $\{b_1^{''}, b_2^{''}, \ldots, b_{p_1}^{''}\}$ be the number of blocks containing at least three edges.

Let $F_b = \{e_t/1 \leq t \leq \lceil \frac{p_1}{2} \rceil - 1\}$ be the edge dominating set of each block $b_i^{''}$ for $1 \leq i \leq p_1$ containing at least three edges of $G \circ H$. Let $F = \{e_1, e_2, \ldots, e_{p_1 t}/1 \leq t \leq \lceil \frac{p_1}{2} \rceil - 1\}$ be the minimum edge dominating set of $G \circ H$ except the path P_{6n-2} for a positive integer $n \geq 1$. In particular, if the path P_{6n-2} for a positive integer $n \geq 1$ then

$F = \{\{e_1, e_2, \ldots, e_{p_1 t}\} \cup \{e_l\}/1 \leq t \leq \lceil \frac{p_1}{2} \rceil - 1, 1 \leq l \leq \frac{p_1}{2}\}$ forms a minimum edge dominating set of $G \circ H$. If the induced subgraph $< E - F >$ has no edge dominating set of cardinality $|F|$ then $A = F$ itself forms a accurate edge dominating set of $G \circ H$ with minimum cardinality. Otherwise, consider $F_1 \subseteq (E - F)$ and $A = F \cup F_1$ forms a accurate edge dominating set of $G \circ H$ with minimum cardinality. Thus,

$$|A| \leq \frac{|\{b_1, b_2, \ldots, b_{2p_1-1}\}|}{2} \frac{|V(P_{p_2})|}{2}$$

Therefore

$$\gamma_{ae}(P_{p_1} \circ P_{p_2}) \leq \lceil \frac{m}{2} \rceil \lceil \frac{p_2}{2} \rceil.$$

Case 2. Suppose $p_1 > p_2$.
Let $\{b_1, b_2, \ldots, b_{2p_1-1}\}$ be the number blocks in $G \circ H$ such that $|\{b_1, b_2, \ldots, b_{2p_1-1}\}| = m = 2p_1 - 1$. We observe that $\{b_1^{'}, b_2^{'}, \ldots, b_{p_1-1}^{'}\}$ be the number of blocks containing exactly one edge and $\{b_1^{''}, b_2^{''}, \ldots, b_{p_1}^{''}\}$ be the number of blocks containing at least three edges.

Let $F = \{e_1, e_2, e_3, \ldots, e_r/1 \leq r \leq q\}$ be the minimum edge dominating set of $G \circ H$ except the path P_{6n-2} for a positive integer $n \geq 1$. For every $e_i \in F$, $1 \leq i \leq r$, there exists at least one edge $e_i \in b_j^{''}$ for $1 \leq i \leq r, 1 \leq j \leq p_1$ but $e_i \notin b_k^{'}$ for $1 \leq i \leq r, 1 \leq k \leq (p_1 - 1)$.

In particular, if the path P_{6n-2} for a positive integer $n \geq 1$ then $F = \{e_1, e_2, \ldots, e_s/1 \leq s \leq q\}$ forms a minimum edge dominating set of $G \circ H$. For every $e_i \in F$, $1 \leq i \leq s$, there exists at least one edge $e_i \in b_j^{''}$ for $1 \leq i \leq s, 1 \leq j \leq p_1$ and $e_i \in b_k^{'}$ for $k = 1, 3, 5, \ldots, (p_1 - 1)$. If the induced subgraph $< E - F >$ has no edge dominating set of cardinality $|F|$ then $A = F$ itself forms a accurate edge dominating set $G \circ H$ with minimum cardinality. Otherwise, consider $F_1 \subseteq (E - F)$ and $A = F \cup F_1$ forms a accurate edge dominating set of $G \circ H$ with minimum cardinality. Thus,

$$|A| \leq \frac{|\{b_1, b_2, \ldots, b_{2p_1-1}\}|}{2} \frac{|V(P_{p_2})|}{2}$$

Therefore

$$\gamma_{ae}(P_{p_1} \circ P_{p_2}) \leq \lceil \frac{m}{2} \rceil \lceil \frac{p_2}{2} \rceil.$$

Hence the proof. ■

Corollary 5.3. Let K_1 and P_{p_1} be the graphs of order one and p_1 then, $\gamma_{ae}(K_1 \circ P_{p_1}) \leq p_1 - 1$.

