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Location-2-Domination in Simple Graphs

G. Rajasekar
Associate Professor, Department of Mathematics,
Jawahar Science College,
Neyveli - 607 803, India.

A. Venkatesan
Assistant Professor, Department of Mathematics,
St. Joseph's College of Arts and Science (Autonomous),
Cuddalore, India.

Abstract

In this paper the definition of Location-2-Domination and minimum cardinality of Location-2-Domination are introduced and denoted by $R_D^2(G)$, and a theorem to find Location-2-dominating set for Path, cycle, Theta graph and Trees in different Kinds are introduced.

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1. Introduction

Throughout this Paper let us follow the terminology and notation of Harary [4]. Thus graph G is undirected and without loops with vertex set V and edge set E, an acyclic graph is called forest and tree is a connected acyclic graph, for safeguard analysis of facility, such as a fire protection study of nuclear power plants [5], the facility can be modeled by a graph or network. For such application a vertex can represent a room, hallway, etc., each edge can connect two areas that are physically adjacent or perhaps two that are within sight or sound of each other. One primary function of safeguard system is detection of some object-perhaps detection of a fire, or perhaps detection of an intruder. Suppose detection device to be located at a vertex and the device can supply three outputs: There is a object at that vertex, there is an object on one the vertex adjacent to that vertex or no objects occupies that vertex or any adjacent one. What is necessary is to determine a collection of location at which to place the detection device so that if an object is placed at any vertex in the graph, then the object can be detected and its position uniquely identified. In order to detect an object which be placed at any vertex of V, its necessary to have a dominating set [6], [7], [8], [9]. There exists the further problem of precisely determining the location of such an object. The concepts of locating sets were introduced in [9], Motivated by the safeguard analysis problem the idea of dominating and locating were merged in [10] as follows.

E. J. Cockayne and S. T. Hedetniemi [7] introduce the concept dominating set A subset S of vertices from V is called a dominating set for G if every vertex of G is either a member of S or adjacent to a member of S. A dominating set of G is called a minimum dominating set if G has no dominating set of smaller cardinality. The cardinality of minimum dominating set of G is called the dominating number for G and it is denoted by $\gamma(G)$ [1].

F. Harary and T.W. Haynes [2] introduce the concepts of double domination in graphs. A dominating set S of$G$ is called double dominating set if every vertex in $V-S$ is adjacent to at least two vertices in S. Given a dominating set$S$ for graph G, for each u in $V-S$ let $S(u)$ denote the set of vertices in S which are adjacent to u. The set S is called locating dominating set, if for any two vertices u and w in $V-S$ one has $S(u)$not equal to $S(w)$ and the minimum cardinality of Location Domination number is denoted by $RD(G)$ [3].

2. Location-2-Domination

Definition 2.1. A subset $S \subseteq V$ is Location-2-Dominating set of G if S is 2-Dominating set of G and if for any two vertices $u, v \in V-S$ such that $N(u) \cap S \neq N(v) \cap S$.The minimum cardinality of Location-2-Dominating set is denoted by $R_D^2(G) = |S|$.

2.1. Location-2-Domination in line graph

Example 2.2. Consider the P_5, $V = \{1, 2, 3, 4, 5\}$. Here $S = \{1, 3, 5\}$ is a Location-2-Dominating set and $V - S = \{2, 4\}$, $N(2) \cap S = \{1, 3\}$ and $N(4) \cap S = \{3, 5\}$ thus $N(2) \cap S \neq N(4) \cap S$. Therefore $R_D^2(G) = |S| = 3$.

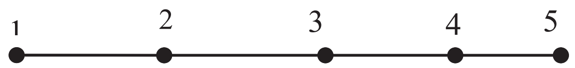

Figure 1: P_5 Graph

Example 2.3. Consider the P_6 Graph. Let V={1,2,3,4,5,6}, here $S = \{1, 2, 4, 6\}$ is a Location-2-Dominating set and $V - S = \{3, 5\}$ and thus $N(3) \cap S \neq N(5) \cap S$. Therefore $R_D^2(G) = |S| = 4$.

