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Integral Transforms and Fractional Integral Operators Associated with S-Generalized Gauss Hypergeometric Function

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Abstract

In this paper, we first find the Euler Beta, Laplace, Whittaker, Sumudu and Hankel integral transforms of the S-Generalized Gauss hypergeometric function. Next, we obtain the image of S-generalized Gauss hypergeometric function under the certain fractional operators (Saigo, Erdélyi, Kober, Riemann–Liouville, Weyl fractional integral operators and Riemann–Liouville fractional Derivative).

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1. Introduction and Definitions

The S-Generalized Gauss hypergeometric function:

$$F_P^{(\alpha,\beta;\tau,\mu)}(a, b; c; z)$$

was introduced and investigated by Srivastava et al. [13, p.350,Eq.(1.12)]. It is represented in the following manner:

$$F_p^{(\alpha,\beta;\tau,\mu)}(a, b; c; z) = \sum_{n=0}^{\infty}(a)_n \frac{B_p^{(\alpha,\beta;\tau,\mu)}(b+n, c-b)}{B(b, c-b)} \frac{z^n}{n!} \qquad (|z| < 1) \tag{1.1}$$

$$(\Re(p) \geq 0; \min\{\Re(\alpha), \Re(\beta), \Re(\tau), \Re(\mu)\} > 0; \Re(c) > \Re(b) > 0)$$

in terms of the classical Beta function $B(\lambda, \mu)$ and the S-generalized Beta function $B_p^{(\alpha,\beta;\tau,\mu)}(x, y)$, which was also defined by Srivastava et al. [13, p.350, Eq. (1.13)] as follows:

$$B_p^{(\alpha,\beta;\tau,\mu)}(x, y) := \int_0^1 t^{x-1}(1-t)^{y-1}{}_1F_1\left(\alpha; \beta; -\frac{p}{t^{\tau}(1-t)^{\mu}}\right) dt \qquad (1.2)$$

$$(\Re(p) \geq 0; \min\{\Re(x), \Re(y), \Re(\alpha), \Re(\beta)\} > 0; \min\{\Re(\tau), \Re(\mu)\} > 0)$$

and $(\lambda)_n$ denotes the Pochhammer symbol defined (for $\lambda \in \mathbb{C}$) by (see [15, p. 2 and pp. 4-6]; see also [14, p. 2]):

$$(\lambda)_n = \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)} = \begin{cases} 1, & (n = 0) \\ \lambda(\lambda+1)...(\lambda+n-1), & (n \in \mathbb{N} := \{1, 2, 3, \cdots\}) \end{cases} \qquad (1.3)$$

provided that the gamma quotient exists (see, for details, [17, p.16 et seq.] and [18, p.22 et seq.]). For $\tau = \mu$, the S-generalized Gauss hypergeometric function defined by (1.1) reduces to the the following generalized Gauss hypergeometric function $F_p^{(\alpha,\beta;\tau)}(a, b; c; z)$studied earlier by parmar[10, p.44]:

$$F_p^{(\alpha,\beta;\tau)}(a, b; c; z) = \sum_{n=0}^{\infty} (a)_n \frac{B_p^{(\alpha,\beta;\tau)}(b+n, c-b)}{B(b, c-b)} \frac{z^n}{n!} \qquad (|z| < 1) \qquad (1.4)$$

$$(\Re(p) \geq 0; \min\{\Re(\alpha), \Re(\beta), \Re(\tau)\} > 0; \Re(c) > \Re(b) > 0)$$

which, in the further special case when τ=1,reduces to the following extension of the generalized Gauss hypergeometric function (see, e.g. [9, p.4606,section 3];see also [8, p.39]):

$$F_p^{(\alpha,\beta)}(a, b; c; z) = \sum_{n=0}^{\infty} (a)_n \frac{B_p^{(\alpha,\beta)}(b+n, c-b)}{B(b, c-b)} \frac{z^n}{n!} \qquad (|z| < 1) \qquad (1.5)$$

