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On extended generalized ϕ-recurrent LP-Sasakian manifolds

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Abstract

The present paper deals with different geometrical properties of m-projective curvature tensor in LP-Sasakian manifolds. We also studied about the extended generalized concircularly ϕ-recurrent LP-Sasakian manifolds.

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1. Introduction

In 1979, Dubey [4] introduced the notion of generalized recurrent manifold and then such a manifold is studied by De and Guha [2]. A Riemannian manifold (M^n, g), $n > 2$, is called generalized recurrent if its curvature tensor R satisfies the condition

$$DR = A \otimes R + B \otimes G, \tag{1.1}$$

where A and B are nowhere vanishing unique 1-forms defined by
$A(X) = g(X, \rho_1)$, $B(X) = g(X, \rho_2)$ and G is a tensor of type $(1, 3)$ given by

$$G(X, Y)Z = g(Y, Z)X - g(X, Z)Y, \tag{1.2}$$

for all vector fields $X, Y, Z \in \chi(M^n)$ and ρ_1, ρ_2 are vector fields associated to the 1-forms A and B, $\chi(M^n)$ being the Lie algebra of all smooth vector fields on M^n and D denotes the covariant differentiation with respect to the metric g.

A Riemannian manifold $(M^n, g), n > 2$, is called generalized Ricci-recurrent [3] if its Ricci tensor S of type $(0, 2)$ satisfies the condition $DS = A \otimes S + B \otimes g$. The notion of generalized ϕ-recurrency to Sasakian manifolds and Lorentzian α-Sasakian manifolds are respectively studied in [7] and [9]. By extending the notion of generalized ϕ-recurrency, Shaikh and Hui [10] introduced the notion of extended generalized ϕ-recurrency to β-Kenmotsu manifolds. Further Shaikh et al. [12] studied this notion for LP-Sasakian manifolds.

The paper is organized as follows: Section 2 deals with some preliminaries of LP-Sasakian manifolds. In section 3, we have shown that m-projectively symmetric LP-Sasakian manifold M^n is Ricci-recurrent. In section 4, we have proved that $\phi - m$-projectively symmetric LP-Sasakian manifold M^n is an Einstein manifold. Again in section 5, we have found that $\phi - m$-projectively flat LP-Sasakian manifold M^n is an η-Einstein manifold. Section 6 is devoted to the study of extended generalized ϕ-recurrent LP-Sasakian manifolds. In the last section, we have shown that an extended generalized concircularly ϕ-recurrent LP-Sasakian manifold is super generalized Ricci-recurrent.

2. Preliminaries

A differentiable manifold M^n of dimension n is called Lorentzian para-Sasakian (briefly LP-Sasakian) [1, 9], if it admits a $(1, 1)$-tensor field ϕ, a contravariant vector field ξ, a 1-form η and a Lorentzian metric g which satisfy

$$\eta(\xi) = -1, \tag{2.1}$$

$$\phi^2(X) = X + \eta(X)\xi, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \tag{2.3}$$

$$g(X, \xi) = \eta(X), \tag{2.4}$$

$$D_X\xi = \phi X, \tag{2.5}$$

$$(D_X\phi)Y = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{2.6}$$

where D denotes the covariant differentiation with respect to the Lorentzian metric g.

It can be easily seen that in an LP-Sasakian manifold the following relations hold:

$$\phi\xi = 0, \eta\phi = 0, \tag{2.7}$$

$$rank\ \phi = n - 1. \tag{2.8}$$

If we put

$$\Phi(X, Y) = g(X, \phi Y) \tag{2.9}$$

for any vector fields X and Y, then the tensor field $\Phi(X, Y)$ is a symmetric (0, 2) tensor field [5].