Now we define the cartesian product of the graphs G and H, written as $G \times H$, is the graph with vertex set $V(G) \times V(H)$, two vertices (u_1, u_2) and (v_1, v_2) being adjacent in $G \times H$ if and only if either $u_1 = v_1$ and $u_2 v_2 \in E(H)$, or $u_2 = v_2$ and $u_1 v_1 \in E(G)$.

Theorem 5.4. Let P_3 be a path of order three and P_{p_1} be any path with $p_1 > 3$. Then accurate edge domination number of the cartesian product of two graphs, $\gamma_{ae}(P_3 \times P_{p_1}) = p_1 + 1$.

Proof. Let G be the cartesian product graph of $P_3 \times P_{p_1}$ with $p_1 > 3$. Consider $P_3 : u_1, u_2, u_3$ and $P_{p_1} : v_1, v_2, \ldots, v_{p_1}$ be the two paths.

$$\begin{aligned} V(G) = \{ \quad & (u_1, v_1), (u_1, v_2), \ldots, (u_1, v_{p_1}), \\ & (u_2, v_1), (u_2, v_2), \ldots, (u_2, v_{p_1}), \\ & (u_3, v_1), (u_3, v_2), \ldots, (u_3, v_{p_1})\} \end{aligned}$$

and the edge set

$$E(G) = \{(u_i, v_j)(u_i, v_{j+1})/1 \leq i \leq 3, 1 \leq j \leq p_1\}.$$

Let

$$\begin{aligned} F = \{&((u_2, v_1)(u_3, v_1)), ((u_1, v_2)(u_2, v_2)), ((u_2, v_3)(u_3, v_3)), \\ &\ldots, ((u_1, v_{p_1-1})(u_2, v_{p_1-1})), ((u_2, v_{p_1})(u_3, v_{p_1}))\} \end{aligned}$$

be the minimum edge dominating set of G such that $|F| = p_1$. Let K be the minimum edge dominating set of the induced subgraph $E - F$. But we observe that $|F| = |K|$. Which is contradiction. So consider $\{(u_1, v_1)(u_2, v_1)\} \in (E - F)$ and $A = F \cup \{(u_1, v_1)(u_2, v_1)\}$ forms a accurate edge dominating set of G with minimum cardinality. Thus $|A| = |F| + |\{(u_1, v_1)(u_2, v_1)\}|$. It implies that $|A| = |F| + 1$. Therefore $\gamma_{ae}(P_3 \times P_{p_1}) = p_1 + 1$. Hence the proof. ■

Theorem 5.5. Let P_{p_1} be a path with $p_1 \geq 3$ vertices and C_3 be any cycle. Then accurate edge domination number of the cartesian product, $\gamma_{ae}(P_{p_1} \times C_3) = p_1 + 1$.

Proof. Let G be the cartesian product of $P_{p_1} \times C_3$ with $p_1 \geq 3$. Consider $P_{p_1} : u_1, u_2, \ldots, u_{p_1}$ and $C_3 : v_1, v_2, v_3$ be the vertices of P_{p_1} and C_3 respectively.

$$\begin{aligned} V(G) = \{ \quad & (u_1, v_1), (u_1, v_2), (u_1, v_3), \\ & (u_2, v_1), (u_2, v_2), (u_2, v_3), \\ & . \\ & . \\ & . \\ & (u_{p_1}, v_1), (u_{p_1}, v_2), (u_{p_1}, v_3)\} \end{aligned}$$

and

$$\begin{aligned} E(G) = \{&((u_1, v_1)(u_1, v_2)), ((u_1, v_1)(u_1, v_3)), ((u_1, v_1)(u_2, v_1)), ((u_1, v_2)(u_2, v_2)), \\ &((u_1, v_3)(u_2, v_3)), ((u_1, v_2)(u_1, v_3)), \ldots\ldots.((u_{p_1}, v_1)(u_{p_1}, v_2)), ((u_{p_1}, v_1)(u_{p_1}, v_3)), \\ &((u_{p_1}, v_1), (u_{p_1-1}, v_1)), ((u_{p_1}, v_2)(u_{p_1-1}, v_2)), ((u_{p_1}, v_2)(u_{p_1}, v_3)), ((u_{p_1}, v_3)(u_{p_1-1}, v_3))\}. \end{aligned}$$

Suppose p_1 is odd then $F = \{((u_1, v_1)(u_1, v_3)), ((u_2, v_1)(u_2, v_2)), ((u_3, v_1)(u_3, v_3)), ((u_4, v_1)(u_4, v_2)), ..., ((u_{p_1-1}, v_1)(u_{p_1-1}, v_2)), ((u_{p_1}, v_1)(u_{p_1}, v_3))\}$ forms a edge dominating set of G with minimum cardinality and $|F| = p_1$.
Suppose p_1 is even then

$$F = \{((u_1, v_1)(u_1, v_3)), ((u_2, v_1)(u_2, v_2)), ((u_3, v_1)(u_3, v_2)),$$

$$((u_4, v_1)(u_4, v_2)), ...((u_{p_1-1}, v_1)(u_{p_1-1}, v_3)), ((u_{p_1}, v_1)(u_{p_1}, v_4))\}$$

forms a edge dominating set of G with minimum cardinality and $|F| = p_1$.

