Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 9 (2017), pp. 5245-5264

http://www.ripublication.com

Optimal Joint Total Cost of an Integrated Supply Chain Model for Deteriorating Inventory Items with backorder through Just in Time; A Fuzzy Approach

J. Jayanthi [1] and M. Maragatham [2]

[1] Department of Mathematics, Periyar Maniammai University,
Thanjavur – 3, India.

[2] P.G & Research Department of Mathematics, Periyar EVR College,
Tiruchirappalli-23, India.

Abstract

In this paper we derive an integrated supply chain model for deteriorating inventory items with backorder through just in time in fuzzy approach using Yager ranking method. Usually, the vendor and the buyer problems treated discretely. But in this paper, we confer a joint integrated vendor buyer model in both crisp and fuzzy sense. Nowadays the integrated vendor-buyer production inventory models are gaining much consequence. Supply Chain Management revolves around competent integration of vendor and buyer, if encompasses the firm's activities at many levels, from the premeditated level through the tactical to the operational level. Here shortages are allowed and lead time is zero. Finally, a numerical example and sensitivity analysis are given to illustrate this model. Yager ranking method is used to defuzzify the results.

Keywords: Supply Chain, Just – In –Time, Quality Assurance, Backorder, Deteriorating items, Integrated Vendor – Buyer, Trapezoidal fuzzy numbers, Yager Ranking Method.

1. INTRODUCTION

Supply chain management plays a pivotal role in ensuring goods, and services are delivered on time to consumers. In supply-chain management, inventory management plays an essential role. Inventory involves various cost, investment, space management, etc. Also there are chances that stored inventory may get smashed or get stolen adding to extra cost to the company. Therefore, it is important to have a vigorous inventory management for an organization.

Numerous established inventory models have focused on deriving optimal order quantities for the consumer. Such models ignore two opportunities. First, it may be possible to decrease costs without altering the ordering policy. Second, the firms can find an order quantity that is jointly optimal for the buyer and vendor.

Supply chain management is principally concerned with the efficient integration of suppliers, factories, warehouses and stores so that product is produced and scattered in the accurate quantities to the right locations and at the correct time, and so as to reduce total cost subject to fulfilling service necessities. Supply chain integration is a new kind of organizational model, taking energetic alliance of supply chain as a subject to grasp global resource integration through interactive collaborate operation of supply chain.

Just-in-time (JIT) is an inventory strategy companies employ to increase competence and decrease waste by receiving goods only as they are needed in the manufacturing process, thereby minimizing inventory costs. This method requires producers to anticipate demand accurately.

Just-in-time inventory control has numerous advantages over conventional models. Production runs remain short, which means manufacturers can move from one type of item for consumption to another very easily. This method reduces costs by eliminating warehouse storage needs. Just in time inventory is intentional to avoid situations in which inventory exceeds demand and places increased burden on your business to deal with the extra inventory. Manufacturers using JIT processes want to use materials for production at levels that convene distributor or retailer demand but not in excess. Retailers only want to acquire and carry inventory that meets immediate customer demand. Excess inventory requires storage and management costs.

Banerjee [1] developed a joint economic lot size model for a single vendor with finite production rate. Goyal [2] first introduced the idea of a joint total cost for a single vendor and a single buyer assuming an infinite production rate for the vendor and lot policy for the shipments from the vendor to the buyer. Ha and Kim [3] obtained the JIT system between the vendor and the buyer using geometric programming. Hwang et all [4] suggests that the climate of increasingly strict regulations for energy efficiency, material composition, waste reduction and product recycling. Kao, C and Hsu, WK [5] investigated that a single period inventory model with fuzzy demand. Li - Hsing Ho and Wei – Feng Kao [6] derives that a supply chain model with inventory and waste reduction considerations. Lee, H.M and Yao, J.S [7] developed an economic production quantity for fuzzy demand and fuzzy production quantity.

Maragatham, M and Jayanthi, J [8] discusses an integrated supply chain model in deteriorating inventory items and waste reduction contemplations through jit with fuzzy approach. Maragatham, M and Jayanthi, J [9] determine an integrated supply chain model for deteriorating inventory items and waste reduction contemplations through JIT with price dependent demand in fuzzy environment. Miltenburg [10] constructed that the term JIT could be adopted to signify techniques, improving quality products and reduce costs by eliminating all waste from the production system. Nagoor Gani and Sabarinathan [11] made an optimal policy for vendor-buyer integrated model with Fuzzy environment situation. Ritha, W and Sagayarani [12] derived a determination of optimal order quantity of an integrated inventory model using yager ranking method. Roy, M.S. and Sana, S.S and Chaudhuri,K [13] discussed an integrated producer buyer relationship in the environment of EMQ and JIT production systems. Yang and Wee [14] derived an integrated approach on economic order policy of deteriorated item for vendor and buyer. Yang, P.C , Wee, H.M and Yang, H.J [15] developed a global optimal policy for vendor-buyer integrated inventory system within just in time environment.

This paper considers a simple and practical situation and derives the minimum optimal solution with deteriorating items which integrates inventory and quality assertion in a JIT supply chain. This model assumes that manufacturing will produce some defective items and those products will not impact the buyer's purchase policy, shortages are allowed and lead time is zero. The vendor absorbs all the scrutiny cost. This approach deals with a single batch, at the end of the 100% screening process, if defective items are found; duplicate costs must be paid by the vendor.

We focus here on one particular ranking method introduced by Yager. This ranking method is based upon the idea of associating with a fuzzy number to a crisp value. Then we will see how to compute the valuation of a trapezoidal fuzzy number. We will analyze and interpret the results to obtain a deeper understanding of the process.

2. METHODOLOGY

2.1 Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function μ_A satisfied the following conditions, is a generalized fuzzy number $\widetilde{A}$.

(i) μ_A is a continuous mapping from R to the closed interval [0, 1].