$$(\Re(p) \geq 0 : \min\{\Re(\alpha), \Re(\beta)\} > 0; \Re(c) > \Re(b) > 0)$$

upon setting $\alpha = \beta$ in (1.5), we arrive at the following extended Gauss hypergeometric function (see [3, p.591, Eqs.(2.1) and (2.2)]):

$$F_p(a, b; c; z) = \sum_{n=0}^{\infty} (a)_n \frac{B_p(b+n, c-b)}{B(b, c-b)} \frac{z^n}{n!} \qquad (|z| < 1) \qquad (1.6)$$

$$(\Re(p) \geq 0; \Re(c) > \Re(b) > 0)$$

2. Integral Transforms associated with S-Generalized Gauss Hypergeometric Function

In this section, we have find the certain integral transform like that Euler, Laplace, Whittaker, Sumudu and Hankel integral transforms of the S-Generalized Gauss hypergeometric functions $F_p^{(\alpha,\beta;\tau,\mu)}$ which is defined by (1.1).

Theorem 2.1. If $\Re(p) \geq 0$, $\min\{\Re(\alpha), \Re(\beta), \Re(\tau), \Re(\mu)\} > 0$, $\Re(c) > \Re(b) > 0$ and $\Re(\ell) > 0$, $\Re(m) > 0$ are parameters. Then, the following Beta transform holds:

$$B\left\{F_p^{(\alpha,\beta;\tau,\mu)}(\ell+m, b; c; xz); \ell, m\right\} = B(\ell, m) F_p^{(\alpha,\beta;\tau,\mu)}(\ell, b; c; x) \qquad (|x| < 1) \tag{2.1}$$

where the beta transform of $f(z)$ defined as (see [12])

$$B\{f(z); \ell, m\} = \int_0^1 z^{\ell-1}(1-z)^{m-1} f(z)dz \tag{2.2}$$

Further, it is assumed that the involved Euler (beta) transforms $F_p^{(\alpha,\beta;\tau,\mu)}(.)$ exist.

Proof. To prove the result (2.1), by taking the Beta transform (2.2) of (1.1), we obtain

$$\Delta = \int_0^1 z^{\ell-1}(1-z)^{m-1} F_p^{(\alpha,\beta;\tau,\mu)}(\ell+m, b; c; xz)dz$$

Next, we express S-Generalized Gauss Hypergeometric Function $F_p^{(\alpha,\beta;\tau,\mu)}$ in the series form with the help of (1.1) and then changing the order of integration and summation (which is permissible under the conditions stated), we get

$$\Delta = \sum_{n=0}^{\infty} (\ell+m)_n \frac{B_p^{(\alpha,\beta;\tau,\mu)}(b+n, c-b)}{B(b, c-b)} \frac{x^n}{n!} \int_0^1 z^{\ell+n-1}(1-z)^{m-1} dz$$

Finally, with the help of Beta function and (1.1), we get the desired result (2.1) after a little simplification. ∎

Theorem 2.2. If $y \geq 0$, $\Re(s) > 0$, $\min\{\Re(\alpha), \Re(\beta), \Re(\tau), \Re(\mu)\} > 0$, $\Re(c) > \Re(b) > 0$, $\Re(p) \geq 0$ and $|\frac{y}{s} < 1|$ are parameters. Then, the following Laplace transform holds:

$$\mathfrak{L}\left[z^{\ell-1} F_p^{(\alpha,\beta;\tau,\mu)}(a, b; c; yz)\right](s) = \frac{\Gamma(\ell)}{s^{\ell}} {}_1F_p^{(\alpha,\beta;\tau,\mu)}\left(a, b, \ell; c; \frac{y}{s}\right) \tag{2.3}$$

where the Laplace transform of $f(z)$ defined as (see [12])

$$\mathfrak{L}[f(z)](s) = \int_0^\infty e^{-st} f(z)dz \qquad (\Re(s) > 0) \tag{2.4}$$

Proof. In order to prove the assertion (2.3), by taking the Laplace transform (2.4) of (1.1), we obtain

$$\Omega = \int_0^\infty e^{-sz} z^{l-1} F_p^{(\alpha,\beta;\tau,\mu)}(a, b; c; yz)dz$$

