Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 9 (2017), pp. 6467-6481

http://www.ripublication.com

Homotopy Analysis to Heat and Mass Transfer in a Visco-Elastic Fluid Flow over Exponentially Stretching Sheet through Porous Medium

Hymavathi. T[1] and B.Akkayya[2]

1,2, Department of Mathematics,
Adikavi Nannaya University, Rajamahemdravaram, India.

Abstract

This paper presents an analytic solution to study two dimensional free boundary layer problem and heat and mass transfer of an incompressible visco-elastic fluid flow over exponentially stretching sheet immersed in a porous medium. The governing equations are converted into non linear ordinary differential equations by using similarity transformations and have been solved by Homotopy Analysis Method (HAM). Later, in the results acquired, we demonstrated the effects of different physical parameters like visco-elasticity, permeability of porous medium, Grashof number, Schmidt number and prandtl number on the flow, heat and mass transfer characteristics. It is observed that this combined effect of thermal diffusion and diffusion of species is to increase the horizontal velocity profile and to decrease the temperature and concentration profiles in the boundary layer flow field.

Keywords: Visco-elastic fluids, porous medium, similarity transformation, boundary layer flow, exponential stretching sheet, HAM.

INTRODUCTION:

The study of visco-elastic fluids has gained its momentum during recent years. This is mainly due to its multiple applications in petroleum drilling, manufacturing of foods and polymer industries. Sakiadis [1] was the first one to study the boundary layer behaviour of a continuous solid surface moving with constant speed. Later, Crane [2] extended the work of Sakiadis [1] over stretching sheet whose velocity is proportional to a distance from the slit. Then, P.S. Gupta and A.S. Gupta [3] argued that realistically stretching of plastic sheet may not necessarily linear. In this point of view, many researchers focussed their attention on the study of boundary layer flow of Newtonian and Non – Newtonian fluids over different stretching sheets i.e., quadratic, power law and non isothermal stretching sheets etc.al [4-7].

The above investigators studied and analysed to flow behaviour in non-porous media. However, some metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. Subsequently, in recent years the interest in heat transfer for Non-Newtonian fluid flows through a porous medium has grown considerably in view of industrial applications to petroleum extraction, filtration and separation process in chemical industries, geophysics and many others. Initially many people came into the front line and conducted researches in their own line. Some of them are, Subhash Abel and Veena [8] Examined heat transfer and flow of a viscoelastic fluid through a porous medium over impermeable stretching sheet. Then, Chauhan and Agrawal [9] investigated non-Newtonian coupled flow in a channel bounded by a stretching sheet and a highly porous medium. Diffusion of chemical reactive species of a non-Newtonian fluid was investigated by Prasad et al. [10] through a porous medium over a stretching sheet. Krishnambal and Anuradha [11] examined radiation effects on the visco-elastic fluid flow and heat transfer in a porous medium, past a stretching sheet. Then, M. Subhas Abel, S.K.Khan and K.V.Prasad [12] considered convective heat and mass transfer in a visco-elastic fluid flow through a porous medium over a stretching sheet.

The solutions given by stretching sheet in the above literature were not unique and derived another closed form of solutions. Then, Troy et al.[13] showed the solution containing exponential terms is physically realistic as slightly elastic fluid produces a boundary layer only slightly altered in its dimensions from the viscous one. In view of the above discussions on boundary conditions, we present in the next section the physically realistic sequential similarity solutions of visco-elastic boundary layer. In view of this point, Elbashbeshy [14] is one who analysed the problem of heat transfer over an exponentially stretching continuous surface with suction. Then Magyari and Keller [15] discussed the heat and mass transfer in boundary layers on an exponentially stretching continuous surface. Consequently, Sanjayanand and Khan [16, 17] extended the work of Elbashbeshy [14] to visco - elastic fluid flow, heat and mass transfer over an exponentially stretching sheet.

The investigation of exact solutions to the problem of nonlinear equations plays an important role in the study of the nonlinear physical phenomena. Firstly, the homotopy analysis method [19, 20] was proposed by Liao in 1992. The HAM was further developed and improved by Liao for nonlinear problems [21]. In this process, Sajid and Hayet [22] discussed the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. Recently, Hymavathi.T [18] et.al considered the flow and heat transfer in visco-elastic fluid flow through a porous medium over exponentially stretching sheet. They used homotopy analysis method (HAM) to solve the problem analytically.