(ii) $\mu_A = 0, -\infty < x \le a_1$,

(iii) $\mu_A =$ L(x) is strictly increasing on $[a_1, a_2]$

(iv) $\mu_A = w_A, a_2 \le x \le a_3$

(v) $\mu_A =$ R(x) is strictly decreasing on $[a_3, a_4]$

(vi) $\mu_A = 0, a_4 \le x < \infty$

Where $0 < w_A \le 1$ and a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number be denoted as $\widetilde{A} = (a_1, a_2, a_3, a_4 : w_A)_{LR}$; When $w_A = 1$, it can be simplified as $\widetilde{A} = (a_1, a_2, a_3, a_4)_{LR}$

2.2 Trapezoidal Fuzzy Number

A trapezoidal fuzzy number $\tilde{A}$ = (a, b, c, d) is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \frac{x-a}{b-a}, & \text{when } a \le x \le b; \\ 1, & \text{when } b \le x \le c; \\ R(x) = \frac{d-x}{d-c}, & \text{when } c \le x \le d; \\ 0, & \text{otherwise} \end{cases}$$

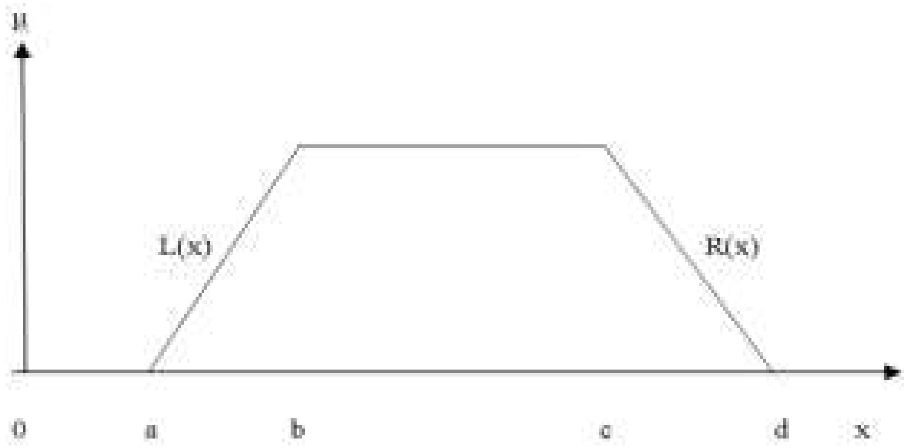

Fig.1: Trapezoidal Fuzzy Number

2.3 The Function Principle

The function principle is used for the operation of addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ *and* $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, then arithmetical operations are defined as:

1. $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
2. $\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$
3. $\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
4. $\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$
5. $\alpha \otimes \tilde{A} = \begin{cases} (\alpha a1, \alpha a2, \alpha a3, \alpha a4), & \alpha \geq 0 \\ (\alpha a4, \alpha a3, \alpha a2, \alpha a1), & \alpha < 0 \end{cases}$

2.4 Yager's Ranking Method

If the α cut of any fuzzy number $\tilde{A}$ is $[A_L(\alpha), A_R(\alpha)]$, then its ranking index $I(\tilde{A})$ is,

$$I(\tilde{A}) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha)]\, d\alpha \ .$$

2.5 Notations and Assumptions

The mathematical model in this paper is developed on the basis of the following notations and assumptions.

2.5.1 Notations

Q - Order Quantity of the buyer
R - Annual demand rate
P - Production rate
n - Total number of shipments per lot from the vendor to the buyer
nQ - Vendor's lot size per delivery
H_V - Vendor's holding cost per unit per unit time
H_B - Buyer's holding cost per unit per unit time
S_V - Production cost paid by the vendor
S_B - Purchasing cost paid by the purchaser
r - Annual inventory holding cost
K - Backordering ratio
1-K - Non-backordering ratio
Π_B - Unit backordering cost of the buyer
TC_V - Total annual cost of the vendor
TC_B - Total annual cost of the buyer
JTC - Joint annual total cost of the vendor and the buyer

α - Screening cost per unit
β - Reworking cost per unit
θ - Percentage of defective items
$\tilde{R}$ - Fuzzy annual demand rate
$\tilde{H}_V$ - Vendor's fuzzy holding cost per unit per unit time
$\tilde{H}_B$ - Buyer's fuzzy holding cost per unit per unit time
$\tilde{S}_V$ - Fuzzy Production cost paid by the vendor
$\tilde{S}_B$ - Fuzzy Purchasing cost paid by the purchaser
$\tilde{r}$ - Fuzzy annual inventory holding cost
$J\tilde{T}C$ - Joint fuzzy integrated total annual cost of the vendor and the buyer

2.5.2 Assumptions

- The integrated system of a single vendor and single buyer for a single product is considered.
- The vendor and the buyer have complete knowledge of each other's information.
- Production rate is uniform and finite.
- Demand rate is constant over time.
- Both the production and demand rates are constant and the production rate is greater than the demand rate.
- Buyer's shortages are allowed.
- Lead time is zero.
- Inventory is continuously reviewed.
- Transportation cost is not considered.
- Sequential conveyances are delivered so that the next one arrives at the buyer when stock from previous freight has just been finished.
- Screening cost and reworking cost is constant.
- In screening process, if defective items are found, then the duplicate costs is paid by the vendor.
- The vendor transferred the extra cost to the purchaser if shortened lead time is entreated.
- Demand rate (R), holding cost of the vendor (H_V), holding cost of the buyer (H_B), production cost of the vendor (S_V), purchasing cost of the buyer (S_B), annual inventory holding cost (r) and unit backordering cost of the buyer (Π_B) are taken as a trapezoidal fuzzy numbers.

3. MODEL FORMULATION

3.1 Proposed Joint Integrated Inventory Model in Crisp Sense

From the above notations and assumptions, total number of shipments, optimal order quantity and the joint total annual cost for the both vendor and buyer are determined.

Then, the joint total annual cost is given by,

JTC (Q, n) = vendor's set up cost + vendor's holding cost + vendor's screening cost + vendor's reworking cost + buyer's ordering cost + buyer's carrying cost + buyer's backordering cost

In vendor's inventory model, the total annual cost of the vendor is given by,

TC_V (Q, n) = set up cost + holding cost + screening cost + reworking cost

In buyer's inventory model, the total annual cost for the buyer is given by,

TC_B (Q) = ordering cost + carrying cost + backordering cost

The average inventory for vendor I_V is determined as follows ,

$$I_V = \frac{\dfrac{nQ^2}{2P} + Q^2\left(\dfrac{1}{R}-\dfrac{1}{P}\right) + 2Q^2\left(\dfrac{1}{R}-\dfrac{1}{P}\right) + 3Q^2\left(\dfrac{1}{R}-\dfrac{1}{P}\right) + \ldots + nQ^2\left(\dfrac{1}{R}-\dfrac{1}{P}\right)}{nQ/R}$$

$$= \frac{R}{nQ}\left[\frac{nQ^2}{2P} + \frac{n\,(n-1)Q^2}{2}\left(\frac{1}{R}-\frac{1}{P}\right)\right]$$

$$I_V = \frac{Q}{2}\left[n\left(1-\frac{R}{P}\right) - 1 + \frac{2R}{P}\right] \qquad (1)$$

