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Explicit Representation and Some Integral Transforms of Sequence of Functions associated with the Wright type Generalized Hypergeometric Function

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Abstract

The principal aim of the present work is to investigate Explicit Representation of a sequence of functions $G_{\tau,n}^{(a,b,c,\delta)}\left(x;\alpha,k,s\right)$ [16] associated with the Generalized Hypergeometric function ${}_2R_1\left(a,b;c;\tau;z\right)$ [6, 24], followed by deduction of several integral transforms of $G_{\tau,n}^{(a,b,c,\delta)}\left(x;\alpha,k,s\right)$.

Keywords: Generalized Hypergeometric Function, Integral Transforms, Sequence of Functions.

AMS Subject Classification (2010): 33C20, 33E20, 44A20, 44A99.

1. INTRODUCTION AND PRELIMINARIES

The Gauss Hypergeometric Function is defined [12] as,

$$ {}_2F_1(a,b;c;z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k k!} z^k, \left(|z|<1, c \neq 0,-1,-2,.....\right); \text{ and} \quad (1.1)$$

The Generalized Hypergeometric Function, in a classical sense has been defined [5] by

$$ {}_pF_q\left[\begin{matrix} a_{1,}.....,a_p\,;z \\ b_{1,}.....,b_q \end{matrix}\right] = {}_pF_q\left[a_{1,}.....,a_p\,;b_{1,}.....,b_q\,;z\right] $$
$$ = \sum_{k=0}^{\infty}\frac{(a_1)_k.....(a_p)_k}{(b_1)_k.....(b_q)_k}\frac{z^k}{k!} \quad ,(\,p=q+1,\,|z|<1\,); \tag{1.2} $$

with no denominator parameter equals zero or negative integer.

Virchenko et al. [24] defined the Generalized Hypergeometric Function in a different sense as:

$$ {}_2R_1^{\tau}(z) = {}_2R_1(a,b;c;\tau;z) = \frac{\Gamma(c)}{\Gamma(b)}\sum_{k=0}^{\infty}\frac{(a)_k\Gamma(b+\tau k)}{\Gamma(c+\tau k)k!}z^k;\ \tau>0, \quad |z|<1. \tag{1.3} $$

On $|z|=1$, ${}_2R_1^{\tau}(z)$ is meaningful for $\mathrm{Re}(c-a-b)>0$.

If $\tau=1$, then (1.3) reduces to a Gauss's hypergeometric function ${}_2F_1(a,b;c;z)$.

One of the most important special functions is the Gauss hypergeometric function ${}_2F_1(a,b;c;z)$ as; many special functions of applied mathematics can be expressed in terms of it. This has inspired the study of several generalizations.

It should be noted that many algebraic or transcendental functions that occur in the problems of applied mathematics can be expressed in terms of the hypergeometric functions. The Legendre, Bessel, Whittaker and other special functions, and the classical orthogonal polynomials are particular cases of the hypergeometric functions or their various combinations. Let us note that in a systematic study of the generalized probability density, in solving problems of the theory of special functions, differential and integral equations, integral transforms, diffraction theory etc., these functions and their applications have played a significant role, see for example [1], [2], [3], [5], [7], [8], [9], [18].

Galue [6], Rao et al. [13, 14, 15, 16, 17] have studied several properties of ${}_2R_1(a,b;c;\tau;z)$ some of them including in the light of Fractional Integral and Differential operators.

Srivastava and Singhal [23] introduced a general class of polynomials in 1971 as

$$ G_n^{(\alpha)}(x,r,p,k) = \frac{x^{-\alpha-kn}}{n!}\exp\left(px^r\right)\left(x^{k+1}D\right)^n\left[x^{\alpha}\exp\left(-px^r\right)\right], \tag{1.4} $$

where Laguerre, Hermite and Konhauser polynomials are the special case of (1.4).

In 1979 Srivastava and Singh [21] introduced a general sequence of functions $\left\{V_n^{(\alpha)}\left(x;a,k,s\right)/\,n=0,1,2,.....\right\}$;

$$V_n^{(\alpha)}\left(x;a,k,s\right)=\frac{x^{-\alpha}}{n!}\exp\left\{p_k\left(x\right)\right\}\left[x^a\left(s+xD\right)\right]^n\left[x^\alpha\exp\left\{-p_k\left(x\right)\right\}\right], \tag{1.5}$$

where $p_k\left(x\right)$ is a polynomial in x of degree k and a and s are constants.