Next, we express S-Generalized Gauss Hypergeometric Function $F_p^{(\alpha,\beta;\tau,\mu)}$ in the series form with the help of (1.1) and then changing the order of integration and summation (which is permissible under the conditions stated), we get

$$\Omega = \sum_{n=0}^{\infty} (a)_n \frac{B_p^{(\alpha,\beta;\tau,\mu)}(b+n, c-b)}{B(b, c-b)} \frac{y^n}{n!} \int_0^\infty e^{-sz} z^{n+l-1} dz$$

Finally, with the help of Gamma function and (1.1), we get the desired result (2.3) after a little simplification. ∎

Theorem 2.3. If $|w/\nu| < 1$, $\Re(\rho) > 0$, $\Re(\nu) > 0$, $\min\{\Re(\alpha), \Re(\beta), \Re(\tau), \Re(\mu)\} > 0$, $\Re(c) > \Re(b) > 0$ and $\Re(p) \geq 0$ are parameters. Then, the following Whittaker transform holds:

$$\mathbb{W}_{k,m}^{\rho}\left[F_p^{(\alpha,\beta;\tau,\mu)}(a, b; c; wz) : \nu\right] = \frac{1}{\nu^\rho} \frac{\Gamma(\frac{1}{2}+m+\rho)\Gamma(\frac{1}{2}-m+\rho)}{\Gamma(\frac{1}{2}-k+\rho)} {}_2F_{p,1}\left[\begin{matrix} a, b, \frac{1}{2}+m+\rho, \frac{1}{2}-m+\rho; & \frac{w}{\nu} \\ c, 1-k+\rho; & \end{matrix}\right] \tag{2.5}$$

where Whittaker transform of $f(z)$ is defined as (see, [6])

$$\mathbb{W}_{k,m}^{\rho}[f(z) : \nu] = \int_0^\infty z^{\rho-1} e^{-\nu z/2} W_{k,m}(\nu z) f(z)dz \tag{2.6}$$

Proof. In order to prove the assertion (2.5), by taking the Whittaker transform (2.6) of (1.1), we obtain

$$\Lambda = \int_0^\infty z^{\rho-1} e^{-\nu z/2} W_{k,m}(\nu z) F_p^{(\alpha,\beta;\tau,\mu)}(a, b; c; wz)dz$$

Next, we express S-Generalized Gauss Hypergeometric Function $F_p^{(\alpha,\beta;\tau,\mu)}$ in the series form with the help of (1.1) and then changing the order of integration and summation (which is permissible under the conditions stated), we get

$$\Lambda = \sum_{n=0}^{\infty}(a)_n \frac{B_p^{(\alpha,\beta;\tau,\mu)}(b+n, c-b)}{B(b, c-b)} \frac{w^n}{n!} \int_0^{\infty} z^{\rho+n-1} e^{-\nu z/2} W_{k,m}(\nu z) dz$$

Now, with the help of result [7, p. 56, Eq. (2.41)], we obtain

$$\Lambda = \frac{1}{\nu^{\rho}} \sum_{n=0}^{\infty}(a)_n \frac{B_p^{(\alpha,\beta;\tau,\mu)}(b+n, c-b)}{B(b, c-b)} \frac{(\frac{w}{\nu})^n}{n!} \frac{\Gamma(\frac{1}{2}+m+n+\rho)\Gamma(\frac{1}{2}-m+n+\rho)}{\Gamma(\frac{1}{2}-k+n+\rho)}$$