Vendor's total annual cost is,

$$TC_V\,(Q,n) = \frac{S_V R}{nQ} + \frac{rQH_V}{2}\left[n\left(1-\frac{R}{P}\right) - 1 + \frac{2R}{P}\right] + \alpha\, nQ + \beta\theta\, nQ \qquad (2)$$

Buyer's total annual cost is,

$$TC_B\,(Q) = \frac{S_B\, R}{Q} + \frac{r(1-K)^2 QH_B}{2} + \frac{K^2 Q\pi_B}{2} \qquad (3)$$

Then, the joint total annual cost is given by,

$JTC\,(Q,n)$ = Vendor's total annual cost + Buyer's total annual cost

$$= \quad TC_V(Q,n) + TC_B(Q)$$

$$JTC(Q,n) = \frac{S_V R}{nQ} + \frac{rQH_V}{2}\left[n\left(1-\frac{R}{P}\right)-1+\frac{2R}{P}\right]+\alpha nQ+\beta\theta nQ+$$

$$\frac{S_B R}{Q} + \frac{r(1-K)^2 QH_B}{2} + \frac{K^2 Q\pi_B}{2} \tag{4}$$

Partially differentiate equation (4) with respect to Q, we get

$$\frac{\partial JTC(Q,n)}{\partial Q} = -\frac{S_V R}{nQ^2} + \frac{rH_V}{2}\left[n\left(1-\frac{R}{P}\right)-1+\frac{2R}{P}\right]+\alpha n+\beta\theta n-\frac{S_B R}{Q^2}+\frac{r(1-K)^2 H_B}{2}$$

$$+\frac{K^2\pi_B}{2}$$

(5)

Again, differentiate equation (5) with respect to Q, we get,

$$\frac{\partial^2 JTC(Q,n)}{\partial^2 Q} = \frac{2S_V R}{nQ^3}+\frac{2S_B R}{Q^3} > 0 \tag{6}$$

Now, set the equation (5) to zero and we compute for Q, then,

$$Q^* = \left[\frac{2R\left(\frac{S_V}{n}+S_B\right)}{r(1-K)^2 H_B + K^2\pi_B + rH_V\left[n\left(1-\frac{R}{P}\right)-1+\frac{2R}{P}\right]+2\alpha n+2\beta\theta n}\right]^{\frac{1}{2}} \tag{7}$$

Substituting equation (7) in (4) , we get,

$$JTC(n) = \left(\frac{2S_V R}{n}+2S_B R\right)^{\frac{1}{2}}\left(rH_V\left[n\left(1-\frac{R}{P}\right)-1+\frac{2R}{P}\right]+2\alpha n+2\beta\theta n+ r(1-K)^2 H_B + K^2\pi_B\right)^{\frac{1}{2}} \tag{8}$$

In equation (8), squaring on both sides , we get

$$[JTC(n)]^2 = 2RS_B rH_V n - \frac{2R^2 S_B rH_V n}{P} - 2RS_B rH_V + \frac{4R^2 S_B rH_V}{P} +$$

$$2RS_B rH_B(1-K)^2 + 2S_B RK^2\pi_B + 4RS_B\alpha n + 4RS_B\beta\theta n +$$

$$2RS_V rH_V - \frac{2R^2 S_V rH_V}{P} - \frac{2RS_V rH_V}{n} + \frac{4R^2 S_V rH_V}{nP} +$$

$$\frac{2RS_V rH_B(1-K)^2}{n} + \frac{2S_V RK^2\pi_B}{n} + 4RS_V\alpha + 4RS_V\beta\theta$$

(9)

Partially differentiate equation (8) with respect to 'n', we have,

$$\frac{\partial JTC(n)}{\partial n} = RS_B rH_V - \frac{2R^2 S_B rH_V}{P} + 2RS_B\alpha + 2RS_B\beta\theta + \frac{RS_V rH_V}{n^2} - \frac{2R^2 S_V rH_V}{n^2 P} - \frac{RS_V rH_B(1-K)^2}{n^2} - \frac{S_V RK^2\pi_B}{n^2} \tag{10}$$

Again, partially differentiate equation (10) with respect to 'n', we have,

$$\frac{\partial^2 JTC(n)}{\partial n^2} = \frac{2RS_V r}{n^3}\left[H_B(1-K)^2 - H_V\right] + \frac{4R^2 S_V rH_V}{n^3 P} + \frac{2S_V RK^2\pi_B}{n^3} > 0 \tag{11}$$

Now, set the equation (10) to zero and we compute for n, then,

$$n = \left\{\frac{S_V r\left[H_B(1-K)^2 - H_V\left(1-\frac{2R}{P}\right) + S_V K^2\pi_B\right]}{S_B\left[rH_V\left(1-\frac{R}{P}\right) + 2\alpha + 2\beta\theta\right]}\right\}^{1/2} \tag{12}$$

The optimal value of n = n∗ is get when,

$$JTC(n^*-1) \le JTC\left(n^*\right) \quad \text{and} \quad JTC(n^*) \le JTC\left(n^*+1\right) \tag{13}$$

From (12) and (13), we get,

$$n^*(n^*-1) \le \left\{\frac{S_V r\left[H_B(1-K)^2 - H_V\left(1-\frac{2R}{P}\right) + S_V K^2\pi_B\right]}{S_B\left[rH_V\left(1-\frac{R}{P}\right) + 2\alpha + 2\beta\theta\right]}\right\}^{1/2} \le n^*(n^*+1) \qquad (14) \qquad (14)$$

3.2 Proposed Joint Integrated Inventory Model in Fuzzy Sense using Yager's Ranking Method

We consider the joint integrated inventory model in fuzzy environment. Here, Demand rate (R), holding cost of the vendor (H_V), holding cost of the buyer (H_B), production cost of the vendor (S_V), purchasing cost of the buyer (S_B), annual inventory holding cost (r) and unit backordering cost of the buyer (Π_B) are taken as a trapezoidal fuzzy numbers.