Rao et al. [16] introduced a sequence of functions containing Generalized

Hypergeometric Function ${}_2R_1\left(a,b;c;\tau;p_k\left(x\right)\right),\left\{G_{\tau,n}^{(a,b,c,\delta)}\left(x;\alpha,k,s\right)/\,n=0,1,2,.....\right\}$;

$$G_{\tau,n}^{(a,b,c,\delta)}\left(x;\alpha,k,s\right)=\frac{x^{-\delta-\alpha n}}{n!}\,{}_2R_1\left(a,b;c;\tau;p_k\left(x\right)\right)\left({}_\alpha T_x^s\right)^n\left[x^\delta\,{}_2R_1\left(a,b;c;\tau;-p_k\left(x\right)\right)\right], \tag{1.6}$$

where $p_k\left(x\right)$ is a polynomial in x of degree k and $x\in\left(0,\infty\right)$, α,δ,s are constants and $\tau\in\mathbb{R}_+=\left(0,\infty\right);a,b,c\in\mathbb{C};\mathrm{Re}\left(a\right)>0,\mathrm{Re}\left(b\right)>0,\mathrm{Re}\left(c\right)>0$; and

$\left({}_\alpha T_x^s\right)^n\equiv x^{\alpha n}\left(s+xD\right)\left(s+\alpha+xD\right).....\left(s+\left(n-1\right)\alpha+xD\right)$ with $D\equiv\dfrac{d}{dx}$.

Some important results are listed below for our further study as:

(i) We are using the operational formula based on [10,11]:

$$\left({}_\alpha T_x^s\right)^n\left(x^a\right)=\alpha^n\left(\frac{a+s}{\alpha}\right)_n x^{a+n\alpha}\ . \tag{1.7}$$

(ii) Srivastava and Manocha [22], gave following result:

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}A\left(k,n\right)=\sum_{n=0}^{\infty}\sum_{k=0}^{n}A\left(k,n-k\right) \tag{1.8}$$

(iii) Some well-known integral transforms [04], [12], [19], [20] are listed as under.

Beta (Eular) Transform:

$$B\left\{f\left(x\right);a,b\right\}=\int_0^1 x^{a-1}\left(1-x\right)^{b-1}f\left(z\right)dz;\qquad \mathrm{Re}\left(a\right),\mathrm{Re}\left(b\right)>0. \tag{1.9}$$

Finite Laplace Transform:

$$L_T\{f(x)\}=\int_0^T e^{-st}f(t)\,dt;\qquad \operatorname{Re}(s)>0,$$

T is a finite positive number. (1.10)

Laplace Transform:

$$L\{f(x)\}=\int_0^\infty e^{-st}f(t)\,dt;\qquad \operatorname{Re}(s)>0. \tag{1.11}$$

Laguerre Transform:

$$L\{f(x)\}=\int_0^\infty e^{-x}x^{\mu}L_n^{\mu}(x)f(x)\,dx, \tag{1.12}$$

where $L_n^{\mu}(x)$ is the Laguerre polynomial of degree $n\geq 0$ and

order $\mu>-1$;

defined by $L_n^{\mu}(x)=\sum_{r=0}^{n}(-1)^r\binom{n+\mu}{n-r}\frac{x^r}{r!}$. (1.13)

2. EXPLICIT REPRESENTATION OF $G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s)$.

As in (1.6), Rao et al. [17] introduced a sequence $G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s)$ of functions containing Generalized Hypergeometric Function.

In this work, we introduce the Explicit Representation of $G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s)$, in terms of following result.