Figure 2: P_6 Graph

Theorem 2.4. Location-2-Domination number of a Path P_n is

$$R_D^2(P_n) = \begin{cases} \frac{n-1}{2} + 1, & \text{if } n \text{ is odd} \\ \frac{n}{2} + 1, & \text{if } n \text{ is even.} \end{cases}$$

Proof. Let P_n be a path on n-vertices with $n - 1$ edges, the vertices of P_n are $P_n = \{v_1, v_2, \ldots, v_{n-1}, v_n\}$. Here v_1 and v_n has degree one and $v_2, \ldots, v_{n-1}$, are of degree two Collect Locating-2-Dominating set and is denoted by S.

Case (i): Suppose n is odd
From the Line graph P_n, we have $n - 1$ even number of edges has adjacent with two vertices, collect S from the vertices $v_1, v_2, \ldots, v_{n-1}, v_n$ alternatively.
$V - S = \{v_2, v_4, \ldots, v_{n-3}, v_{n-1}\}$ and $N(v_2) \cap S \neq, \ldots, \neq N(v_{n-1}) \cap S$. Then the Locating-2-dominating set contains $\frac{n-1}{2} + 1$ vertices. i.e., $R_D^2(G) = \frac{n-1}{2} + 1$, for n is odd

Case (ii): Suppose n is even
Clearly $n + 1$ is odd, then there are two cases

(a) The additional vertex belongs to S by the Case (i)
$R_D^2(P_n) = \frac{(n+1)-1}{2} + 1 = \frac{n}{2} + 1.$

(b) Suppose the additional vertex does not belong to S, in this situation this vertex could not be included in the 2-Domninating set and hence there is no question of Location-2-Domination.

2.2. Location-2-Domination for Cycle

Theorem 2.5. For the cycle C_n where $n \neq 4$,

$$R_D^2(C_n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ \frac{n-1}{2} + 1, & \text{if } n \text{ is odd.} \end{cases}$$

Proof. Let C_n be a cycle having n vertices with n edges in each vertex has degree two, each vertex dominates two vertices. The Vertices of C_n are $v_1, v_2, \ldots, v_n$.

Case (i) Suppose n is even ($n \neq 4$).
Let $v_1, v_2, \ldots, v_n$ are the vertices of C_n, Here v_1 is adjacent to $v_2$and v_n and also v_i is adjacent to v_{i+i} and $v_{i-1} (i = 2, 3, \ldots, n-1)$. Now collect S be the set of alternative vertices from v_1 to v_n. such a collection forms $\frac{n}{2}$ number of vertices. Hence the Location-2-Domination set containing $\frac{n}{2}$ vertices. i.e.; $R_D^2(G) = \frac{\text{n}}{2}$, for n is even

Case (ii) For the cycle C_4 the Location-2-Domination Does not Exist.
$n = 4$ is even, by the Case (i) $R_D^2(G) = 2$.

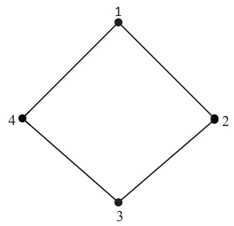

Figure 3: C_4 Graph

Let C_4 be a cycle, the vertices of C_4 are $\{1, 2, 3, 4\}$. Suppose $S = \{1, 3\}$ then $V - S = \{2, 4\}$ but $N(2) = N(4)$. Therefore it is a 2-Dominating set but not a Location-2-Dominating set. Even the selection of any other two Dominating set S the results contradicts to the Case (i).

Case (iii) Suppose n is odd
Clearly $n - 1$ is even, in this case, the removal of one vertex from C_n results a new graph forms P_{n-1}. From the Theorem 2.4, Location-2-Domination set containing $\frac{n-1}{2} + 1$ vertices ie; $R_D^2(G) = \frac{n-1}{2} + 1$ for n is odd. ■

Theorem 2.6. For any two cycle C_n, C_m $(n, m > 2)$ having same order with a common vertex then

$$R_D^2(G) = \begin{cases} n, & \text{if } n \text{ is odd} \\ n-1, & \text{if } n \text{ is even.} \end{cases}$$

Proof. Let C_n be a cycle of n vertices with n edges the vertices of C_n are $\{v_1, v_2, \ldots, v_n\}$ and let C_m be a cycle of m vertices with m edges the vertices of C_m are $\{u_1, u_2, \ldots, u_n\}$. Let these two cycle be fused possibly at $v_1 = u_1$, the new graph form G from a cycle along a cut vertex v_1 with $2n - 1$ vertices where $2n - 2$ vertices with degree two and one vertex with degree four.