Apply Yager Ranking method to defuzzify the joint total annual cost of the vendor and the buyer, total number of shipments and optimal order quantity .

Let $\tilde{R}, \tilde{H}_V, \tilde{H}_B, \tilde{S}_V, \tilde{S}_B, \tilde{r}$, $\tilde{\pi}_B$ be trapezoidal fuzzy numbers and they are defined as follows. [(ie) they are described by the α – cuts]

$$R(\alpha_R)=\left[L_R^{-1}(\alpha_R),\ R_R^{-1}(\alpha_R)\right]$$
$$H_V(\alpha_{H_V})=\left[L_{H_V}^{-1}(\alpha_{H_V}),\ R_{H_V}^{-1}(\alpha_{H_V})\right]$$
$$H_B(\alpha_{H_B})=\left[L_{H_B}^{-1}(\alpha_{H_B}),\ R_{H_B}^{-1}(\alpha_{H_B})\right]$$
$$S_V(\alpha_{S_V})=\left[L_{S_V}^{-1}(\alpha_{S_V}),\ R_{S_V}^{-1}(\alpha_{S_V})\right]$$
$$S_B(\alpha_{S_B})=\left[L_{S_B}^{-1}(\alpha_{S_B}),\ R_{S_B}^{-1}(\alpha_{S_B})\right]$$
$$r(\alpha_r)=\left[L_r^{-1}(\alpha_r),\ R_r^{-1}(\alpha_r)\right]$$
$$\pi_B(\alpha_{\pi_B})=\left[L_{\pi_B}^{-1}(\alpha_{\pi_B}), R_{\pi_B}^{-1}(\alpha_{\pi_B})\right]$$

The joint total annual cost is given by,

$J\widetilde{TC}(Q,n)$ = vendor's set up cost + vendor's holding cost +
vendor's screening cost + vendor's reworking cost +
buyer's ordering cost + buyer's carrying cost +
buyer's backordering cost

In vendor's inventory model, the total annual cost of the vendor is given by,

$\widetilde{TC}_V(Q,n)$ = set up cost + holding cost + screening cost + reworking cost

In buyer's inventory model, the total annual cost for the buyer is given by,

$\widetilde{TC}_B(Q)$ = ordering cost + carrying cost + backordering cost

The average inventory for vendor I_V is determined as follows ,

$$I_V = \frac{\dfrac{nQ^2}{2P}+Q^2\left(\dfrac{1}{\widetilde{R}}-\dfrac{1}{P}\right)+2Q^2\left(\dfrac{1}{\widetilde{R}}-\dfrac{1}{P}\right)+3Q^2\left(\dfrac{1}{\widetilde{R}}-\dfrac{1}{P}\right)+\ldots+nQ^2\left(\dfrac{1}{\widetilde{R}}-\dfrac{1}{P}\right)}{nQ/\widetilde{R}}$$

$$= \frac{\widetilde{R}}{nQ}\left[\frac{nQ^2}{2P}+\frac{n\,(n-1)\,Q^2}{2}\left(\frac{1}{\widetilde{R}}-\frac{1}{P}\right)\right]$$

$$I_V = \frac{Q}{2}\left[n\left(1-\frac{\tilde{R}}{P}\right)-1+\frac{2\tilde{R}}{P}\right] \tag{15}$$

Vendor’s total annual cost is,

$$T\tilde{C}_V(Q,n) = \frac{\tilde{S}_V\tilde{R}}{nQ}+\frac{\tilde{r}Q\tilde{H}_V}{2}\left[n\left(1-\frac{\tilde{R}}{P}\right)-1+\frac{2\tilde{R}}{P}\right]+\alpha nQ+\beta\theta nQ \tag{16}$$

Buyer’s total annual cost is,

$$T\tilde{C}_B(Q) = \frac{\tilde{S}_B\tilde{R}}{Q}+\frac{\tilde{r}(1-K)^2Q\tilde{H}_B}{2}+\frac{K^2Q\tilde{\pi}_B}{2} \tag{17}$$

Then, the joint total annual cost is given by,

$$J\tilde{T}C(Q,n) = \text{Vendor's total annual cost + Buyer's total annual cost}$$

$$= T\tilde{C}_V(Q,n)+T\tilde{C}_B(Q)$$

$$J\tilde{T}C(Q,n) = \frac{\tilde{S}_V\tilde{R}}{nQ}+\frac{\tilde{r}Q\tilde{H}_V}{2}\left[n\left(1-\frac{\tilde{R}}{P}\right)-1+\frac{2\tilde{R}}{P}\right]+\alpha nQ+\beta\theta nQ+\frac{\tilde{S}_B\tilde{R}}{Q}+\frac{\tilde{r}(1-K)^2Q\tilde{H}_B}{2}+\frac{K^2Q\tilde{\pi}_B}{2} \tag{18}$$

The above equation can be rewritten as,

$$J\tilde{T}C(Q,n)=\frac{\tilde{S}_V\tilde{R}}{nQ}+(n-1)\frac{\tilde{r}Q\tilde{H}_V}{2}+(2-n)\frac{\tilde{r}Q\tilde{H}_V\tilde{R}}{2P}+\frac{\tilde{S}_B\tilde{R}}{Q}+\frac{\tilde{r}(1-K)^2Q\tilde{H}_B}{2}++\frac{K^2Q\tilde{\pi}_B}{2}+\alpha nQ+\beta\theta nQ \tag{19}$$

Using Yaker Ranking Method for equation (19), we have,

$$K_1\left(\alpha_{S_V},\alpha_R\right) = \frac{1}{4}\left\{\int_0^1 L_{S_V}^{-1}\left(\alpha_{S_V}\right)d\alpha_{S_V}.\int_0^1 L_R^{-1}(\alpha_R)d\alpha_R+\int_0^1 R_{S_V}^{-1}\left(\alpha_{S_V}\right)d\alpha_{S_V}.\int_0^1 R_R^{-1}(\alpha_R)d\alpha_R\right\}$$

$$K_2\left(\alpha_r,\alpha_{H_V}\right) = \frac{1}{4}\left\{\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 L_{H_V}^{-1}\left(\alpha_{H_H}\right)d\alpha_{H_V}+\int_0^1 R_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 R_{H_V}^{-1}\left(\alpha_{H_V}\right)d\alpha_{H_V}\right\}$$