Also in an LP-Sasakian manifold, the following relations hold:

$$\eta(R(X, Y)Z) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y), \tag{2.10}$$

$$R(\xi, X)Y = g(X, Y)\xi - \eta(Y)X, \tag{2.11}$$

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.12}$$

$$R(\xi, X)\xi = X + \eta(X)\xi, \tag{2.13}$$

$$S(X, \xi) = (n - 1)\eta(X), \tag{2.14}$$

$$S(\phi X, \phi Y) = S(X, Y) + (n - 1)\eta(X)\eta(Y), \tag{2.15}$$

and

$$(D_W R)(X, \xi)Z = g(Z, \phi W)X - g(X, Z)\phi W - R(X, \phi W)Z, \tag{2.16}$$

for any vector fields X, Y, Z, W where R and S are the Riemannian curvature and the Ricci tensor of the manifold respectively.

An LP-Sasakian manifold M^n is said to be η-Einstein if its Ricci tensor S is of the form

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y), \tag{2.17}$$

for any vector fields X, Y, where a and b are functions on M^n [1].

Definition 2.1. The m-projective curvature tensor W^* on an LP-Sasakian manifold with respect to Levi-Civita connection D is defined as [8]

$$\begin{aligned} W^*(X, Y)Z &= R(X, Y)Z - \frac{1}{2(n-1)}\{S(Y, Z)X - S(X, Z)Y \\ &+ g(Y, Z)QX - g(X, Z)QY\}. \end{aligned} \tag{2.18}$$

3. m-projectively symmetric LP-Sasakian manifold

Definition 3.1. An LP-Sasakian manifold M^n is said to be m-projectively symmetric if the m-projective curvature tensor W^* satisfies the relation

$$(D_U W^*(X, Y, Z)) = 0, \tag{3.1}$$

for all X, Y, Z and U.

Theorem 3.2. An m-projectively symmetric LP-Sasakian manifold M^n is Ricci-recurrent.

Proof. Let M^n be an m-projectively symmetric LP-Sasakian manifold. Then by the equation (3.1) and (2.18), we find

$$\begin{aligned} g((D_W R)(X, Y, Z), U) &= \frac{1}{2(n-1)}\{(D_W S)(Y, Z)g(X, U) \\ &- (D_W S)(X, Z)g(Y, U) + (D_W S)(X, U)g(Y, Z) \\ &- (D_W S)(Y, U)g(X, Z)\}. \end{aligned} \tag{3.2}$$

Taking contraction over X and U, we secure

$$\begin{aligned} (D_W S)(Y, Z) &= \frac{1}{2(n-1)}\{n(D_W S)(Y, Z) - (D_W S)(Y, Z) \\ &+ dr(W)g(Y, Z) - (D_W S)(Y, Z)\}, \end{aligned} \tag{3.3}$$

which implies

$$(D_W S)(Y, Z) = \left\{\frac{dr(W)}{n}\right\}g(Y, Z). \tag{3.4}$$

Hence the manifold is Ricci-recurrent. ■

Suppose the scalar curvature r is constant then we mention the following corollary:

Corollary 3.3. An m-projectively symmetric LP-Sasakian manifold M^n with constant scalar curvature is Einstein.

4. $\phi - m$-projectively symmetric LP-Sasakian manifold

Definition 4.1. An LP-Sasakian manifold M^n is said to be $\phi-m$-projectively symmetric, if the m-projective curvature W^* satisfies the relation

$$\phi^2((D_V W^*)(X, Y, Z)) = 0, \tag{4.1}$$

for all vector feilds X, Y, Z and V.

Theorem 4.2. A $\phi-m$-projectively symmetric LP-Sasakian manifold M^n is an Einstein.