Theorem 2.1

$$G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s)$$

$$=\frac{\alpha^n}{n!}\sum_{m=0}^{\infty}\sum_{l=0}^{m}\left(\frac{(a)_{m-l}\Gamma(c)\Gamma(b+\tau(m-l))}{\Gamma(c+\tau(m-l))\Gamma(b)}\frac{x^{km}}{(m-l)!\,l!}\frac{(-1)^l(a)_l\Gamma(c)\Gamma(b+\tau l)}{\Gamma(c+\tau l)\Gamma(b)}\left(\frac{\delta+s+kl}{\alpha}\right)_n\right),$$

where $p_k(x)$ is a polynomial in x of degree k and $x\in(0,\infty)$, α,δ,s are constants and $\tau\in\mathbb{R}_+=(0,\infty);a,b,c\in\mathbb{C};\operatorname{Re}(a)>0,\operatorname{Re}(b)>0,\operatorname{Re}(c)>0$; and

$$\left({}_{\alpha}T_x^s\right)^n\equiv x^{\alpha n}(s+xD)(s+\alpha+xD).....(s+(n-1)\alpha+xD)\ \text{with}\ D\equiv\frac{d}{dx}.$$

Proof: For simplicity purpose, putting $p_k(x) = x^k$ in (1.6), we get

$$G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s) = \frac{x^{-\delta-\alpha n}}{n!}\,{}_2R_1\left(a,b;c;\tau;p_k(x)\right)\left({}_\alpha T_x^s\right)^n\left[x^\delta\,{}_2R_1\left(a,b;c;\tau;-p_k(x)\right)\right]$$

$$= \frac{x^{-\delta-\alpha n}}{n!}\,{}_2R_1\left(a,b;c;\tau;x^k\right)\left({}_\alpha T_x^s\right)^n\left[x^\delta\,{}_2R_1\left(a,b;c;\tau;-x^k\right)\right]$$

$$= \frac{x^{-\delta-\alpha n}}{n!}\,{}_2R_1\left(a,b;c;\tau;x^k\right)\sum_{l=0}^{\infty}\frac{(-1)^l(a)_l\,\Gamma(c)\Gamma(b+\tau l)}{\Gamma(c+\tau l)\Gamma(b)\,l!}\left({}_\alpha T_x^s\right)^n\left(x^{\delta+kl}\right)$$

Using, (1.7) yields

$$= \frac{x^{-\delta-\alpha n}}{n!}\,{}_2R_1\left(a,b;c;\tau;x^k\right)\sum_{l=0}^{\infty}\frac{(-1)^l(a)_l\,\Gamma(c)\Gamma(b+\tau l)}{\Gamma(c+\tau l)\Gamma(b)\,l!}\alpha^n\left(\frac{\delta+s+kl}{\alpha}\right)_n x^{\delta+kl+\alpha n}$$

$$= \frac{\alpha^n}{n!}\,{}_2R_1\left(a,b;c;\tau;x^k\right)\sum_{l=0}^{\infty}\frac{(-1)^l(a)_l\,\Gamma(c)\Gamma(b+\tau l)}{\Gamma(c+\tau l)\Gamma(b)\,l!}x^{kl}\left(\frac{\delta+s+kl}{\alpha}\right)_n$$

$$= \frac{\alpha^n}{n!}\left[\sum_{m=0}^{\infty}\frac{(a)_m\,\Gamma(c)\Gamma(b+\tau m)}{\Gamma(c+\tau m)\Gamma(b)}\frac{x^{km}}{m!}\right]\left[\sum_{l=0}^{\infty}\frac{(-1)^l(a)_l\,\Gamma(c)\Gamma(b+\tau l)}{\Gamma(c+\tau l)\Gamma(b)\,l!}x^{kl}\left(\frac{\delta+s+kl}{\alpha}\right)_n\right]$$

$$= \frac{\alpha^n}{n!}\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}\left(\frac{(a)_m\,\Gamma(c)\Gamma(b+\tau m)}{\Gamma(c+\tau m)\Gamma(b)}\frac{x^{km}}{m!}\frac{(-1)^l(a)_l\,\Gamma(c)\Gamma(b+\tau l)}{\Gamma(c+\tau l)\Gamma(b)}\frac{x^{kl}}{l!}\left(\frac{\delta+s+kl}{\alpha}\right)_n\right)$$

Applying, (1.8) gives us

$$G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s)$$

$$= \frac{\alpha^n}{n!}\sum_{m=0}^{\infty}\sum_{l=0}^{m}\left(\frac{(a)_{m-l}\,\Gamma(b+\tau(m-l))\Gamma(c)}{\Gamma(b)\,\Gamma(c+\tau(m-l))}\frac{x^{km}}{(m-l)!\,l!}\frac{(-1)^l(a)_l\,\Gamma(c)\Gamma(b+\tau l)}{\Gamma(c+\tau l)\Gamma(b)}\left(\frac{\delta+s+kl}{\alpha}\right)_n\right)$$

. (2.1)

Equation (2.1) gives the required result.