Case (i): Suppose n is odd
Let C_n and C_m are both odd cycle now we join these cycle by our hypothesis. In the

new graph G we collect the S-set as follows. Take one complete cycle along with the common vertex and remove adjacent vertices of the common vertex in the second cycle so the second cycle can be reformed by a path of length $n-3$.Therefore the number vertices belongs to S- set is Location-2-Dominating set in one odd length of cycle and Location-2-Dominating set in even length of Path, then by Theorem 2.4 and Theorem 2.5. we followed by $R_D^2(G) = \frac{n-1}{2} + 1 + \frac{n-3}{2} + 1 = n$, i.e. $R_D^2(G) = n$, for n is odd.

Case (ii): Suppose the common vertex does not belongs to the S, they present in $V - S$ so we easily consider that one is cut vertex the graph can be portioned in to two even path of length $n-1$ so $R_D^2(G) = \frac{n-1}{2} + 1 + \frac{n-1}{2} + 1 = n+1$. Which not the minimum cardinality. Therefore the good result is $R_D^2(G) = n$.
Case (iii): Suppose n is even
As per the Hypothesis we collect S- set as followed by Case (i) Therefore the set S contains Location-2-Dominating set for cycle of even length and location-2-Dominating set for path of odd length then by Theorem 2.4 and Theorem 2.5. followed by
$R_D^2(G) = \frac{n}{2} + \frac{n-4}{2} + 1 = n-1.$ ■

Theorem 2.7. For any two cycle $C_n,\ C_m\ (n, m > 2)$ having different order with a common vertex then

$$R_D^2\,(C_n) = \begin{cases} \dfrac{n+m-1}{2}, & \text{if } n \text{ is even and } m \text{ odd} \\ \dfrac{n+m}{2}, & \text{if } n \neq m, \text{ both } n \text{ and } m \text{ are odd} \\ \dfrac{n+m-2}{2}, & \text{if } n \neq m, \text{ both } n \text{ and } m \text{ are even, for } (n \neq 4, m \neq 4). \end{cases}$$

Proof. Let C_n be a cycle of n vertices with $n$edges and let C_m be cycle of m vertices with m edges the vertices of C_n and C_m are $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_m$ respectively.

Case (i): Suppose n is even and m is odd.
Therefore C_n is a cycle of even length and C_m is a cycle of odd length, now Location-2-Dominating set, S is collected as follows. On considering the cycle of even length along with cut vertex, we get an even cycle of length n. While removing the vertices adjacent with common vertex other than the vertices in the even cycle, they form a path of length $m-2$. Therefore the S- set contains Location-2-Dominating set of even cycle and Location-2-Dominating set of odd Path. so by the Theorem 2.4 and Theorem 2.5

$$R_D^2(G) = \frac{n}{2} + \frac{m-3}{2} + 1 = \frac{n+m-1}{2}$$

Case (ii): Suppose both n are m are odd.
Therefore C_n and C_m are both cycles of length odd. Now collect S set as follows.

Consider any one cycle of odd length along cut vertex. Thus we get a cycle of length n and remaining vertices from a path of length $m-3$. Therefore the S set contains Location-2-Dominating set of $C_n$and location-2-Dominating Set of P_{n-3} so by the Theorem 2.4 and Theorem 2.5

$$\therefore R_D^2(G) = \frac{n-1}{2} + 1 + \frac{m-3}{2} + 1 = \frac{n+m}{2}$$

where both m and n are even.

Case (iii): Suppose both n and m are even.

