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Optimizing Transportation Problem with Multiple Objectives by Hierarchical Order Goal Programming Model

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Abstract

Various techniques have been developed to solve multiple objective transportation problems (MOTP) by researchers. One of these is goal programming (GP) method. In goal programming, there is a need to set up a hierarchy of importance among goals so that the lower order goals are considered only after the higher order goal are satisfied. Goal programming technique helps in complete the satisfactory level of all objectives. In this paper, we are developing, optimizing transportation problem (TP) with multiple objectives by hierarchical order goal programming model. And also extension of optimizing transportation problems with multiple objectives.

Keyword: Transportation Problem**,** Multi Objective Transportation Problem, Goal Programming problem.

INTRODUCTION

GP is a branch of multi objective optimization, which in turn is a branch of multi-criteria decision analysis (MCDA), also known as multiple-criteria decision making (MCDM). G.P is the extension or generalization of linear programming to handle multiple, normally conflicting objective measures. Each of these measures is given a goal or target value to be achieved. Unwanted deviations from this set of target values are then minimized in an achievement function. Goal programming was first used by Charnes, Cooper and Ferguson [1] although the actual name first appeared in a 1961 by Charnes and Cooper [2]. Contributing later development works by Lee [6], Ignizio [7], Ignizio and Cavalier and Romero [8] followed. The formulation of goal programming problem is similar to that of linear programming problems. According to Charnes and Cooper [2], goal programming extends the linear programming formulation to accommodate mathematical programming with multiple objectives. It was refined by Ijiri in 1965 [10]. The major differences are an explicit consideration of goals and the various priorities associated with the different goals. Sang Moon Lee [3] is a pioneer of the solution technique of multi-objective transportation problem which is solved by using goal programming techniques in 1972. Most of the techniques for transportation problem have focused upon the optimization of a single objective condition, namely the minimization of total transportation cost, time etc. used before seventeen century. They have generally neglected the multiple objectives, i.e various environmental constraints, unique organizational value of the firm, and bureaucratic decision structures involved in the problem. But in reality, these are important factors which greatly control the decision in organization. They studied these entire situations, and then developed new technique to solve MOTP by using goal programming.

In the formulation two types of variable are used decision variables and deviational variables. There are two categories of system constraints and goal constraints, which are expressions of the original functions with target goals, set priorities and positive and negative deviational variables. When we deal with goals on the same priority level, our approach is just like the one described for non-preemptive goal programming. The goal programming model may be categorized in terms of how the goals are of roughly comparable importance, goal programming is known as non preemptive.

In another case, goal programming is known as called preemptive goal programming, there is a hierarchy of priority levels for the goals. Thus, the initial focus should be on achieving, as closely as possible, these first-priority goals. The other goals also might naturally divide further into second-priority goals, third-priority goals, and so on. After finding an optimal solution with respect to the first-priority goals, we can break any ties for the optimal solution by considering the second-priority goals. Any ties that remain after this re-optimization can be broken by considering the third-priority goals, and so on. Any of the same three types of goals (lower one-sided, two-sided, upper one-sided) can arise. Different penalty weights for deviations from different goals still can be included, if desired. The same formulation technique of introducing supporting variables again is used to reformulate this portion of the problem to fit the linear programming format. There are two basic methods based on linear programming for solving preemptive goal programming problems. One is called the sequential procedure, and the other is the streamlined procedure. Goal programming is used to perform three types of analysis: Determining the required resources to achieve a desired set of objectives, determining the degree of attainment of the goals with the available resources and providing the best satisfying solution under a varying amount of resources and priorities of the goals.

The basic approach of goal programming is to establish a specific numeric goal for each of the objectives, formulate an objective function for each objective, and then seek a solution that minimizes the (weighted) sum of deviations of these objective functions from their respective goals. There are three possible types of goals- a lower, one-sided goal sets a lower limit that we do not want to fall under (but exceeding the limit is fine), an upper, one-sided goal sets an upper limit that we do not want to exceed (but falling under the limit is fine) and a two-sided goal sets a specific target that we do not want to miss on either side.