After a little simplification, we get the desired result (2.3). ■

Theorem 2.4. Suppose that $|uy| < 1$, $\min\{\Re(\alpha), \Re(\beta), \Re(\tau), \Re(\mu)\} > 0$, $\Re(c) > \Re(b) > 0$ and $\Re(p) \geq 0$ are parameters. Then, the following Sumudu transform holds:

$$\mathfrak{S}\left[F_p^{(\alpha,\beta;\tau,\mu)}(a, b; c; yz); u\right] = {}_1F_p^{(\alpha,\beta;\tau\,\mu)}(a, b, 1; c; uy) \tag{2.7}$$

where the Sumudu Transform of $f(z)$ defined as (see, [20])

$$\mathfrak{S}[f(z); u] = \int_0^{\infty} e^{-z} f(uz) dz \qquad [u \in (-\tau_1, \tau_2); \tau_1, \tau_2 > 0] \tag{2.8}$$

Proof. In order to prove the assertion (2.7), by taking the Sumudu transform (2.8) of (1.1), we obtain

$$\Xi = \int_0^{\infty} e^{-z} F_p^{(\alpha,\beta;\tau,\mu)}(a, b; c; uyz) dz$$

Next, we express S-Generalized Gauss Hypergeometric Function $F_p^{(\alpha,\beta;\tau,\mu)}$ in the series form with the help of (1.1) and then changing the order of integration and summation (which is permissible under the conditions stated), we get

$$\Xi = \sum_{n=0}^{\infty}(a)_n \frac{B_p^{(\alpha,\beta;\tau,\mu)}(b+n, c-b)}{B(b, c-b)} \frac{(uy)^n}{n!} \int_0^{\infty} e^{-z} z^n dz$$

Finally, with the help of Gamma function and (1.1), we get the desired result (2.7) after a little simplification. ■

Theorem 2.5. Suppose that $\Re(p) \geq 0$, $\min\{\Re(\alpha), \Re(\beta), \Re(\tau), \Re(\mu)\} > 0$, $\Re(c) > \Re(b) > 0$ are parameters. Then, the following Hankel transform holds:

$$\mathfrak{F}_k\left[F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;ut):\nu\right] = \frac{2\Gamma(\beta)}{k^2\Gamma(\alpha)\Gamma(a)B(b,c-b)} H^{0,1:1,2;1,2}_{1,1:3,1;3,1}\left[\begin{array}{c|c} -\dfrac{16u}{k^4} & (1-b;1,\tau):A^* \\ \dfrac{1}{p} & (1-c;1,\tau+\mu):B^* \end{array}\right] \tag{2.9}$$

where

$$A^* = (1-a,1), \left(-\frac{\nu}{2}, \frac{1}{2}\right), \left(\frac{\nu}{2}, \frac{1}{2}\right); (1,1), (1-c+b,\mu), (\beta,1)$$

$$B^* = (0,1); (\alpha,1)$$

and Hankel transform defined of $f(t)$ defined as (see, [5])

$$\mathfrak{F}_k[f(t):\nu] = \int_0^\infty tJ_\nu(kt)f(t)dt \tag{2.10}$$

Proof. In order to prove the assertion (2.9), by taking the Hankel transform (2.10) of (1.1), we obtain

$$\Upsilon = \int_0^\infty tJ_\nu(kt)F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;ut)dt$$

Next, with the help of result [16, p.18, Eq.(2.6.5)], we get

$$\Upsilon = \int_0^\infty F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;ut)tH^{1,0}_{0,2}\left[\left(\frac{kt}{2}\right)^2 \middle| \begin{array}{c} \\ (\frac{\nu}{2},1), (-\frac{\nu}{2},1)\end{array}\right]dt$$

Further, with the help of result given by Bansal and Jain [2, p. 118, Eq.(2.1)], we get the desired result (2.9) after a little simplification. ■

Remark 2.6. If we take $\tau = \mu$ in Theorems 2.1 to 2.3 its reduces to the corresponding results due to Choi [4]. It may be remarked that the known result of Theorems 2.1 to 2.3 when $\tau = \mu = 1$ immediately reduce to the corresponding results due to Agarwal [1].

3. Fractional Calculus of the S-Generalized Gauss Hypergeometric Function

In this section, we establish image formulas for the S-Generalized Gauss Hypergeometric Function under the Saigo fractional integral operators and also point out their special cases which are believed to be new.