But ($n \neq 4, m \neq 4$). Therefore C_n and C_m are cycle length even. Now collect the S set as follows. consider either C_n or C_m thus get a cycle of length is even, the remaining vertices forms a path of length is odd. Therefore the S set contains Location-2-Dominating set of C_n and location-2-Dominating set of P_{m-3} then by the Theorem 2.4 and Theorem 2.5. ■

Remark 2.8. For the Cycle C_2,C_3 having a common vertex then Location-2-Domination is Three.

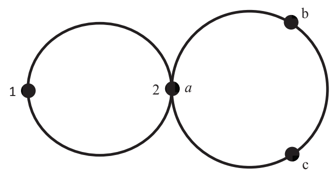

Figure 4: C_2C_3 Graph

(1, b, c) are the Location-2-dominating set as shown in Figure 4.

Remark 2.9. For the Cycle C_n,C_4 having a common vertex we have

$$R_D^2(G) = R_D^2(C_n) + 2.$$

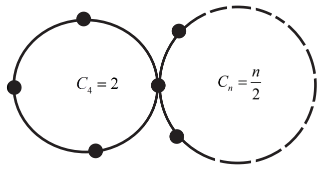

Figure 5: C_4C_n Graph

Theorem 2.10. For the Theta Graph

$$R_D^2(G) = \begin{cases} n-1, & \text{if } n = m \text{ and } n = m \neq 3 \\ \frac{n+m-2}{2}, & \text{if } n \neq m \text{ and both } n, m \text{ are either even or odd} \\ \frac{n+m-1}{2}, & \text{if } n \neq m \text{ and } n \text{ is odd and } m \text{ is even and vice versa.} \end{cases}$$

Proof. Let C_n be a cycle of n vertices with n edges the vertices of C_n are $v_1, v_2, \ldots, v_n$. Let C_m be a cycle of m vertices with m edges, the vertices of C_m are $u_1, u_2, \ldots, u_m$ now join these two cycle with a common edge possibly at $v_1 = u_1$ *and* $v_n = u_m$ (form of Theta graph). The new graph G has $2n-2$ vertices with $2n-1$ edges. Let S be the collection of Location-2-Dominating set.

Case (i): Suppose $n = m$ $((n = m) \neq 3)$ both odd and even.
As per the hypothesis in the resultant Graph G, remove the edge from common v_1 to v_n then G is a block. In this case G form a cycle of length $2n-2$ with $2n-2$ edges for both odd and even number of vertices from C_n and C_m, so the Graph G forms a cycle of even length then by the Theorem 2.5. S be the collection of $\frac{2n-2}{2} = n-1$ vertices. Therefore $R_D^2(G) = n-1$.

Case (ii): For two cycle C_3, C_3 with two common vertex has an edge (Theta Graph) then Location-2-Domination Does not exist.

Let the vertices of $C_1 = \{1, 2, 3\}$ be a cycle of 3 vertices with 3 edges and let vertices of $C_2 = \{a, b, c\}$ be a cycle of 3 vertices with 3 edges, now we join these two cycle with a common vertex possibly $1 = a$ and $3 = c$ (form of Theta graph) the new graph has 4 vertices with 5 edges. Suppose $S = (2, b)$, $V - S = ((1 = a), (3 = c))$ then, clearly $N(2) = N(b)$. Therefore, it is not a Location-2-Domination. Even the selection of any other two dominating set S the results contradicts the case (i).

Case (iii): Suppose both n and m are odd for $n \neq m$,
let C_n be a cycle with n vertices are $v_1, v_2, \ldots, v_n$ and C_m be a cycle with m vertices are $u_1, u_2, \ldots, u_m$, both C_n and C_m are odd length cycle. Join these two cycle possibly at $v_1 = u_1$ and $v_n = u_m$. Now let us consider the C_n along with two common vertex thus we get a cycle of length n also remove the vertex adjacent to v_1 and v_n in C_m. Therefore the remaining vertices of C_m they form a path of length $m-4$. Therefore the set S contains Location-2-Dominating set for cycle of length odd and a Path of length odd. Then by the Theorem 2.4 and Theorem 2.5 we get