$$K_3\left(\alpha_r,\alpha_{H_V},\alpha_R\right) = \frac{1}{8}\left\{\begin{array}{l}\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 L_{H_V}^{-1}\left(\alpha_{H_H}\right)d\alpha_{H_V}.\int_0^1 L_R^{-1}(\alpha_R)d\alpha_R\\ +\int_0^1 R_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 R_{H_V}^{-1}\left(\alpha_{H_H}\right)d\alpha_{H_V}.\int_0^1 R_R^{-1}(\alpha_R)d\alpha_R\end{array}\right\}$$

$$K_4\left(\alpha_{S_B},\alpha_R\right) = \frac{1}{4}\left\{\int_0^1 L_{S_B}^{-1}\left(\alpha_{S_B}\right)d\alpha_{S_B}.\int_0^1 L_R^{-1}(\alpha_R)d\alpha_R+\int_0^1 R_{S_B}^{-1}\left(\alpha_{S_B}\right)d\alpha_{S_B}.\int_0^1 R_R^{-1}(\alpha_R)d\alpha_R\right\}$$

$$K_5\left(\alpha_r,\alpha_{H_B}\right) = \frac{1}{4}\left\{\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 L_{H_B}^{-1}\left(\alpha_{H_B}\right)d\alpha_{H_B} + \int_0^1 R_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 R_{H_B}^{-1}\left(\alpha_{H_B}\right)d\alpha_{H_B}\right\}$$

$$K_6\ (\alpha_{\pi_B}) = \frac{1}{2}\left\{\int_0^1 L_{\pi_B}^{-1}(\alpha_{\pi_B})d\alpha_{\pi_B} + \int_0^1 R_{\pi_B}^{-1}(\alpha_{\pi_B})d\alpha_{\pi_B}\right\}$$

Therefore, equation (19) becomes,

$$\begin{aligned} J\widetilde{T}C(Q,n) = {} & \frac{K_1\left(\alpha_{S_V},\alpha_R\right)}{nQ} + (n-1)\frac{QK_2\left(\alpha_r,\alpha_{H_V}\right)}{2} + (2-n)\frac{QK_3\left(\alpha_r,\alpha_{H_V},\alpha_R\right)}{2P} + \\ & \frac{K_4\left(\alpha_{S_B},\alpha_R\right)}{Q} + \frac{Q(1-K)^2K_5\left(\alpha_r,\alpha_{H_B}\right)}{2} + \frac{K^2QK_6\left(\alpha_{\pi_B}\right)}{2} + \alpha nQ + \beta\theta nQ \end{aligned} \tag{20}$$

To determine the optimal order quantity,
Partially differentiate equation (20) with respect to Q, we get

$$\begin{aligned} \frac{\partial J\widetilde{T}C(Q,n)}{\partial Q} = {} & \frac{-K_1\left(\alpha_{S_V},\alpha_R\right)}{nQ^2} + (n-1)\frac{K_2\left(\alpha_r,\alpha_{H_V}\right)}{2} + (2-n)\frac{K_3\left(\alpha_r,\alpha_{H_V},\alpha_R\right)}{2P} \\ & - \frac{K_4\left(\alpha_{S_B},\alpha_R\right)}{Q^2} + \frac{(1-K)^2K_5\left(\alpha_r,\alpha_{H_B}\right)}{2} + \frac{K^2K_6\left(\alpha_{\pi_B}\right)}{2} + \alpha n + \beta\theta n \end{aligned} \tag{21}$$

Again, differentiate equation (21) with respect to Q, we get,

$$\frac{\partial^2 J\widetilde{T}C(Q,n)}{\partial^2 Q} = \frac{2K_1(\alpha_{S_V},\alpha_R)}{nQ^3} + \frac{2K_4\left(\alpha_{S_B},\alpha_R\right)}{Q^3} \ > \ 0 \tag{22}$$

Now, set the equation (21) to zero and we compute for Q, then,

$$Q^* = \left[\frac{2P\left(\frac{K_1\left(\alpha_{S_V},\alpha_R\right)}{n} + K_4\left(\alpha_{S_B},\alpha_R\right)\right)}{\begin{array}{r}(n-1)PK_2\left(\alpha_r,\alpha_{H_V}\right) + (2-n)K_3\left(\alpha_r,\alpha_{H_V},\alpha_R\right) + (1-K)^2PK_5(\alpha_r,\alpha_{H_B}) \\ + K^2PK_6\left(\alpha_{\pi_B}\right) + 2P\alpha n + 2P\beta\theta n\end{array}}\right]^{\frac{1}{2}} \tag{23}$$

Now, to determine the number of shipments,

$$n = \left\{\frac{P\tilde{r}\,\tilde{S}_V\tilde{H}_B(1-K)^2 - P\tilde{r}\,\tilde{S}_V\tilde{H}_V + 2\tilde{r}\tilde{S}_V\tilde{H}_V\tilde{R} + PK^2\tilde{S}_V\tilde{\pi}_B}{P\tilde{r}\,\tilde{S}_B\tilde{H}_V - \tilde{r}\tilde{S}_B\tilde{H}_V\tilde{R} + 2P\alpha\,\tilde{S}_B + 2P\beta\theta\,\tilde{S}_B}\right\}^{1/2} \tag{24}$$

Using Yager Ranking method for the above equation, we get,

$$K_7\left(\alpha_r,\alpha_{S_V},\alpha_{H_B}\right) = \frac{1}{8}\left\{\begin{array}{l}\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 L_{S_V}^{-1}\left(\alpha_{S_H}\right)d\alpha_{S_V}.\int_0^1 L_{H_B}^{-1}(\alpha_{H_B})d\alpha_{H_B} \\ + \int_0^1 R_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 R_{S_V}^{-1}\left(\alpha_{S_H}\right)d\alpha_{S_V}.\int_0^1 R_{H_B}^{-1}(\alpha_{H_B})d\alpha_{H_B}\end{array}\right\}$$

$$K_8\left(\alpha_r,\alpha_{S_V},\alpha_{H_V}\right)=\frac{1}{8}\left\{\begin{array}{l}\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 L_{S_V}^{-1}\left(\alpha_{S_H}\right)d\alpha_{S_V}.\int_0^1 L_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V}\\+\int_0^1 R_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 R_{S_V}^{-1}\left(\alpha_{S_H}\right)d\alpha_{S_V}.\int_0^1 R_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V}\end{array}\right\}$$