GENERAL GOAL PROGRAMMING MODEL

Charnes and Cooper presented the general goal programming model which can be expressed mathematically as:

$$Minimize: Z = \sum_{i=1}^{m} d_i^+ + d_i^- \tag{1}$$

Subject to constraints:

Goal Constraints:

$$\sum_{j=1}^{n} a_{ij}\, x_j - d_i^+ + d_i^- = b_i \quad for \quad i=1,2,\ldots\ldots\ldots m \tag{2}$$

System Constraints:

$$\sum_{j=1}^{n} a_{ij}x_j \begin{vmatrix} \leq \\ = \\ \geq \end{vmatrix} b_i \; ; \quad for\ i = m+1, \ldots\ldots m+p \tag{3}$$

With $d_i^+, d_i^-, x_j \geq 0$, for i=1,2,…….m; j=1,2……n.

Where there are m goals, p system constraints and n decision variables.

Z= objective function =summation of all deviation.

a_{ij} = the coefficient associated with variable j in i^{th} goals.

x_j= the j^{th} decision variable.

b_i = the associated right hand side value.

d_i^-= negative deviational variable from the i^{th} goal (under achievement)

d_i^+= positive deviational variable from the i^{th} goal (over achievement)

Both over achievement and under achievement of a goal cannot occur simultaneously. Hence, either one or both of these variable must have a zero value; that is,

$$d_i^+ \times d_i^- = 0$$

Both variables apply for the non-negativity requirement as to all other linear programming variables; that is,

$$d_i^+, d_i^- = 0$$

Table 1 shows three basic options to achieve various goals:

Procedure for Achieving a Goal

Minimize	**Goal**	**If goal is achieved**
d_i^-	Minimize the underachievement	d_i^- =0, $d_i^+ \geq 0$
d_i^+	Minimize the over achievement	$d_i^+ = 0, d_i^- \geq 0$
$d_i^+ + d_i^-$	Minimize both under and over achievement	$d_i^- = 0, d_i^+ = 0$

The deviational variables are related to the functional algebraically as:

$$d_i^+ = \frac{1}{2}\left[\left|\sum_{j=1}^{n} a_{ij}x_j - b_i\right| + \left(\sum_{j=1}^{n} a_{ij}x_j - b_i\right)\right] \tag{4}$$

And

$$d_i^- = \frac{1}{2}\left[\left|\sum_{j=1}^{n} a_{ij}x_j + b_i\right| - \left(\sum_{j=1}^{n} a_{ij}x_j + b_i\right)\right] \tag{5}$$

The GP model in (1) has an objective function, constraints (called goal constraints) and the same nonnegative restriction on the decision variables as the LP model.

MODEL FORMULATION

Secondary data we are collected for this model by egg dealer. The main office of dealer is located at ABC place. Owner has been collecting egg by poultry farms namely S_1, S_2, S_3, S_4 respectively and then he supply four customer namely D_1, D_2, D_3, D_4 respectively. During the planning period of egg dealer is unable to meet whole demand of customer. However, dealer has determined that demand of customer D_4 must be satisfied as compare to other customer, union agreement to customer D_1, minimize the rejection during transportation from supplier S_2 to customer $D_{4,}$ satisfy minimum 80% demand of each customer and minimize the total transportation cost. Transportation cost per 10 stray from i^{th} source to j^{th} destination given in the following table.

Destination→ Sources ↓	D_1	D_1	D_3	D_4	Supply
S_1	14 (x_{11})	21 (x_{12})	18 (x_{13})	13 (x_{14})	210
S_2	24 (x_{21})	13 (x_{22})	21 (x_{13})	23 (x_{34})	240
S_3	12 (x_{31})	30 (x_{32})	9 (x_{33})	11 (x_{34})	180
S_4	13 (x_{41})	22 (x_{42})	1 9 (x_{43})	14 (x_{44})	300
Demand	130	220	260	300	930

Let P be the priority level of goal. Here we assume that P_1, P_2, P_3, P_4 and P_5 are the priority levels of the goals associated with hierarchical order goal. X_{ij} be the amount to be transported from i^{th} supplier to j^{th} destination.

P_1 : Guaranteed delivery to customer D_4.

P_2 : Supply at least 10 egg stray from supplier S_4 to customer D_1.

P_3 : Satisfy the demand minimum 80% of each customer.

P_4 : Minimize the rejection during transportation form supplier S_2 to customer D_4.

P_5 : Minimize the total transportation cost.

Let;

$$d_i^+ = over\ acchivement\ of\ the\ goals\ or\ constaints\ in\ the\ i^{th}\ equation$$

$$d_i^- = under\ acchivement\ of\ the\ goals\ or\ constaints\ in\ the\ i^{th}\ equation$$

Optimizing transportation problem with multiple objectives by hierarchical order goal programming model, first we have to formulate the model constraints on the basis of our goals.