Proof. Let us consider M^n is a $\phi - m$-projectively symmetric LP-Sasakian manifold. Then by the equations (4.1) and (2.2), we get

$$g((D_V W^*)(X, Y, Z), U) = -\eta((D_V W^*)(X, Y, Z))g(\xi, U). \tag{4.2}$$

The existence of the relation (2.18), the above equation becomes

$$\begin{aligned} g((D_V R)(X, Y, Z), U) - \frac{1}{2(n-1)}\{(D_V S)(Y, Z)g(X, U) \\ -(D_V S)(X, Z)g(Y, U) + g(Y, Z)(D_V S)(X, U) - g(X, Z)(D_V S)(Y, U)\} \\ = -g((D_V R)(X, Y, Z), \xi)g(\xi, U) + \frac{1}{2(n-1)}\{(D_V S)(Y, Z)g(X, \xi) \\ -(D_V S)(X, Z)g(Y, \xi) + (D_V S)(X, \xi)g(Y, Z) \\ -(D_V S)(Y, \xi)g(X, Z)\}g(\xi, U). \end{aligned} \tag{4.3}$$

After contraction over X and Z, we secure

$$\begin{aligned} (D_V S)(Y, U) + (D_V S)(Y, \xi)\eta(U) &= \left[\frac{dr(V)}{n}\right]\{g(Y, U) \\ &+ \eta(Y)\eta(U)\}. \end{aligned} \tag{4.4}$$

Putting $Y = \xi$, we get

$$(D_V S)(\xi, U) = 0. \tag{4.5}$$

By virtue of the relation (4.5), we have

$$S(\phi U, V) = (n-1)g(\phi U, V). \tag{4.6}$$

We put $U = \phi U$ in the above relation and then using the equation (2.2), we find

$$S(U, V) = (n-1)g(U, V). \tag{4.7}$$

This completes the proof. ■

5. $\phi - m$-projectively flat LP-Sasakian manifold

Definition 5.1. An LP-Sasakian manifold M^n is said to be $\phi - m$-projectively flat if the m-projective curvature tensor W^* satisfies the relation

$$\phi^2(W^*(\phi X, \phi Y, \phi Z)) = 0, \tag{5.1}$$

for all vector fields X, Y and Z.

Theorem 5.2. A $\phi - m$-projectively flat LP-Sasakian manifold M^n is an η-Einstein manifold.

Proof. Let us assume that M^n be a $\phi - m$-projectvely flat LP-Sasakian manifold. Then by virtue of the relations (5.1) and (2.2), we have

$$W^*(\phi X, \phi Y, \phi Z) = -\eta(W^*(\phi X, \phi Y, \phi Z))\xi, \tag{5.2}$$

which implies

$$g(W^*(\phi X, \phi Y, \phi Z), \phi U) = -\eta(W^*(\phi X, \phi Y, \phi Z))g(\xi, \phi U). \tag{5.3}$$

Inconsequence of (2.7) the equation (5.3) yields

$$g(W^*(\phi X, \phi Y, \phi Z), \phi U) = 0. \tag{5.4}$$

Making use of (2.7) and (2.18) in the equation (5.4) we obtain

$$\begin{aligned} g(R(\phi X, \phi Y, \phi Z), \phi U) &= \frac{1}{2(n-1)}\{S(\phi Y, \phi Z)g(\phi X, \phi U) \\ &- S(\phi X, \phi Z)g(\phi Y, \phi U) + g(\phi Y, \Phi U)S(\phi X, \phi U) \\ &- g(\phi X, \phi Z)S(\phi Y, \phi U)\}. \end{aligned} \tag{5.5}$$

Let $\{e_1, e_2, \ldots, e_{n-1}, \xi\}$ be a local orthonormal basis of vector fields in M^n. By using the fact that $\{\phi e_1, \phi e_2, \ldots, \phi e_{n-1}, \xi\}$ is an orthonormal basis, if we put $X = U = e_i$ in the above relation and taking summation over i, $1 \leq i \leq n-1$, then we have

$$\begin{aligned} \sum_{i=1}^{n-1} g(R(\phi e_i, \phi Y, \phi Z), \phi e_i) &= \frac{1}{2(n-1)}\Big[\sum_{i=1}^{n-1} S(\phi Y, \phi Z)g(\phi e_i, \phi e_i) \\ &- \sum_{i=1}^{n-1} S(\phi e_i, \phi Z)g(\phi Y, \phi e_i) \\ &+ \sum_{i=1}^{n-1} g(\phi Y, \phi Z)S(\phi e_i, \phi e_i) \\ &- \sum_{i=1}^{n-1} g(\phi e_i, \phi Z)S(\phi Y, \phi e_i)\Big]. \end{aligned} \tag{5.6}$$