<u>Some Particular cases</u> **:**

(i) For $a = b = c = \tau = 1$;

$$G_{1,n}^{(1,1,1,\delta)}(x;\alpha,k,s) = \frac{\alpha^n}{n!}\left(\frac{1}{1-x^k}\right)\sum_{l=0}^{\infty}(-1)^l x^{kl}\left(\frac{\delta+kl+s}{\alpha}\right)_n$$

$$=\frac{\alpha^n}{n!}\left[\sum_{m=0}^{\infty}x^{km}\right]\left[\sum_{l=0}^{\infty}(-1)^l\,x^{kl}\left(\frac{\delta+kl+s}{\alpha}\right)_n\right]$$

$$=\frac{\alpha^n}{n!}\sum_{m=0}^{\infty}\sum_{l=0}^{m}(-1)^l\,x^{km}\left(\frac{\delta+kl+s}{\alpha}\right)_n \tag{2.2}$$

(ii) Taking $\delta=s=0$ and $\alpha=k$ in (2.2), gives

$$G_{1,n}^{(1,1,1,0)}(x;k,k,0)=\frac{k^n}{n!}\left[\sum_{m=0}^{\infty}x^{km}\right]\left[\sum_{l=0}^{m}(-1)^l\,(l)_n\right]$$

$$=\frac{k^n}{n!}\left[\frac{1}{1-x^k}\right]\left[\sum_{l=0}^{m}(-1)^l\,\frac{\Gamma(l+n)}{\Gamma(l)}\right]=\frac{k^n}{n}\left[\frac{1}{1-x^k}\right]\left[\sum_{l=0}^{m}(-1)^l\,\frac{1}{\beta(l,n)}\right]$$

3. INTEGRAL TRANSFORMS OF $G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s)$.

(I) <u>The Beta Transform</u>:

Consider, $\int_0^t x^{d-1}(t-x)^{e-1}\,G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s)\,dx$, where k is a positive integer.

From (2.1) we have

$$\int_0^t x^{d-1}(t-x)^{e-1}\,G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s)\,dx$$

$$=\frac{\alpha^n}{n!}\sum_{m=0}^{\infty}\sum_{l=0}^{m}\left(\frac{(a)_{m-l}\,\Gamma(c)\Gamma(b+\tau(m-l))}{\Gamma(c+\tau(m-l))\Gamma(b)}\,\frac{(-1)^l\,(a)_l\,\Gamma(c)\Gamma(b+\tau l)}{(m-l)!\,l!\,\Gamma(c+\tau l)\Gamma(b)}\left(\frac{\delta+s+kl}{\alpha}\right)_n\right)\int_0^t x^{km+d-1}(t-x)^{e-1}\,dx,$$

substituting $x=tu$, yields

$$=\frac{\alpha^n}{n!}\sum_{m=0}^{\infty}\sum_{l=0}^{m}\left(\frac{(a)_{m-l}\,\Gamma(c)\Gamma(b+\tau(m-l))}{\Gamma(c+\tau(m-l))\Gamma(b)}\,\frac{(-1)^l\,(a)_l\,\Gamma(c)\Gamma(b+\tau l)}{(m-l)!\,l!\,\Gamma(c+\tau l)\Gamma(b)}\left(\frac{\delta+s+kl}{\alpha}\right)_n\right)t^{km+d+e-1}\,\frac{\Gamma(km+d)\Gamma(e)}{\Gamma(km+d+e)}$$

$$=\frac{\alpha^n}{n!}\beta(d,e)\sum_{m=0}^{\infty}\sum_{l=0}^{m}\left(\frac{(a)_{m-l}\,\Gamma(c)\Gamma(b+\tau(m-l))}{\Gamma(c+\tau(m-l))\Gamma(b)}\,\frac{(-1)^l\,(a)_l\,\Gamma(c)\Gamma(b+\tau l)}{(m-l)!\,l!\,\Gamma(c+\tau l)\Gamma(b)}\right)\frac{(d)_{km}}{(d+e)_{km}}\,t^{km+d+e-1}\left(\frac{\delta+s+kl}{\alpha}\right)_n \tag{3.1}$$