$$K_9\left(\alpha_r,\alpha_{S_V},\alpha_{H_V},\alpha_R\right)=\frac{1}{16}\left\{\begin{array}{l}\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 L_{S_V}^{-1}\left(\alpha_{S_H}\right)d\alpha_{S_V}.\int_0^1 L_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V}.\int_0^1 L_R^{-1}(\alpha_R)d\alpha_R\\+\int_0^1 R_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 R_{S_V}^{-1}\left(\alpha_{S_H}\right)d\alpha_{S_V}.\int_0^1 R_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V}.\int_0^1 R_R^{-1}(\alpha_R)d\alpha_R\end{array}\right\}$$

$$K_{10}\left(\alpha_{S_V},\alpha_{\pi_B}\right)=\frac{1}{4}\left\{\int_0^1 L_{S_V}^{-1}\left(\alpha_{S_V}\right)d\alpha_{S_V}.\int_0^1 L_{\pi_B}^{-1}\left(\alpha_{\pi_B}\right)d\alpha_{\pi_B}+\int_0^1 R_{S_V}^{-1}\left(\alpha_{S_V}\right)d\alpha_{S_V}.\int_0^1 R_{\pi_B}^{-1}\left(\alpha_{\pi_B}\right)d\alpha_{\pi_B}\right\}$$

$$K_{11}\left(\alpha_r,\alpha_{S_B},\alpha_{H_V}\right)=\frac{1}{8}\left\{\begin{array}{l}\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 L_{S_B}^{-1}\left(\alpha_{S_B}\right)d\alpha_{S_B}.\int_0^1 L_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V}\\+\int_0^1 R_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 R_{S_B}^{-1}\left(\alpha_{S_B}\right)d\alpha_{S_B}.\int_0^1 R_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V}\end{array}\right\}$$

$$K_{12}\left(\alpha_r,\alpha_{S_B},\alpha_{H_V},\alpha_R\right)=\frac{1}{16}\left\{\begin{array}{l}\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 L_{S_B}^{-1}\left(\alpha_{S_B}\right)d\alpha_{S_B}.\int_0^1 L_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V}.\int_0^1 L_R^{-1}(\alpha_R)d\alpha_R\\+\int_0^1 R_r^{-1}(\alpha_r)d\alpha_r.\int_0^1 R_{S_B}^{-1}\left(\alpha_{S_B}\right)d\alpha_{S_B}.\int_0^1 R_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V}.\int_0^1 R_R^{-1}(\alpha_R)d\alpha_R\end{array}\right\}$$

$$K_{13}(\alpha_{S_B})=\frac{1}{2}\left\{\int_0^1 L_{S_B}^{-1}(\alpha_{S_B})+\int_0^1 R_{S_B}^{-1}(\alpha_{S_B})\right\}$$

Substitute these ranking values in equation (24) , we have

$$n^*=\left\{\frac{P(1-K)^2K_7\left(\alpha_r,\alpha_{S_V},\alpha_{H_B}\right)-PK_8\left(\alpha_r,\alpha_{S_V},\alpha_{H_V}\right)+2K_9\left(\alpha_r,\alpha_{S_V},\alpha_{H_V},\alpha_R\right)+PK^2K_{10}\left(\alpha_{S_V},\alpha_{\pi_B}\right)}{PK_{11}\left(\alpha_r,\alpha_{S_B},\alpha_{H_V}\right)-K_{12}\left(\alpha_r,\alpha_{S_B},\alpha_{H_V},\alpha_R\right)+\left(2P\alpha+2P\beta\theta\right)K_{13}(\alpha_{S_B})}\right\}^{1/2} \quad (25)$$

The optimal value of n = n∗ is get when,

$$J\tilde{T}C(n^*-1)\le J\tilde{T}C\left(n^*\right) \quad \text{and} \quad J\tilde{T}C(n^*)\le J\tilde{T}C\left(n^*+1\right) \quad (26)$$

From (25) and (26), we get,

$$n^*(n^*-1)\le$$

$$n^* = \left\{ \frac{P(1-K)^2 K_7\left(\alpha_r, \alpha_{S_V}, \alpha_{H_B}\right) - PK_8\left(\alpha_r, \alpha_{S_V}, \alpha_{H_V}\right) + 2K_9\left(\alpha_r, \alpha_{S_V}, \alpha_{H_V}, \alpha_R\right) + PK^2 K_{10}\left(\alpha_{S_V}, \alpha_{\pi_B}\right)}{PK_{11}\left(\alpha_r, \alpha_{S_B}, \alpha_{H_V}\right) - K_{12}\left(\alpha_r, \alpha_{S_B}, \alpha_{H_V}, \alpha_R\right) + \left(2P\alpha + 2P\beta\theta\right) K_{13}(\alpha_{S_B})} \right\}^{1/2}$$

$$\leq n^*(n^*+1) \qquad (27)$$

4. NUMERICAL EXAMPLE

4.1 Numerical Example in Crisp Sense

The annual demand of an item is 840 unit / year and the production rate is 3200 unit / year. Annual inventory holding cost is Rs. 5 per unit. The holding cost for the vendor is Rs.14 / unit, the holding cost for the buyer is Rs.20 / unit, the production cost for the vendor is Rs. 440 / set up and the production cost for the buyer is Rs. 35 / order. If there is 5 % defective items then the duplicate cost for the defective items is Rs. 2 / unit and the screening cost is Rs. 3 / unit. Total number of shipments, order quantity, joint total annual cost for both the vendor and the buyer are determined.

Sol:

R = Rs. 840 unit / year
P = Rs. 3200 unit / year
r = Rs. 1 / unit
H_V = Rs. 14 / unit
H_B = Rs. 20 / unit
S_V = Rs. 440 / setup
S_B = Rs. 35 / order
K = 10%
Π_B = 5
θ = 5 %
α = Rs. 3 / unit
β = Rs. 2 / unit

4.1.1 Total Number of Shipments

$$n = \left\{ \frac{S_V r \left[H_B(1-K)^2 - H_V\left(1-\frac{2R}{P}\right) + S_V K^2 \pi_B \right]}{S_B \left[rH_V\left(1-\frac{R}{P}\right) + 2\alpha + 2\beta\theta \right]} \right\}^{1/2}$$