Now we find that [6]

$$\sum_{i=1}^{n-1} g(R(\phi e_i, \phi Y, \phi Z), \phi e_i) = S(\phi Y, \phi Z) + g(\phi Y, \phi Z), \tag{5.7}$$

$$\sum_{i=1}^{n-1} S(\phi e_i, \phi e_i) = r + n - 1, \tag{5.8}$$

$$\sum_{i=1}^{n-1} g(\phi e_i, \phi Z) S(\phi Y, \phi e_i) = S(\phi Y, \phi Z), \tag{5.9}$$

and

$$\sum_{i=1}^{n-1} g(\phi e_i, \phi e_i) = n + 1. \tag{5.10}$$

By virtue of the relations (5.7)–(5.10), the equation (5.6) reduces to

$$S(\phi Y, \phi Z) = \left[\frac{r}{n-1} - 1\right] g(\phi Y, \phi Z). \tag{5.11}$$

Then by making use of (2.3) and (2.15), the equation (5.11) takes the form

$$S(Y, Z) = \left\{\frac{r}{n-1} - 1\right\} g(Y, Z) + \left\{\frac{r}{n-1} - n\right\} \eta(Y)\eta(Z), \tag{5.12}$$

which implies from (2.17) that M^n is an η-Einstein manifold.
This completes the proof of the theorem. ■

6. An extended generalized ϕ-recurrent LP-Sasakian manifold

Definition 6.1. An LP-Sasakian manifold M^n is said to be extended generalized ϕ-recurrent if its curvature tensor R satisfies the relation

$$\phi^2((D_W R)(X, Y)Z) = A(W)\phi^2(R(X, Y)Z) + B(W)\phi^2(G(X, Y)Z), \tag{6.1}$$

where A and B are two 1-forms, B is non zero and these are defined by
$g(W, \rho_1) = A(W)$, $g(W, \rho_2) = B(W)$, and

$$G(X, Y, Z) = g(Y, Z)X - g(X, Z)Y,$$

for all $X, Y, Z, W \in \chi(M^n)$ and ρ_1, ρ_2 being vector fields associated to the 1-forms A and B respectively.

Lemma 6.2. In an extended generalized ϕ-recurrent LP-Sasakian manifold

$$(D_W S)(Y, \xi) = (n-1)g(Y, \phi W) - S(Y, \phi W). \tag{6.2}$$

Proof. We know that

$$(D_W S)(Y, \xi) = D_W S(Y, \xi) - S(D_W Y, \xi) - S(Y, D_W \xi). \tag{6.3}$$

Using (2.5)and (2.14) in the above relation, we can easily derive the expression (6.2).■

Theorem 6.3. In an extended generalized ϕ-recurrent LP-Sasakian manifold M^n, the 1-forms A and B are in opposite direction.

Proof. Let us consider that M^n be an extended generalized ϕ-recurrent LP-Sasakian manifold.
Then by virtue of relations (2.1), (2.2) and (2.7), the equation (6.1) becomes

$$\begin{aligned}&(D_W R)(X, Y, Z) + \eta((D_W R)(X, Y, Z))\xi \\ &= A(W)\{R(X, Y, Z) + \eta(R(X, Y, Z))\xi\} \\ &+B(W)\{G(X, Y, Z) + \eta(G(X, Y, Z))\xi\}.\end{aligned} \tag{6.4}$$

Taking inner product of the above relation with U we get

$$\begin{aligned}&g((D_W R)(X, Y, Z), U) + g((D_W R)(X, Y, Z), \xi)g(U, \xi) \\ &= A(W)[g(R(X, Y, Z), U) + g(R(X, Y, Z), \xi)g(U, \xi)] \\ &+B(W)[g(G(X, Y, Z), U) + g(G(X, Y, Z), \xi)g(U, \xi)].\end{aligned} \tag{6.5}$$