Let K be the edge dominating set of the induced subgraph $< E-F >$. But we observe that $|F| = |K|$ which is a contradiction. So consider one edge $\{(u_2, v_1)(u_2, v_3)\} \in < E - F >$ and $A = F \cup \{(u_2, v_1)(u_2, v_3)\}$ be the minimum accurate edge dominating set of G. Thus, $|A| = |F| + |\{(u_2, v_1)(u_2, v_3)\}|$. Therefore $\gamma_{ae}(P_{p_1} \times C_3) = p_1 + 1$. Hence the proof. ∎

Corollary 5.6. Let P_{p_1} be a path with $p_1 \geq 3$ vertices and C_4 be any cycle with four vertices. Then accurate edge domination number of the cartesian product, $\gamma_{ae}(P_{p_1} \times C_4) = 2p_1 - 1$.

Theorem 5.7. Let K_{p_1} be a complete graph with p_1 vertices and K_{p_2} be any complete graph with p_2 vertices where $p_1 < p_2$ and $p_1 \geq 2, p_2 \leq 6$. Then accurate edge domination number of the cartesian product of two complete graphs,

$$\gamma_{ae}(K_{p_1} \times K_{p_2}) \leq \begin{cases} \dfrac{p_1 p_2}{2} + p_1 & \text{i}fp_2 \equiv 0(mod3) \\ \dfrac{p_1 p_2}{2} + \dfrac{p_1}{2} & \text{i}fp_2 \equiv 1or2(mod3) \end{cases}$$

Proof. Let $G = K_{p_1}$ and $H = K_{p_2}$ be two complete graph of order p_1 and p_2 respectively with $p_1 < p_2$. Let G_1 be a cartesian product of $G \times H$. Let $V[K_{p_1}] = \{u_1, u_2, ...u_{p_1}\}$ and $V[K_{p_2}] = \{v_1, v_2, ...v_{p_2}\}$ be the vertex set of K_{p_1} and K_{p_2} respectively. Consider the vertex set

$$\begin{aligned} V(G_1) = \{ \quad & (u_1, v_1), (u_1, v_2), ..., (u_1, v_{p_2}), \\ & (u_2, v_1), (u_2, v_2), ..., (u_2, v_{p_2}), \\ & . \\ & . \\ & . \\ & (u_{p_1}, v_1), (u_{p_1}, v_2), ..., (u_{p_1}, v_{p_2})\} \end{aligned}$$

and $E(G_1) = \{(u_i, v_j)(u_{i+1}, v_{j+1})/1 \leq i \leq p_1, 1 \leq j \leq p_2\}$ be the edge set of G_1 when two vertices (u_i, v_j) and (u_{i+1}, v_{j+1}) being adjacent in $G \times H$ if and only if either $u_i = u_{i+1}$ and $(v_j v_{j+1}) \in E(H)$ or $v_j = v_{j+1}$ and $(u_i u_{i+1}) \in E(G)$. We have the following cases.

Case 1. Suppose $p_2 \equiv 0(mod3)$.
Let

$$\begin{aligned} F = \{ \quad & ((u_1, v_1)(u_1, v_2)), ((u_1, v_{p_2})(u_1, v_{p_2-1})), \\ & ((u_2, v_1)(u_2, v_{p_2-1})), ((u_2, v_{p_2-3})(u_2, v_{p_2})), \\ & . \\ & . \\ & . \\ & ((u_{p_1}, v_1)(u_{p_1}, v_{p_2-1})), ((u_{p_1}, v_{p_2-3})(u_{p_1}, v_{p_2}))\} \end{aligned}$$

be the minimum edge dominating set of G_1 and let $M \subseteq (E - F)$ be the minimum edge dominating set of the induced subgraph $< E - F >$. But we observe that $|F| = |M|$ which is a contradiction. So consider $F_1 \subseteq (E-F)$ and $A = F \cup F_1$ forms a accurate edge dominating set of G_1 with minimum cardinality. Therefore, $|A| \leq \frac{|V(G)||V(H)|}{2} + |V(G)|$. It implies that