Theorem 3.1. If $x > 0, \lambda, \nu, \eta \in \mathbb{C}$ be parameters such that, $\Re(\rho) > 1, \Re(\rho-\nu+\eta) > 1$ and $\Re(\rho + \lambda + \eta) > 1$. Then, the following Saigo fractional integral formula holds:

$$\left(I_{0,x}^{\lambda,\nu,\eta} t^{\rho-1} F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;t)\right)(x) = \frac{x^{\rho-\nu-1}\Gamma(\rho)\Gamma(\rho-\nu+\eta)}{\Gamma(\rho-\nu)\Gamma(\rho+\lambda+\eta)} {}_2F_{p,2}^{(\alpha,\beta;\tau,\mu)}\left[\begin{matrix} a, b, \rho-\nu+\eta, \rho; \\ c, \rho-\nu, \rho+\lambda+\eta; \end{matrix} \; x\right] \tag{3.1}$$

where Saigo hypergeometric fractional integral operators of $f(t)$ is defined as (see, [7])

$$\left(I_{0,x}^{\lambda,\nu,\eta} f(t)\right)(x) = \frac{x^{-\lambda-\nu}}{\Gamma(\lambda)} \int_0^x (x-t)^{\lambda-1} {}_2F_1\left(\lambda+\nu, -\eta; \lambda; 1-\frac{t}{x}\right) f(t)dt \tag{3.2}$$

$$(x > 0 \quad \text{and} \quad \lambda, \nu, \eta \in \mathbb{C})$$

Proof. In order to prove the assertion (3.1), by taking the Saigo fractional integral (3.2) of (1.1), we obtain

$$\Delta_1 = \frac{x^{-\lambda-\nu}}{\Gamma(\lambda)} \int_0^x (x-t)^{\lambda-1} {}_2F_1\left(\lambda+\nu, -\eta; \lambda; 1-\frac{t}{x}\right) t^{\rho-1} F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;t)dt$$

Next, we express S-Generalized Gauss Hypergeometric Function $F_p^{(\alpha,\beta;\tau,\mu)}(.)$ in the series form with the help of (1.1) and then changing the order of integration and summation (which is permissible under the conditions stated), we get

$$\Delta_1 = \sum_{r=0}^{\infty} (a)_r \frac{B_p^{(\alpha,\beta;\tau,\mu)}(b+r, c-b)}{B(b,c-b)r!} \frac{x^{-\lambda-\nu}}{\Gamma(\lambda)} \times \int_0^x t^{r+\rho-1}(x-t)^{\lambda-1} {}_2F_1\left(\lambda+\nu, -\eta; \lambda; 1-\frac{t}{x}\right) dt$$

Now, with the help of result [11], we obtain

$$\Delta_1 = \sum_{r=0}^{\infty} (a)_r \frac{B_p^{(\alpha,\beta;\tau,\mu)}(b+r, c-b)}{B(b,c-b)r!} \frac{\Gamma(r+\rho)\Gamma(\rho+r+\eta-\nu)}{\Gamma(\rho+r-\nu)\Gamma(\rho+r+\lambda+\eta)} x^{r+\rho-\nu-1}$$

Finally, with the help of (1.1), we get the desired result (3.1) after a little simplification. ■

Theorem 3.2. If $x > 0$, $\lambda, \nu, \eta \in \mathbb{C}$ be parameters such that, $\Re(\rho) > 1$, $\Re(\rho + \nu) > 1$ and $\Re(\lambda + \nu + \rho + \eta) > 1$. Then, the following Saigo fractional integral formula holds:

$$\left(J_{x,\infty}^{\lambda,\nu,\eta} t^{-\rho} F_p^{(\alpha,\beta;\tau,\mu)}\left(a, b; c; \frac{1}{t}\right)\right)(x) = \frac{\Gamma(\nu+\rho)\Gamma(\eta+\rho)}{\Gamma(\rho)\Gamma(\lambda+\nu+\eta+\rho)} x^{-\rho-\nu} \; {}_2F_{p,2}^{(\alpha,\beta;\tau,\mu)}\left[\begin{matrix} a, b, \nu+\rho, \eta+\rho; \\ c, \rho, \nu+\lambda+\eta+\rho; \end{matrix} \; \frac{1}{x}\right] \tag{3.3}$$