Let us suppose that $\{e_1, e_2, \ldots, e_n\}$ be an orthonormal basis of tangent space at any point of the manifold. Setting $X = U = e_i$ in the relation (6.5) and taking summation over i, $1 \le i \le n$, we obtain

$$\begin{aligned}(D_W S)(Y, Z) + \eta((D_W R)(\xi, Y, Z)) &= A(W)[S(Y, Z) + \eta(R(\xi, Y, Z))] \\ &+ B(W)[(n-2)g(Y, Z) \\ &- \eta(Y)\eta(Z)].\end{aligned} \tag{6.6}$$

Putting $Z = \xi$ in the above equation, we find

$$(D_W S)(Y, \xi) = \{A(W) + B(W)\}(n-1)\eta(Y) + g(Y, \phi W). \tag{6.7}$$

By virtue of the Lemma (6.2) in the above relation, we have

$$\begin{aligned}(n-1)g(Y, \phi W) - S(Y, \phi W) = (n-1)\eta(Y)[A(W) \\ +B(W)] + g(Y, \phi W).\end{aligned} \tag{6.8}$$

Replacing Y by ξ in equation (6.8) and after using the relations (2.7) and (2.14), we get

$$A(W) + B(W) = 0. \tag{6.9}$$

Hence we prove our theorem. ■

Theorem 6.4. An extended generalized ϕ-recurrent LP-Sasakian manifold M^n is an η-Einstein manifold.

Proof. Let us consider M^n be an extended generalized ϕ-recurrent LP-Sasakian manifold. In the theorem 6.3, we have proved that in an extended generalized ϕ-recurrent

LP-Sasakian manifold M^n, the 1-forms A and B are in opposite direction and so the relation (6.9) holds.
Now making use of (6.9) in (6.8) we have

$$(n-2)g(Y,\phi W) - S(Y,\phi W) = 0. \tag{6.10}$$

Replacing W by ϕW in the above relation and then using (2.2) and (2.14), we obtain

$$S(Y,W) = (n-2)g(Y,W) - \eta(Y)\eta(W). \tag{6.11}$$

Hence the manifold is η-Einstein. ■

7. Extended generalized concircularly φ-recurrent LP-Sasakian manifolds

Definition 7.1. An extended genaralized ϕ-recurrent LP-Sasakian manifold M^n is said to be an extended generalized concircularly ϕ-recurrent LP-Sasakian manifold, if the concircular curvature C satisfies the relation

$$\phi^2((D_W C)(X,Y,Z)) = A(W)\phi^2(C(X,Y,Z)) + B(W)\phi^2(G(X,Y,Z)), \tag{7.1}$$

where A and B are two 1-forms, B is non-zero and

$$C(X,Y,Z) = R(X,Y,Z) - \frac{r}{n(n-1)}G(X,Y,Z) \tag{7.2}$$

for all $X, Y, Z \in \chi(M^n)$ and r is the scalar curvature.

Theorem 7.2. An extended genaralized concircularly ϕ-recurrent LP-Sasakian manifold M^n, $n \geq 3$, is extended generalized ϕ-recurrent if and only if

$$\frac{[dr(W) - rA(W)]}{n(n-1)}\big\{g(Y,Z)X + g(Y,Z)\eta(X)\xi - g(X,Z)Y - g(X,Z)\eta(Y)\xi\big\} = 0. \tag{7.3}$$

Proof. Let us consider an extended generalized concircularly ϕ-recurrent LP-Sasakian manifold M^n, $n \geq 3$. Hence the defining condition of an extended generalized concircularly ϕ-recurrent LP-Sasakian manifold yields by virtue of (7.2) that

$$\begin{aligned}\phi^2((D_W R)(X,Y)Z) - A(W)\phi^2(R(X,Y)Z) - B(W)\phi^2(G(X,Y)Z)\\ = \frac{[dr(W) - rA(W)]}{n(n-1)}\{g(Y,Z)X + g(Y,Z)\eta(X)\xi\\ - g(X,Z)Y - g(X,Z)\eta(Y)\xi\}.\end{aligned} \tag{7.4}$$