$$\gamma_{ae}(K_{p_1} \times K_{p_2}) \leq \frac{p_1 p_2}{2} + p_1.$$

Case 2. Suppose $p_2 \equiv 1 or 2(mod3)$.
Let

$$\begin{aligned} F = \{ \quad & ((u_1, v_1)(u_1, v_2)), ((u_1, v_{p_2})(u_1, v_{p_2-1})), \\ & ((u_2, v_1)(u_2, v_{p_2-1})), ((u_2, v_{p_2-3})(u_2, v_{p_2})), \\ & . \\ & . \\ & . \\ & ((u_{p_1}, v_1)(u_{p_1}, v_{p_2-1})), ((u_{p_1}, v_{p_2-3})(u_{p_1}, v_{p_2}))\} \end{aligned}$$

be the minimum edge dominating set of G_1. If the induced subgraph $< E - F >$ has no edge dominating set of cardinality $|F|$ then $A = F$ itself forms a accurate edge dominating set of G_1 with minimum cardinality. Otherwise, consider $\{(u_i v_j)(u_{i+1} v_{j+1})\} \in (E - F)$ for $1 \leq i \leq p_1, 1 \leq j \leq p_2$ such that $F \cap \{(u_i v_j)(u_{i+1} v_{j+1})\} = \emptyset$ and $A = F \cup \{(u_i v_j)(u_{i+1} v_{j+1})\}$ for $1 \leq i \leq p_1, 1 \leq j \leq p_2$ forms a accurate edge dominating set of G_1 with minimum cardinality. Therefore,

$$|A| \leq \frac{|V(G)||V(H)|}{2} + \frac{|V(G)|}{2}.$$

It implies that

$$\gamma_{ae}(K_{p_1} \times K_{p_2}) \leq \frac{p_1 p_2}{2} + \frac{p_1}{2}.$$

Hence the proof. ■

Theorem 5.8. For any complete graph K_p with $p \geq 3$, $\gamma_{ae}(K_p \times K_p) = (p-1)^2$.

Proof. Let $G_1 = K_{p_1}$ and $G_2 = K_{p_2}$ be any two complete graphs of order p_1 and p_2 respectively and $p_1 = p = p_2 \geq 3$. Let $G(p, q)$ be the cartesian product of $G = G_1 \times G_2$. Consider

$$V(G) = \{ \quad (u_1, v_1), (u_1, v_2), ..., (u_1, v_{p_2}),$$
$$(u_2, v_1), (u_2, v_2), ..., (u_2, v_{p_2}),$$
$$.$$
$$.$$
$$.$$
$$(u_{p_1}, v_1), (u_{p_1}, v_2), ..., (u_{p_1}, v_{p_2})\}.$$

Let $E(G) = \{x_{ij}/1 \leq i \leq p_1, 1 \leq j \leq p_2\}$ be the edge set such that $\{x_{ij}\} = \{(u_i, v_j)(u_{i+1}, v_{j+1})/1 \leq i \leq p_1, 1 \leq j \leq p_2\}$ if two vertices (u_i, v_j) and (u_{i+1}, v_{j+1}) being adjacent in $G_1 \times G_2$ if and only if either $u_i = u_{i+1}$ and $(v_j v_{j+1}) \in E(G_2)$ or $v_j = v_{j+1}$ and $(u_i u_{i+1}) \in E(G_1)$. Here, G is formed by p_1 copies of p_2 with above condition for adjacency between edges and in each copies of p_2 of G we should take $(p_1 - 2)$ edges in minimum edge dominating set. Let $F = \{y_1, y_2, ...y_{p_1-2}\}$ forms a edge dominating set of G with minimum cardinality. where, $y = \{x_{ij}\}$. Let $M \subseteq (E - F)$ be the minimum edge dominating set of $E - F$. But $|F| = |M|$ which is a contradiction. So consider an edge $y' \in (E - F)$ and $A = F \cup y'$ be the minimum accurate edge dominating set of G. Therefore, $|A| = |F| + |\{y'\}|$. It implies that $\gamma_{ae}(K_p \times K_p) = (p-1)^2$. Hence the proof. ■

Corollary 5.9. Let P_{p_1} and P_{p_2} be any path with $p_1 = p_2$ vertices. Then accurate edge domination number of the cartesian product, $\gamma_{ae}(P_{p_1} \times P_{p_2}) \geq 2p_1$.