where Saigo hypergeometric fractional integral operators of $f(t)$ is defined as (see, [7])

$$\left(J_{x,\infty}^{\lambda,\nu,\eta} f(t)\right)(x) = \frac{1}{\Gamma(\lambda)} \int_x^\infty (t-x)^{\lambda-1} t^{-\lambda-\nu} {}_2F_1\left(\lambda+\nu, -\eta; \lambda; 1-\frac{x}{t}\right) f(t)dt \tag{3.4}$$

$$(x > 0 \quad \text{and} \quad \lambda, \nu, \eta \in \mathbb{C})$$

Proof. In order to prove the assertion (3.3), by taking the Saigo fractional integral (3.4) of (1.1), we obtain

$$\Xi_1 = \frac{1}{\Gamma(\lambda)} \int_x^\infty (t-x)^{\lambda-1} t^{-\lambda-\nu} {}_2F_1\left(\lambda+\nu, -\eta; \lambda; 1-\frac{x}{t}\right) t^{-\rho} F_p^{(\alpha,\beta;\tau,\mu)}\left(a, b; c; \frac{1}{t}\right) dt$$

Next, we express S-Generalized Gauss Hypergeometric Function $F_p^{(\alpha,\beta;\tau,\mu)}$ in the series form with the help of (1.1) and then changing the order of integration and summation (which is permissible under the conditions stated), we get

$$\Xi_1 = \sum_{r=0}^\infty (a)_r \frac{B_p^{(\alpha,\beta;\tau,\mu)}(b+r, c-b)}{B(b, c-b) r!} \frac{1}{\Gamma(\lambda)} \times \int_x^\infty (t-x)^{\lambda-1} t^{-\rho-\lambda-\nu-r} {}_2F_1\left(\mu+\nu, -\eta; \mu; 1-\frac{x}{t}\right) dt$$

Now, with the help of result [11], we obtain

$$\Xi_1 = \sum_{r=0}^\infty (a)_r \frac{B_p^{(\alpha,\beta;\tau,\mu)}(b+r, c-b)}{B(b, c-b) r!} \frac{\Gamma(\nu+r+\rho)\Gamma(\eta+r+\rho)}{\Gamma(r+\rho)\Gamma(\lambda+\nu+\eta+r+\rho)} x^{-r-\rho-\nu}$$

Finally, with the help of (1.1), we get the desired result (3.3) after a little simplification. ■

Corollary 3.3. If we put $\nu = 0$ in (3.1), then Saigo hypergeometric fractional integral operator reduces to Erdélyi fractional integral operator of S-Generalized Gauss Hypergeometric Function

$$\left(E_{0,x}^{\lambda,\eta} t^{\rho-1} F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;t)\right)(x) = \frac{x^{\rho-1}\Gamma(\rho+\eta)}{\Gamma(\rho+\lambda+\eta)}\, {}_1F_{p,1}^{(\alpha,\beta;\tau,\mu)}\left[\begin{matrix} a, b, \rho+\eta; \\ c, \rho+\lambda+\eta; \end{matrix}\; x\right] \tag{3.5}$$

where Erdélyi fractional integral operator of $f(t)$ is defined as (see, [19])

$$\left(E_{0,x}^{\lambda,\eta} f(t)\right)(x) = \frac{x^{-\lambda-\eta}}{\Gamma(\lambda)} \int_0^x (x-t)^{\lambda-1} t^{\eta} f(t)dt \qquad (\Re(\lambda) > 0, \quad \Re(\eta) > 0) \tag{3.6}$$

provided that the conditions are easily obtainable from the existing conditions of (3.1) are satisfied.