$n^* = 3$

4.1.2 Order Quantity

$$Q^* = \left[\frac{2R\left(\frac{S_V}{n} + S_B\right)}{r(1-K)^2 H_B + K^2 \pi_B + rH_V \left[n\left(1 - \frac{R}{P}\right) - 1 + \frac{2R}{P} \right] + 2\alpha n + 2\beta\theta n} \right]^{\frac{1}{2}}$$

$$Q^* = 37.14$$

4.1.3 Joint Total Annual Cost of the Vendor and the Buyer

$$JTC(Q,n) = \frac{S_V R}{nQ} + \frac{rQH_V}{2}\left[n\left(1 - \frac{R}{P}\right) - 1 + \frac{2R}{P} \right] + \alpha nQ + \beta\theta nQ + \frac{S_B R}{Q} + \frac{r(1-K)^2 QH_B}{2} + \frac{K^2 Q\pi_B}{2}$$

$$JTC(Q,n) = \text{Rs. } 8217.86$$

4.2 Numerical Example in Fuzzy Sense

To validate the proposed model consider the data

Let $\tilde{R}$ = (760, 780, 820, 840) unit / year

P = Rs. 3200 unit / year

$\tilde{r}$ = (Rs.1, Rs.2, Rs.4, Rs.5) unit/year

$\tilde{H}_V$ = (Rs.6, Rs.8, Rs.12, Rs.14) unit/year

$\tilde{H}_B$ = (Rs.12, Rs.14, Rs.18, Rs.20) unit/ year

$\tilde{S}_V$ = (Rs.360, Rs.380, Rs.420, Rs.440) / setup

$\tilde{S}_B$ = (Rs.15, Rs.20, Rs.30, Rs.35) / order

K = 10%

$\tilde{\pi}_B$ = (RS. 3, Rs. 4, Rs.6, Rs.7)

θ = 5 %

α = Rs. 3 / unit

β = Rs. 2 / unit

Sol:

$$\tilde{R}(\alpha_R) = (760+20\alpha,\ 840-20\alpha)$$

$$P = 3200$$

$$\tilde{r}(\alpha_r) = (1+\alpha,\ 5-\alpha)$$

$$\tilde{H}_V(\alpha_{H_V}) = (6+2\alpha, 14-2\alpha)$$

$$\tilde{H}_B(\alpha_{H_B}) = (12+2\alpha, 20-2\alpha)$$

$$\tilde{S}_V(\alpha_{S_V}) = (360+20\alpha, 440-20\alpha)$$

$$\tilde{S}_B(\alpha_{S_B}) = (15+5\alpha, 35-5\alpha)$$

$$K = 10\%$$

$$\tilde{\pi}_B(\alpha_{\pi_B}) = (3+\alpha, 7-\alpha)$$

$$\theta = 5\,\%$$

$$\alpha = \text{Rs. } 3 \text{ / unit}$$

$$\beta = \text{Rs. } 2 \text{ / unit}$$

$$K_1(\alpha_{S_V}, \alpha_R) = \frac{1}{4}\left\{\int_0^1 L_{S_V}^{-1}(\alpha_{S_V})d\alpha_{S_V} . \int_0^1 L_R^{-1}(\alpha_R)d\alpha_R + \int_0^1 R_{S_V}^{-1}(\alpha_{S_V})d\alpha_{S_V} . \int_0^1 R_R^{-1}(\alpha_R)d\alpha_R\right\}$$

$$= 160450$$

$$K_2(\alpha_r, \alpha_{H_V}) = \frac{1}{4}\left\{\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 L_{H_V}^{-1}(\alpha_{H_H})d\alpha_{H_V} + \int_0^1 R_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 R_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V}\right\}$$

$$= 17.25$$

$$K_3(\alpha_r, \alpha_{H_V}, \alpha_R) = \frac{1}{8}\left\{\begin{array}{l}\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 L_{H_V}^{-1}(\alpha_{H_H})d\alpha_{H_V} . \int_0^1 L_R^{-1}(\alpha_R)d\alpha_R \\ + \int_0^1 R_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 R_{H_V}^{-1}(\alpha_{H_H})d\alpha_{H_V} . \int_0^1 R_R^{-1}(\alpha_R)d\alpha_R\end{array}\right\}$$

$$= 7080$$

$$K_4(\alpha_{S_B}, \alpha_R) = \frac{1}{4}\left\{\int_0^1 L_{S_B}^{-1}(\alpha_{S_B})d\alpha_{S_B} . \int_0^1 L_R^{-1}(\alpha_R)d\alpha_R + \int_0^1 R_{S_B}^{-1}(\alpha_{S_B})d\alpha_{S_B} . \int_0^1 R_R^{-1}(\alpha_R)d\alpha_R\right\}$$

$$= 10112.5$$

$$K_5(\alpha_r, \alpha_{H_B}) = \frac{1}{4}\left\{\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 L_{H_B}^{-1}(\alpha_{H_B})d\alpha_{H_B} + \int_0^1 R_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 R_{H_B}^{-1}(\alpha_{H_B})d\alpha_{H_B}\right\}$$

$$= 26.25$$

$$K_6\ (\alpha_{\pi_B}) \quad = \quad \frac{1}{2}\left\{\int_0^1 L_{\pi_B}^{-1}\ (\alpha_{\pi_B})\, d\alpha_{\pi_B} + \int_0^1 R_{\pi_B}^{-1}\ (\alpha_{\pi_B})\, d\alpha_{\pi_B}\right\}$$

$$= \quad 5$$

$$K_7\left(\alpha_r, \alpha_{S_V}, \alpha_{H_B}\right) = \frac{1}{8}\left\{\begin{array}{l}\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 L_{S_V}^{-1}\left(\alpha_{S_H}\right)d\alpha_{S_V} . \int_0^1 L_{H_B}^{-1}(\alpha_{H_B})d\alpha_{H_B} \\ + \int_0^1 R_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 R_{S_V}^{-1}\left(\alpha_{S_H}\right)d\alpha_{S_V} . \int_0^1 R_{H_B}^{-1}(\alpha_{H_B})d\alpha_{H_B}\end{array}\right\}$$

$$= \ 5497.5$$

$$K_8\left(\alpha_r, \alpha_{S_V}, \alpha_{H_V}\right) = \frac{1}{8}\left\{\begin{array}{l}\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 L_{S_V}^{-1}\left(\alpha_{S_H}\right)d\alpha_{S_V} . \int_0^1 L_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V} \\ + \int_0^1 R_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 R_{S_V}^{-1}\left(\alpha_{S_H}\right)d\alpha_{S_V} . \int_0^1 R_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V}\end{array}\right\}$$