6. Accurate edge domination number on strong product of Graphs

Strong product of two graphs G_1 and G_2 is a graph $G = G_1 \boxtimes G_2$ is defined as the graph on the vertex set $V(G_1) \times V(G_2)$ with vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ connected by an edge if and only if either $u_1 = v_1$ and $u_2 v_2 \in E(G_2)$ or $u_2 = v_2$ and $u_1 v_1 \in E(G_1)$ or $u_1 v_1 \in E(G_1)$ and $u_2 v_2 \in E(G_2)$.

Theorem 6.1. Let P_{p_1} and P_{p_2} be any two paths of order p_1 and p_2 respectively with $p_1 > p_2$ and $p_1 \geq 3$ then

$$\gamma_{ae}(P_{p_1} \boxtimes P_{p_2}) = \begin{cases} \mathrm{p}_1 + 1 & if p_2 \equiv 0 (mod 3) \\ \mathrm{p}_1 & if p_2 \equiv -1 (mod 3) \end{cases}$$

Proof. Let $G_1 = P_{p_1}$ and $G_2 = P_{p_2}$ be any two graph of order $p_1 \geq 3$ with $p_1 > p_2$. The strong product of G_1 and G_2 is $G = G_1 \boxtimes G_2$ with $p = p_1 \times p_2$ vertices and q edges. Let $V(G) = \{x_1, x_2, ...x_i/1 \leq i \leq p\}$ be the vertex set of G and let $E(G) = \{y_1, y_2, ...y_j/1 \leq j \leq q\}$ be the edge set of G. Where, $\{x_i/1 \leq i \leq p\} = \{(u_i, v_j)/1 \leq i \leq p_1, 1 \leq j \leq p_2\}$ and $\{y_j/1 \leq j \leq q\} = \{(u_i, v_j)(u_{i+1}, v_{j+1})/1 \leq i \leq p_1, 1 \leq j \leq p_2\}$ such that (u_i, u_{i+1}) and (v_j, v_{j+1}) connected by an edge if and only if either $u_i = v_j$ and $u_{i+1}v_{j+1} \in E(G_2)$ or $u_{i+1} = v_{j+1}$ and $u_iv_j \in E(G_1)$ or $u_iv_j \in E(G_1)$ and $u_{i+1}v_{j+1} \in E(G_2)$. We have the following cases.

Case 1. Suppose $p_2 \equiv 0(mod 3)$
Let $F = \{y_1, y_2, ...y_{p_1}\}$ be the edge dominating set of G with minimum cardinality such that $|F| = p_1$. Let $K \subseteq (E - F)$ be the minimum edge dominating set of G. But $|F| = |K|$ which is a contradiction. So consider an edge $y \in (E - F)$ and $A = F \cup \{y\}$ forms a accurate edge dominating set of G with minimum cardinality. Therefore,

$$\begin{aligned} |A| &= |F| + |\{y\}|, \\ |A| &= |F| + 1, \\ \gamma_{ae}(P_{p_1} \boxtimes P_{p_2}) &= p_1 + 1. \end{aligned}$$

Case 2. Suppose $p_2 \equiv -1(mod 3)$
Let $F = \{y_1, y_2, ...y_{p_1}\}$ be the edge dominating set of G with minimum cardinality such that $|F| = p_1$. Let $K \subseteq (E - F)$ be the minimum edge dominating set of G and $|F| \neq |K|$. Therefore $A = F$ itself forms a accurate edge dominating set of G with minimum cardinality such that the induced subgraph $< E - A >$ has no edge dominating set of cardinality $|A|$. Therefore,

$$\begin{aligned} |A| &= |F|, \\ \gamma_{ae}(P_{p_1} \boxtimes P_{p_2}) &= p_1. \end{aligned}$$

Hence the proof. ■

Theorem 6.2. For any cycle C_p with $p \geq 3$, $\gamma_{ae}(C_p \boxtimes C_p) \leq p^2 - \lfloor \frac{2p}{3} \rfloor$.