Setting $t=1$ in (3.1) reduces to Beta transform (1.9) and gives the following result in the form of (3.2):

$$B\left(G_{\tau,n}^{(a,b,c,\delta)}\left(x;\alpha,k,s\right);d,e\right)$$

$$=\frac{\alpha^n}{n!}\beta\left(d,e\right)\sum_{m=0}^{\infty}\sum_{l=0}^{m}\left(\frac{(a)_{m-l}\Gamma(c)\Gamma\left(b+\tau(m-l)\right)}{\Gamma\left(c+\tau(m-l)\right)\Gamma(b)}\frac{(-1)^l(a)_l\Gamma(c)\Gamma(b+\tau l)}{(m-l)!\,l!\,\Gamma(c+\tau l)\Gamma(b)}\right)\frac{(d)_{km}}{(d+e)_{km}}\left(\frac{\delta+s+kl}{\alpha}\right)_n. \quad \ldots(3.2)$$

(II) <u>The Finite Laplace Transform</u>:

Using (1.10), where $f(x)=G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s)$, we have

$$L_T\left\{G_{\tau,n}^{(a,b,c,\delta)}\left(x;\alpha,k,s\right)\right\}=\int_0^T e^{-st}G_{\tau,n}^{(a,b,c,\delta)}\left(t;\alpha,k,s\right)dt$$

$$=\frac{\alpha^n}{n!}\sum_{m=0}^{\infty}\sum_{l=0}^{m}\left(\frac{(a)_{m-l}\Gamma(c)\Gamma\left(b+\tau(m-l)\right)}{\Gamma\left(c+\tau(m-l)\right)\Gamma(b)}\frac{(-1)^l(a)_l\Gamma(c)\Gamma(b+\tau l)}{(m-l)!\,l!\Gamma(c+\tau l)\Gamma(b)}\left(\frac{\delta+s+kl}{\alpha}\right)_n\int_0^T e^{-st}t^{km}dt\right)$$

$$=\frac{\alpha^n}{n!}\sum_{m=0}^{\infty}\sum_{l=0}^{m}\left(\frac{(a)_{m-l}\Gamma(c)\Gamma\left(b+\tau(m-l)\right)}{\Gamma\left(c+\tau(m-l)\right)\Gamma(b)}\frac{(-1)^l(a)_l\Gamma(c)\Gamma(b+\tau l)}{(m-l)!\,l!\Gamma(c+\tau l)\Gamma(b)}\left(\frac{\delta+s+kl}{\alpha}\right)_n\frac{1}{s^{km+1}}\gamma\left(km+1,st\right)\right), \quad \ldots(3.3)$$

where, $\gamma(\alpha,x)$ is an incomplete Gamma function [12].

Thus the finite Laplace transform of $G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s)$ is given by (3.3).

<u>Particular case</u>: Setting, $a=b=c=\tau=1,\delta=s=0$ and $\alpha=k$

$$L_T\left\{G_{1,n}^{(1,1,1,0)}\left(x;k,k,0\right)\right\}$$

$$=\frac{k^n}{n!}\sum_{m=0}^{\infty}\frac{1}{s^{km+1}}\gamma\left(km+1,st\right)\sum_{l=0}^{m}\left((-1)^l(l)_n\right).$$

(III) <u>Laplace Transform</u>:

Using (1.11), with $f(x)=G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s)$, gives

$$L\left\{G_{\tau,n}^{(a,b,c,\delta)}\left(x;\alpha,k,s\right)\right\}=\int_0^\infty e^{-st}G_{\tau,n}^{(a,b,c,\delta)}\left(t;\alpha,k,s\right)dt$$

$$=\frac{\alpha^n}{n!}\sum_{m=0}^{\infty}\sum_{l=0}^{m}\left(\frac{(a)_{m-l}\Gamma(c)\Gamma\left(b+\tau(m-l)\right)}{\Gamma\left(c+\tau(m-l)\right)\Gamma(b)}\frac{(-1)^l(a)_l\Gamma(c)\Gamma(b+\tau l)}{(m-l)!\,l!\Gamma(c+\tau l)\Gamma(b)}\left(\frac{\delta+s+kl}{\alpha}\right)_n\int_0^\infty e^{-st}t^{km}dt\right)$$

$$=\frac{\alpha^n}{n!}\sum_{m=0}^{\infty}\sum_{l=0}^{m}\left(\frac{(a)_{m-l}\,\Gamma(c)\Gamma(b+\tau(m-l))}{\Gamma(c+\tau(m-l))\Gamma(b)}\frac{(-1)^l(a)_l\,\Gamma(c)\Gamma(b+\tau l)}{(m-l)!\,l!\Gamma(c+\tau l)\Gamma(b)}\left(\frac{\delta+s+kl}{\alpha}\right)_n\frac{\Gamma(km+1)}{s^{km+1}}\right). \quad \ldots(3.4)$$

Thus the Laplace transform of $G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s)$ is given by (3.4).

(IV) Laguerre Transform:

For $f(x)=G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s)$, using (1.12)

$$L\{f(x)\}=L\left\{G_{\tau,n}^{(a,b,c,\delta)}(x;\alpha,k,s)\right\}$$

$$=\frac{\alpha^n}{n!}\sum_{m=0}^{\infty}\sum_{l=0}^{m}\left(\frac{(a)_{m-l}\,\Gamma(b+\tau(m-l))\Gamma(c)}{\Gamma(b)\,\Gamma(c+\tau(m-l))}\frac{(-1)^l(a)_l\,\Gamma(c)\Gamma(b+\tau l)}{(m-l)!l!\,\Gamma(c+\tau l)\Gamma(b)}\left(\frac{\delta+s+kl}{\alpha}\right)_n\right)L\left\{x^{km}\right\}$$

$$=\frac{\alpha^n}{n!}\sum_{m=0}^{\infty}\sum_{l=0}^{m}\left(\frac{(a)_{m-l}\,\Gamma(b+\tau(m-l))\Gamma(c)}{\Gamma(b)\,\Gamma(c+\tau(m-l))}\frac{(-1)^l(a)_l\,\Gamma(c)\Gamma(b+\tau l)}{(m-l)!l!\,\Gamma(c+\tau l)\Gamma(b)}\left(\frac{\delta+s+kl}{\alpha}\right)_n\right)\int_0^{\infty}e^{-x}x^{\mu+km}L_n^{\mu}(x)\,dx,$$

further simplification of this equation gives

$$=\frac{\alpha^n}{n!}\sum_{m=0}^{\infty}\sum_{l=0}^{m}\left(\frac{(a)_{m-l}\,\Gamma(b+\tau(m-l))\Gamma(c)}{\Gamma(b)\,\Gamma(c+\tau(m-l))}\frac{(-1)^l(a)_l\,\Gamma(c)\Gamma(b+\tau l)}{(m-l)!l!\,\Gamma(c+\tau l)\Gamma(b)}\left(\frac{\delta+s+kl}{\alpha}\right)_n\right)\frac{\Gamma(km+1+\mu)\Gamma(n-km)}{n!\Gamma(-km)}$$

$$=\frac{\alpha^n}{n!}\sum_{m=0}^{\infty}\left[\frac{\Gamma(km+1+\mu)\Gamma(n-km)}{n!\Gamma(-km)}\sum_{l=0}^{m}\left(\frac{(a)_{m-l}\,\Gamma(b+\tau(m-l))\Gamma(c)}{\Gamma(b)\,\Gamma(c+\tau(m-l))}\frac{(-1)^l(a)_l\,\Gamma(c)\Gamma(b+\tau l)}{(m-l)!l!\,\Gamma(c+\tau l)\Gamma(b)}\left(\frac{\delta+s+kl}{\alpha}\right)_n\right)\right].$$

REFERENCES

[1] Al-Musallam F. and Kalla S.L., 1997, "Asymptotic expansions for generalized gamma and incomplete gamma functions", Appl. Anal., 66, pp. 173-187.