Using (6.1) in the above relation, we get

$$\frac{[dr(W) - rA(W)]}{n(n-1)}\{g(Y,Z)X + g(Y,Z)\eta(X)\xi - g(X,Z)Y - g(X,Z)\eta(Y)\xi\} = 0. \tag{7.5}$$

This completes the proof. ■

Theorem 7.3. If an extended generalized concircularly ϕ-recurrent LP-Sasakian manifold M^n, $n \geq 3$, is an extended generalized ϕ-recurrent LP-Sasakian manifold, then the associated vector field corresponding to the 1-form A is given by $\rho_1 = \frac{1}{r} grad\, r$, r being the non-zero and non-constant scalar curvature of the manifold.

Proof. Taking inner product of the equation (7.5) with U, we obtain

$$\begin{aligned} \frac{[dr(W) - rA(W)]}{n(n-1)} & \{g(Y,Z)g(X,U) + g(Y,Z)\eta(X)g(\xi,U) \\ & -g(X,Z)g(Y,U) - g(X,Z)\eta(Y)g(\xi,U)\} = 0. \end{aligned} \tag{7.6}$$

Taking contraction over X and U, we get

$$[dr(W) - rA(W)]\{(n-2)g(Y,Z) - \eta(Y)\eta(Z)\} = 0. \tag{7.7}$$

Again contracting the equation (7.7) with respect to Y and Z, we obtain,

$$[dr(W) - rA(W)]\{(n-2)n + 1\} = 0 \tag{7.8}$$

which implies that

$$A(W) = \frac{dr(W)}{r}, \tag{7.9}$$

for all vector field W and $r \neq 0$
i.e.,

$$\rho_1 = \frac{1}{r} grad\, r,$$

where $A(W) = g(W, \rho_1)$.
Our theorem is thus proved. ■

Theorem 7.4. In an extended generalized concircularly ϕ-recurrent LP-Sasakian manifold M^n, $n \geq 3$, the associated 1-forms A and B are related by the relation

$$dr(W) = A(W)[r - n(n-1)] + B(W)n(n-1)^2. \tag{7.10}$$

Proof. By virtue of (2.2), it follows from (7.4) that

$$\begin{aligned} (D_W R)(X,Y)Z &= -\eta((D_W R)(X,Y)Z)\xi + A(W)[R(X,Y)Z \\ &+ \eta(R(X,Y)Z)\xi] + B(W)[G(X,Y)Z + \eta(G(X,Y)Z)\xi] \\ &+ \frac{\{dr(W) - rA(W)\}}{n(n-1)}[g(Y,Z)X \\ &+ g(Y,Z)\eta(X)\xi - g(X,Z)Y - g(X,Z)\eta(Y)\xi]. \end{aligned} \tag{7.11}$$

Taking inner product of the above relation with U and then contracting over X and U, and then using (2.16), we get

$$\begin{aligned}(D_W S)(Y,Z) &= A(W)S(Y,Z) + [(n-2)B(W) - A(W)]g(Y,Z) \\ &+ \frac{dr(W)}{n(n-1)}\{(n-2)g(Y,Z) - \eta(Y)\eta(Z)\} \\ &- A(W)\Big[\Big\{1 - \frac{r}{n(n-1)}\Big\}\eta(Y)\eta(Z) \\ &+ \Big\{\frac{(n-2)r}{n(n-1)}\Big\}g(Y,Z)\Big] - B(W)\eta(Y)\eta(Z). \end{aligned} \tag{7.12}$$