Corollary 3.4. If we put $\nu = 0$ in (3.3), then Saigo hypergeometric fractional integral operator reduces to Kober fractional integral operator of S-Generalized Gauss Hypergeometric Function

$$\begin{aligned} &\left(K_{x,\infty}^{\lambda,\eta} t^{-\rho} F_p^{(\alpha,\beta;\tau,\mu)}\left(a,b;c;\frac{1}{t}\right)\right)(x) \\ &= \frac{\Gamma(\eta+\rho)}{\Gamma(\lambda+\eta+\rho)} x^{-\rho}\, {}_1F_{p,1}^{(\alpha,\beta;\tau,\mu)}\left[\begin{matrix} a, b, \eta+\rho; \\ c, \lambda+\eta+\rho; \end{matrix}\; \frac{1}{x}\right] \end{aligned} \tag{3.7}$$

where Kober fractional integral operator of $f(t)$ is defined as (see, [19])

$$\left(K_{x,\infty}^{\lambda,\eta} f(t)\right)(x) = \frac{x^{\eta}}{\Gamma(\lambda)} \int_x^{\infty} (t-x)^{\lambda-1} t^{-\lambda-\eta} f(t)dt \qquad (\Re(\lambda) > 0, \quad \Re(\eta) > 0) \tag{3.8}$$

provided that the conditions are easily obtainable from the existing conditions of (3.3) are satisfied.

Corollary 3.5. If we put $\nu = -\lambda$ in (3.1), then Saigo hypergeometric fractional integral operator reduces to Riemann-liouville fractional integral operator of S-Generalized Gauss Hypergeometric Function

$$\left(R_{0,x}^{\lambda} t^{\rho-1} F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;t)\right)(x) = \frac{x^{\rho+\lambda-1}\Gamma(\rho)}{\Gamma(\rho+\lambda)}\, {}_1F_{p,1}^{(\alpha,\beta;\tau,\mu)}\left[\begin{matrix} a, b, \rho; \\ c, \rho+\lambda+\eta; \end{matrix}\; x\right] \tag{3.9}$$

where Riemann Liouville Fractional integral Operator of $f(t)$ is defined as (see, [19])

$$\left(R_{0,x}^{\lambda} f(t)\right)(x) = \frac{1}{\Gamma(\lambda)} \int_0^x (x-t)^{\lambda-1} f(t)dt \qquad (\Re(\lambda) > 0) \tag{3.10}$$

Corollary 3.6. If we put $\nu = -\lambda$ in (3.3), then Saigo hypergeometric fractional integral operator reduces to Weyl type fractional integral operator of S-Generalized Gauss Hypergeometric Function

$$\left(W_{x,\infty}^{\lambda} t^{-\rho} F_p^{(\alpha,\beta;\tau,\mu)}\left(a, b; c; \frac{1}{t}\right)\right)(x) = \frac{\Gamma(\rho-\lambda)}{\Gamma(\rho)} x^{\lambda-\rho} {}_1F_{p,1}^{(\alpha,\beta;\tau,\mu)} \left[\begin{matrix} a, b, \rho-\lambda; \\ c, \rho; \end{matrix} \; \frac{1}{x}\right] \tag{3.11}$$

where Weyl Fractional integral Operator of $f(t)$ is defined as (see, [19])

$$\left(W_{x,\infty}^{\mu} f(t)\right)(x) = \frac{1}{\Gamma(\mu)} \int_x^{\infty} (t-x)^{\mu-1} f(t)dt \qquad (\Re(\mu) > 0) \tag{3.12}$$

Corollary 3.7. If we replace λ by $-\lambda$ in (3.9), then Riemann-Liouville Fractional Integral Operator reduces to Riemann-Liouville fractional derivative operator of S-Generalized Gauss Hypergeometric Function

$$\left(D_{0,x}^{\lambda} t^{\rho-1} F_p^{(\alpha,\beta;\tau,\mu)}(a, b; c; t)\right)(x) = \frac{x^{\rho-\lambda-1}\Gamma(\rho)}{\Gamma(\rho-\lambda)} {}_1F_{p,1}^{(\alpha,\beta;\tau,\mu)} \left[\begin{matrix} a, b, \rho; \\ c, \rho-\lambda+\eta; \end{matrix} \; x\right] \tag{3.13}$$

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