[2] Al-Musallam F. and Kalla S.L., 1998, "Further results on a generalized gamma function occurring in diffraction theory", Integral Transforms and Special Functions,7, pp. 175-190.

[3] Aomoto K., 1996, "Hypergeometric functions: The past, today and . . . . . ", Sugaku Expositions, 9, pp. 99-116.

[4] Debnath L., 1995, "Integral transforms and their applications", CRC press, New York-London-Tokyo.

[5] Erdelyi et al.(Editors), 1953- 1954, "Higher Transcendental Functions", McGaw-Hill, New York.

[6] Galue L., Al-Zamel A., Kalla S.L., 2003, "Further results on generalized hypergeometric functions", Applied Mathematics and Computation, 136, pp. 17-25.

[7] Kalla S. L., Bader N. Al-Sagabi, 2001, "Further results on a unified form of gamma-type distributions", Fract. Calculus and Appl. Anal., 4(1), pp. 91-100.

[8] Kilbas A.A. and Saigo M., 2004, "H-Transforms. Theory and Applications. Ser. Analytic Methods and Special Functions", Vol. 9, CRC Press, London and New York.

[9] Kiryakova V.S., 1994, "Generalized Fractional Calculus and Applications", Piman Research Notes in Math., Vol. 301, Longman & John Wiley and Sons, New York.

[10] Mittal H.B., Bilinear and bilateral generating relations, 1977, "American Journal of Mathematics", 99, pp. 23-55.

[11] Patil K.R. and Thakare N.K., 1975, "Operational formulas for a function defined by a generalized Rodrigues formula-II", Scientific Journal of Shivaji University, 15, pp.1-10.

[12] Rainville E. D., 1960, "Special Functions", The Macmillan Company, New York.

[13] Rao S. B., Shukla A.K., 2013, "Note On Generalized Hypergeometric Function", Integral Transforms and Special Functions, 24(11), pp. 896–904.

[14] Rao S. B., Prajapati J. C., Shukla A. K., 2013, "Wright type Hypergeometric Function and its properties", Advances in Pure Mathematics, 3(3), PP. 335-342.

[15] Rao S. B., Patel A. D., Prajapati J. C., Shukla A. K., 2013, "Some Properties of Generalized Hypergeometric Function", Commun. Korean Math. Soc. 28(2), pp. 303-317.

[16] Rao S. B., Salehbhai I. A., Shukla A. K., 2013, "On Sequence of Functions Containing Generalized Hypergeometric Function", Mathematical Sciences Research Journal, 17(4), pp. 98-110.

[17] Rao S.B., Prajapati J.C., Patel A.D., Shukla A.K., 2014, "Some properties of Wright-type Generalized Hypergeometric Function via fractional calculus", Advances in Difference Equations, 2014:119.

[18] Samko S.G., Kilbas A.A., Marichev O.I., 1993, "Fractional integrals and derivatives. Theory and Applications". Gordon and Breach, New York.

[19] Sneddon I.N., 1951, "Fourier transform", McGaw-Hill, New York.

[20] Sneddon I.N., 1979, "The use of integral transforms", Tata McGaw-Hill pub. Co. Ltd., New Delhi.

[21] Srivastava A.N. and Singh S.N., 1979, “Some generating relations connected with a function defined by a generalized Rodrigues formula”, Indian Journal of Pure and Applied Mathematics, 10(10), pp. 1312-1317.

[22] Srivastava H.M. and Manocha, H.L., 1984, “A treatise on generating functions”, Ellis Horwood Limited, Chichester.

[23] Srivastava H.M. and Singhal J.P., 1971, “A class of polynomials defined by generalized Rodrigues formula”, Annalidi Matematica Puraed Applicata, 90(4), pp. 75-85.

[24] Virchenko N., Kalla S.L., A.Al-Zamel, 2001, “Some Results On a Ggeneralized Hypergeometric Function”, Integral Transforms and Special Functions, 12, No.1, pp. 89-100.