P_1 : Guaranteed delivery to customer D_4 i.e it is assumed that the supplier never whishes to overfill a customer demand, so positive deviations can be excluded from demand constraints. However, since demand cannot be satisfied in all cases then negative deviation must be included to identified the under achievement of demand goals.

$$x_{14} + x_{24} + x_{34} + x_{44} + d_1^- - d_1^+ = 300 \quad (6)$$

P_2 : Supply at least 10 egg stray from supplier S_4 to customer D_1, i.e Supply at least 10 egg stray from supplier S_4 to customer D_1,

$$x_{14} + d_2^- - d_2^+ = 10 \quad (7)$$

P_3 : Satisfy the demand minimum 80% of each customer.

$$x_{11} + x_{21} + x_{31} + x_{41} + d_3^- - d_3^+ = 104 \quad (8)$$

$$x_{12} + x_{22} + x_{32} + x_{42} + d_4^- - d_4^+ = 176 \quad (9)$$

$$x_{13} + x_{23} + x_{33} + x_{34} + d_5^- - d_5^+ = 208 \quad (10)$$

$$x_{14} + x_{24} + x_{34} + x_{44} + d_6^- - d_6^+ = 204 \quad (11)$$

P_4 : Minimize the rejection during transportation form supplier S_2 to customer D_4.

$$x_{24} + d_7^- - d_7^+ = 0 \quad (12)$$

P_5 : Minimize the total transportation cost.

Minimize the total transportation cost,

$$\sum x_{ij}c_{ij} + d_8^- - d_8^+ = 0;\ for\ i,j \tag{13}$$

i.e

$14x_{11} + 21x_{12} + 18x_{13} + 13x_{14} + 24x_{21} + 13x_{22} + 21x_{23} + 23x_{24} + 12x_{31} + 30x_{32} + 9x_{33} + 11x_{34} + 13x_{41} + 22x_{42} + 19x_{43} + 14x_{44} + d_8^- - d_8^+ = 0$

$x_{ij}, d_i^- d_i^+ \geq 0$

Hierarchical order goal programming model as follows from above information,

Minimize

$$Z = P_1 d_1^- + P_2 d_2^- + P_3(d_3^- + d_4^- + d_5^- + d_6^-) + P_4 d_7^+ + P_5 d_8^+$$

Subject to;

$x_{14} + x_{24} + x_{34} + x_{44} + d_1^- - d_1^+ = 300$

$x_{14} \qquad + d_2^- - d_2^+ = 10$

$x_{11} + x_{21} + x_{31} + x_{41} + d_3^- - d_3^+ = 104$

$x_{12} + x_{22} + x_{32} + x_{42} + d_4^- - d_4^+ = 176$

$x_{13} + x_{23} + x_{33} + x_{34} + d_5^- - d_5^+ = 208$

$x_{14} + x_{24} + x_{34} + x_{44} + d_6^- - d_6^+ = 204$

$x_{24} \qquad + d_7^- - d_7^+ = 0$

$14x_{11} + 21x_{12} + 18x_{13} + 13x_{14}$

$+24x_{21} + 13x_{22} + 21x_{23} + 23x_{24}$

$+12x_{31} + 30x_{32} + 9x_{33} + 11x_{34}$

$+13x_{41} + 22x_{42} + 19x_{43} + 14x_{44} + d_8^- - d_8^+ = 0$

$x_{ij}, d_i^- d_i^+ \geq 0$

Z= {96,0,0,0,0,0,0,484}, these are the values ofd1M, d2M, d3M,d4M, d5M,d6M, d7P and d8P. In this result, we have to observe that goal 1 and goal 2 cannot be achieved because value of d1M is 96, d8P is 484 respectively, i.e, in priority 1the demand of customer D4 has not been satisfied and in priority 5 total transportation cost cannot be minimized. Goal 2 is achieved, i.e., supplier S4 satisfies the complete demand of customer D1, goal 3 is to achieve each customer's satisfaction upto minimum 80% of his demand, and goal 4 has also been achieved i.e. zero rejection during transportation from supplier S2 to the customer.

CONCLUSION:

In this paper we are solving transportation problem with multiple objective by goal programming using LINDO 14.0. A simple example is presented in this study and formulated the multi objective transportation problem in goal programming model in simple way. In this paper most useful tips to those researchers working on multi objective transportation problem by goal programming and other techniques.

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