$$R_D^2(G) = \frac{n-1}{2} + 1 + \frac{(m-4)-1}{2} + 1$$

$$R_D^2(G) = \frac{n+m-2}{2}.$$

Case (iv): Suppose both n and m are even for $n \neq m$,
let C_n be a cycle with n vertices are $v_1, v_2, \ldots, v_n$ and C_m be a cycle with m vertices are $u_1, u_2, \ldots, u_m$, both C_n and C_m are cycle of length even. Join these two cycle possibly at $v_1 = u_1$ and $v_n = u_m$. Now let us consider the C_n along with common edge. Thus we get a cycle of length n first collect S set for cycle C_n also remove the vertex adjacent to v_1 in C_m, if v_1 belong to S otherwise remove the adjacent to v_n, if v_n belong to S. Therefore the remaining vertices in C_m form a path of length $m-3$. Therefore in the new graph

G, S set contains Location-2-Dominating set for cycle of length even and Location-2-Dominating set for path of length odd. Then by the Theorem 2.4 and Theorem 2.5 we have

$$R_D^2(G) = \frac{n}{2} + \frac{(m-3)-1}{2} + 1$$

i.e. $R_D^2(G) = \dfrac{n+m-2}{2}$.

Case (v): Suppose n is odd and m is even,
let C_n be a cycle with n vertices are $v_1, v_2, \ldots, v_n$ and C_m be a cycle with m vertices are $u_1, u_2, \ldots, u_m$, are the cycles of length odd and even respectively, now join these two cycle with a common vertex Possibly $v_1 = u_1$ and $v_n = u_m$ (form of Theta graph) the new graph form G form a cycle of length odd the collection of location-2-Dominating set as follow by first fix C_n along the common vertex and edge, and remove the vertices adjacent with v_1 and v_n in C_m. Therefore C_m form by a path of length $m-4$. Then location-2-dominating set for G contains the Location-2-Dominating set for cycle of odd length and location-2-Dominating set for path of length even then by the Theorem 2.4 and Theorem 2.5. Then,

$$R_D^2(G) = \frac{n-1}{2} + 1 + \frac{m-4}{2} + 1$$

i.e. $R_D^2(G) = \dfrac{n+m-1}{2}$. ■

3. Location-2-Domination for Trees

Theorem 3.1. For any Tree $T_n, n > 2$ then

$$R_D^2(T_n) > \frac{n}{2}.$$

Proof. Let G be a any Tree of order n and it has $n-1$ edges and Let S be the collection Location-2 Dominating set. Suppose $|S| \leq \frac{n}{2}$, clearly at most $\frac{n}{2}$ vertices of $V-S$ have $|S| = 2$, and thus at least $\frac{n}{2}$, vertices of $V-S$ have $|S| \geq 2$ so we have n vertices with n edges which is a contradiction to Tree. Therefore $R_D^2(T_n) > \frac{n}{2}$. ■

References

[1] F. Harary and T.W. Haynes, Double Domination in Graphs Ars Combin., 55(2000) 201–213.

[2] Haynes, S.T. Hedetniemi and P.J. Salter, Fundamental of domination in Graphs, Marcel Decker, Inc., Newyork 1997.

[3] P.J. Salter Domination and Location in Acyclic Graphs vol-17(1987) 55–64.

[4] F. Haray, Graph Theory. Addsion Wesley, Reading MA (1969).

[5] B.L. Hulme, A.W. Shiver, and P.J. Salter, Fire: A Subroutine for Fire Protection Network Analysis. SAND 81-1261, Sandia National Laboratories, Albuquerque, NM (1981).

[6] E. J. Cockayne, S. E. Goodman, and S. T. Hedetniemi, A Linear algorithm for domination number of a tree. Inform.proc.Lett. 4(1975) 41–44.

[7] E. J. Cockayne and S. T. Hedetniemi, Towards a theory of domination in graphs.

[8] D.E. Knuth, The Art of Computer Programming, Vol. I, Fundamental Algorithms. Advision-Wesley, Reading, MA(1968) 334–338.

[9] P.J. Salter, Leaves of tree. Proc. of the sixth S.E. Conf. on combinatoriecs, Graph theory and computing, Utlitas Math (1975) 549–559.

[10] P.J. Salter, Dominating and Refrence sets in Graphs, J. Math. Phy. Sci. 22(1988) 445–455.