Proof. Let $G_1 = C_{p_1}$ and $G_2 = C_{p_2}$ be any two cycle of order p_1 and p_2 respectively with $p_1 = p_2$. The strong product of G_1 and G_2 is $G = G_1 \boxtimes G_2$ with $p = p_1 \times p_2$ vertices and q edges. Consider an edge set $E(G) = \{y_1, y_2, ...y_j/1 \leq j \leq q\}$. Let $F(G) = \{y_1, y_2, ...y_r/1 \leq r \leq q\}$ be the minimum edge dominating set of G such that $|F| = p_1$ and let $K = \{y_1, y_2, ...y_s/1 \leq s \leq q\}$ be the minimum edge dominating set of $E - F$. But $|F| = |K|$ which is a contradiction. We consider any one edge $y \in (E - F)$ and $A = F \cup \{y\}$ forms a accurate edge dominating set of G with minimum cardinality. Therefore,

$$\begin{aligned} |A| &\leq |F| + |\{y\}|, \\ &\leq p_1^2 - \lfloor \frac{2p_1}{3} \rfloor, \\ \gamma_{ae}(C_p \boxtimes C_p) &\leq p^2 - \lfloor \frac{2p}{3} \rfloor. \end{aligned}$$

Hence the proof. ∎

Corollary 6.3. Let P_3 and P_{p_1} be any path of order three and $p_1 > 3$. Then accurate edge domination number of the strong product, $\gamma_{ae}(P_3 \boxtimes P_{p_1}) \geq p_1 - 1$.

Theorem 6.4. Let P_3 and P_{p_1} be any path of order three and $p_1 > 3$. Then accurate edge domination number of the strong product, $\gamma_{ae}(P_3 \boxtimes P_{p_1}) \geq diam(P_{p_1})$.

Proof. Let $G_1(p^{'}, q^{'}) = P_3$ and $G_2(p^{''}, q^{''}) = P_{p_1}$ be any path of order three and p_1 respectively with $p_1 > 3$. Let $N = \{e_1, e_2, ...e_t/1 \leq t \leq q^{''}\} \subseteq E(P_{p_1})$ be the minimum set of edges which constitute the longest path between any two distinct vertices $v_1, v_2 \in V(P_{p_1})$ such that $d(v_1, v_2) = diam(P_{p_1})$ and $diam(P_{p_1}) = p_1 - 1$. The strong product of G_1 and G_2 is $G = G_1 \boxtimes G_2$ with $p = p_1 \times p_2$ vertices and q edges. Consider an edge set $E(G) = \{y_1, y_2, ...y_j/1 \leq j \leq q\}$. Let $A(G) = \{y_1, y_2, ...y_r/1 \leq r \leq q\}$ forms a accurate edge dominating set of G with minimum cardinality. Thus,

$$\begin{aligned} |A| &\geq p_1 - 1, \\ |A| &\geq diam(P_{p_1}), \\ \gamma_{ae}(P_3 \boxtimes P_{p_1}) &\geq diam(P_{p_1}). \end{aligned}$$

Hence the proof. ∎

7. Conclusion

In this paper we discussed the relation between cartesian product, corona, compositions, join, and strong product of a accurate edge domination number of a graph.

References

[1] Arumugam. S and Velammal., Edge domination in graphs, Taiwanese Journal of mathematics, Vol. 2, No. 2, 173–179, (1998).

[2] C.J. Cockayne, R.M. Dawes and S.T. Hedetniemi, Total domination in graphs, networks, Vol.10, 211–219 (1980).

[3] F. Harary, Graph theory, adison wesley, reading mass, (1969).

[4] H.B. Walikar, B.D. Acharya and E. Sampathkumar, Recent developments in the theory of domination in graphs. In MRI Lecture Notes in Math. Mehta research Instit., Allahbad No.1 (1979).

[5] O. Ore, Theory of graphs, Amer. Math. Soc. Colloq. Publ., 38, Providence (1962).

[6] S.L. Mitchell and S.T. Hedetniemi, Edge domination tree, Congr. No.19, 489–509 (1977).

[7] S.R. Jayaram, Line domination in graphs, Graph Combin. 3, 357–363 (1987).

[8] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Fundamentals of domination in graphs, Marcel Dekker, Newyork, (1997).

[9] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Domination in graphs: advanced topics, Marcel Dekker, Newyork, (1998).

[10] V.R. Kulli and M.B. Kattimani, Accurate edge domination number in graph, Vishwa International publications, 2012.

[11] V.R. Kulli and M.B. Kattimani, Accurate domination number in graph, Vishwa International publications, 2012.

[12] V.R. Kulli and S.C. Srgarkanti, The connected edge domination number of a graph, Technical Report 8801, 1988.

[13] Venkatesh S.H, V.R. Kulli, Venkanagouda M Goudar and Venkatesha, Results on Accurate Edge Domination Number in Graphs, Journal of ultra scientist of physical sciences., Vol. 29, 21–29, 2017.