$$= \ 3630$$

$$K_9\left(\alpha_r, \alpha_{S_V}, \alpha_{H_V}, \alpha_R\right) = \frac{1}{16}\left\{\begin{array}{l}\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 L_{S_V}^{-1}\left(\alpha_{S_H}\right)d\alpha_{S_V} . \int_0^1 L_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V} . \int_0^1 L_R^{-1}(\alpha_R)d\alpha_R \\ + \int_0^1 R_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 R_{S_V}^{-1}\left(\alpha_{S_H}\right)d\alpha_{S_V} . \int_0^1 R_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V} . \int_0^1 R_R^{-1}(\alpha_R)d\alpha_R\end{array}\right\}$$

$$= \quad 1491881.25$$

$$K_{10}\left(\alpha_{S_V}, \alpha_{\pi_B}\right) = \frac{1}{4}\left\{\int_0^1 L_{S_V}^{-1}\left(\alpha_{S_V}\right)d\alpha_{S_V} . \int_0^1 L_{\pi_B}^{-1}\left(\alpha_{\pi_B}\right)d\alpha_{\pi_B} + \int_0^1 R_{S_V}^{-1}\left(\alpha_{S_V}\right)d\alpha_{S_V} . \int_0^1 R_{\pi_B}^{-1}\left(\alpha_{\pi_B}\right)d\alpha_{\pi_B}\right\}$$

$$= \quad 1022.5$$

$$K_{11}\left(\alpha_r, \alpha_{S_B}, \alpha_{H_V}\right) = \frac{1}{8}\left\{\begin{array}{l}\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 L_{S_B}^{-1}\left(\alpha_{S_B}\right)d\alpha_{S_B} . \int_0^1 L_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V} \\ + \int_0^1 R_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 R_{S_B}^{-1}\left(\alpha_{S_B}\right)d\alpha_{S_B} . \int_0^1 R_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V}\end{array}\right\}$$

$$= \quad 260.625$$

$$K_{12}\left(\alpha_r, \alpha_{S_B}, \alpha_{H_V}, \alpha_R\right) = \frac{1}{16}\left\{\begin{array}{l}\int_0^1 L_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 L_{S_B}^{-1}\left(\alpha_{S_B}\right)d\alpha_{S_B} . \int_0^1 L_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V} . \int_0^1 L_R^{-1}(\alpha_R)d\alpha_R \\ + \int_0^1 R_r^{-1}(\alpha_r)d\alpha_r . \int_0^1 R_{S_B}^{-1}\left(\alpha_{S_B}\right)d\alpha_{S_B} . \int_0^1 R_{H_V}^{-1}(\alpha_{H_V})d\alpha_{H_V} . \int_0^1 R_R^{-1}(\alpha_R)d\alpha_R\end{array}\right\}$$

$$= \quad 107470.3125$$

$$K_{13}(\alpha_{S_B}) = \frac{1}{2}\left\{\int_0^1 L_{S_B}^{-1}(\alpha_{S_B}) + \int_0^1 R_{S_B}^{-1}(\alpha_{S_B})\right\} = 25$$

4.2.1 Total Number of Shipments

$$n^* = \left\{\frac{P(1-K)^2 K_7(\alpha_r,\alpha_{S_V},\alpha_{H_B}) - PK_8(\alpha_r,\alpha_{S_V},\alpha_{H_V}) + 2K_9(\alpha_r,\alpha_{S_V},\alpha_{H_V},\alpha_R) + PK^2 K_{10}(\alpha_{S_V},\alpha_{\pi_B})}{PK_{11}(\alpha_r,\alpha_{S_B},\alpha_{H_V}) - K_{12}(\alpha_r,\alpha_{S_B},\alpha_{H_V},\alpha_R) + (2P\alpha + 2P\beta\theta)K_{13}(\alpha_{S_B})}\right\}^{1/2}$$

$$n^* = 3$$

4.2.2 Fuzzy Order Quantity

$$Q^* = \left[\frac{2P\left(\dfrac{K_1(\alpha_{S_V},\alpha_R)}{n} + K_4(\alpha_{S_B},\alpha_R)\right)}{(n-1)PK_2(\alpha_r,\alpha_{H_V}) + (2-n)K_3(\alpha_r,\alpha_{H_V},\alpha_R) + (1-K)^2 PK_5(\alpha_r,\alpha_{H_B}) + K^2 PK_6(\alpha_{\pi_B}) + 2P\alpha n + 2P\beta\theta n}\right]^{\frac{1}{2}}$$

$$Q^* = 21$$

4.2.3 Joint Fuzzy Total Annual Cost of the Vendor and the Buyer

$$J\widetilde{T}C(Q,n) = \frac{K_1(\alpha_{S_V},\alpha_R)}{nQ} + (n-1)\frac{QK_2(\alpha_r,\alpha_{H_V})}{2} + (2-n)\frac{QK_3(\alpha_r,\alpha_{H_V},\alpha_R)}{2P} + \frac{K_4(\alpha_{S_B},\alpha_R)}{Q} + \frac{Q(1-K)^2 K_5(\alpha_r,\alpha_{H_B})}{2} + \frac{K^2 QK_6(\alpha_{\pi_B})}{2} + \alpha nQ + \beta\theta nQ$$

$$J\widetilde{T}C(Q,n) = \text{Rs. } 3762.91$$

5. SENSITIVITY ANALYSIS

	Q^*	JTC(Q, n)
n = 3	**21**	**3762.91**
n = 4	**16.88**	**3785.40**
n = 5	**14.26**	**3842.41**
n = 6	**12.45**	**3915.56**
n = 7	**11.11**	**3996.85**
n = 8	**10.09**	**4082.32**

6. CONCLUSION

In this paper we studied the ranking process based on Yager's ranking method. In this model, we develop a Joint integrated total annual inventory cost of the vendor and the buyer with backorder in the crisp sense as well as fuzzy sense. Annual demand, Annual inventory holding cost, Holding cost and purchasing cost of the vendor and buyer are taken as a fuzzy numbers. Here we take the defective items in terms of percentage. When the number of shipments (n) is increasing, the joint total integrated cost is increasing. It implies that the dealer will order more number of quantities to reduce the annual inventory cost. Finally, the proposed model has been verified by the numerical example along with the sensitivity analysis. In the future study, we apply the fuzzy concept for all provisions in this proposed model.

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