Again contraction over Y and Z in (7.12) yields

$$dr(W) = [r - n(n-1)]A(W) + n(n-1)^2 B(W). \tag{7.13}$$

Corollary 7.5. In an extended generalized concircularly ϕ-recurrent LP-Sasakian manifold M^n, $n \geq 3$, with constant scalar curvature, the associated 1-forms A and B are related by

$$\{r - n(n-1)\}A + n(n-1)^2 B = 0. \tag{7.14}$$

■

Definition 7.6. [11] An n-dimensional Riemannian manifold M^n, $n > 2$, is called a super generalized Ricci-recurrent if its Ricci tensor S of type (0, 2) satisfies the relation

$$DS = \alpha \otimes S + \beta \otimes g + \gamma \otimes \pi, \tag{7.15}$$

where α, β, γ are nowhere vanishing unique 1-forms and $\pi = \eta \otimes \eta$.

Theorem 7.7. An extended generalized concircularly ϕ-recurrent LP-Sasakian manifold M^n, $n \geq 3$, is super generalized Ricci recurrent manifold.

Proof. Using the equation (7.13) in (7.12), we get

$$\begin{aligned}(D_W S)(Y,Z) &= A(W)S(Y,Z) + n(n-2)B(W)g(Y,Z) \\ &- (n-1)A(W)g(Y,Z) - nB(W)\eta(Y)\eta(Z), \end{aligned} \tag{7.16}$$

From (7.16), it follows that the Ricci tensor S satisfies the condition

$$DS = \alpha \otimes S + \beta \otimes g + \gamma \otimes \pi, \tag{7.17}$$

where $\alpha(W) = A(W)$, $\beta(W) = n(n-2)B(W) - (n-1)A(W)$, $\gamma(W) = -nB(W)$ and $\pi = \eta \otimes \eta$.
This completes the proof of our theorem. ■

Theorem 7.8. In an extended generalized concircularly ϕ-recurrent LP-Sasakian manifold M^n, $n \geq 3$, the Ricci tensor in the direction of ρ_1 is given by

$$\begin{aligned} S(Y, \rho_1) &= \Big[\frac{r - (n-1)(n-2)}{2}\Big]A(Y) \\ &+ \Big[\frac{n(n^2 - 4n + 5)}{2}\Big]B(Y) - n\eta(Y)B(\xi). \end{aligned} \tag{7.18}$$

Proof. Taking contraction of (7.16) over W and Z, we get

$$\frac{1}{2}dr(Y) = S(Y, \rho_1) + n(n-2)B(Y) - (n-1)A(Y) + n\eta(Y)B(\xi). \tag{7.19}$$

By virtue of (7.13), the above relation takes the form

$$\begin{aligned} S(Y, \rho_1) &= \Big[\frac{r - (n-1)(n-2)}{2}\Big]A(Y) \\ &+ \Big[\frac{n(n^2 - 4n + 5)}{2}\Big]B(Y) - n\eta(Y)\beta(\xi). \end{aligned} \tag{7.20}$$

Hence we prove our theorem. ■

Theorem 7.9. In an extended generalized concircularly ϕ-recurrent LP-Sasakian manifold M^n, $n \geq 3$, the vector field ρ_2 associated with the 1-form B and the characteristic vector field ξ are in opposite direction.

Proof. By setting $Z = \xi$ in (7.16) and then using (6.2) and (2.14) we obtain

$$S(Y, \phi W) = (n-1)g(Y, \phi W) - n(n-1)B(W)\eta(Y). \tag{7.21}$$

Making replace of Y by ϕY in the equation (7.21) and using (2.3) and (2.15), we have

$$S(Y, W) = (n-1)g(Y, W). \tag{7.22}$$

Replacing W by ϕW in the above relation (7.21) and then using (2.2), we get

$$S(Y, W) = (n-1)g(Y, W) - n(n-1)B(\phi W)\eta(Y). \tag{7.23}$$

From (7.22) and (7.23) we have

$$B(\phi W) = 0, \tag{7.24}$$

which implies that

$$B(W) = -\eta(W)B(\xi). \tag{7.25}$$

This completes the proof. ■

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