In view of the importance of porous medium, we considered the study of heat and mass transfer of visco-elastic fluid flow over exponentially stretching sheet in porous medium. Here we employed Homotopy Analysis Method to solve converted non-linear differential equations and discussed the effects of visco – elastic parameter (k_1), porosity parameter (k_2), Grashof number (G_r), modified Grashof number (G_c),Prandtl number (Pr) and Schmidt number (Sc) on velocity, temperature and concentration profiles. Comparison of the present analysis is also made with the existing results in the literature and is seen in good agreement.

MATHEMATICAL FORMULATION:

Consider a steady state two-dimensional visco-elastic fluid flow through a porous medium over exponentially stretching sheet and the flow being confined to y > 0. Boundary is assumed to be moving axially with a velocity of exponential order in axial distance by the application of two equal and opposite forces along x-axis keeping the origin fixed and generating the boundary layer type of flow. So, Buoyancy effects due to both temperature and concentration gradients and stretching of the wall provide the driving force for the flow. Under these assumptions and neglecting Soret and Dufour effects, the governing equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\nu}{k^*}u+g\beta^*\left(T-T_\infty\right)+g\beta^{**}\left(C-C_\infty\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

Where u, v are velocity components along x, y directions respectively, T and C are temperature and concentration of chemical species in the fluid, g is the acceleration due to gravity, v is the kinematic viscosity, k_0 is the visco-elastic parameter, k^* is the permeability coefficient of porous medium, β^* is the volumetric coefficient of thermal expansion and β^{**} is the volumetric coefficient. Other quantities have usual meanings.

The boundary conditions governing the flow are:

$$u = U_w(x) = U_0\, e^{x/l},\ v = 0,\ T = T_w = T_\infty + T_0\, e^{x/2l},\quad C = C_w + C_0\, e^{x/2l}\quad at\ \ y = 0$$

$$u = 0,\ u_y = 0,\ T = T_\infty,\ C = C_\infty \qquad as\ \ y \rightarrow \infty \tag{5}$$

Here U_0 is the constant, l is the reference length, suffix y denotes the differentiation with respect to y, T_0 is the parameter of the temperature distribution and T_∞, C_∞ are the temperature and concentration far away from the stretching sheet.

Solution of the problem:

The velocity components u and v in terms of stream function $\psi(x, y)$ can be written as:

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

For solving momentum equation, introduce a similarity variable η such that

$$\eta = y\sqrt{\frac{U_0}{2vl}}\, e^{x/2l} \tag{7}$$

$$\psi(x,y) = \sqrt{2vlU_0}\, f(x,\eta)e^{x/2l},\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{8}$$

Here f is the dimensionless stream function and considering $f(x, \eta) = f(\eta)$. Using (6)-(8), the equations (2)-(4) results in a fourth order non- linear ordinary differential equation of the form

$$2f'^2 - ff'' = f''' - k_1\left[3f'f''' - \frac{1}{2}ff'''' - \frac{3}{2}f''^2\right] - k_2 f' + Gr\theta + Gc\phi \tag{9}$$

$$\theta'' + \Pr\left(f\theta' - f'\theta\right) = 0 \tag{10}$$

$$\phi'' - Sc\left(f'\phi - f\phi'\right) = 0 \tag{11}$$

Where $k_1 = \dfrac{k_0 U_w}{vl}$, $k_2 = \dfrac{2vl}{k^* U_w}$, $Gr = \dfrac{2g\beta^*\left[T - T_\infty\right]}{{U_w}^2}$, $Gc = \dfrac{2g\beta^{**}\left[C - C_\infty\right]}{U_w^2}$ are visco-elastic, porosity and free convection parameters respectively and $\Pr = \dfrac{\mu c_p}{k}$, $Sc = \dfrac{v}{D}$ are the Prandtl number and Schmidt number respectively.

Subject to the boundary conditions

$$f = 0,\ f' = 1,\quad \theta = 1,\quad \phi = 1 \qquad at\ \eta = 0$$

$$f' = f'' = \theta = \phi = 0 \qquad as\ \eta \to \infty \tag{12}$$

Analytic solution (HAM):

In this section, we employ HAM to solve the equations (9) to (11) subject to the boundary conditions (12). We choose the initial guesses f_0, θ_0 and ϕ_0 of f, θ and ϕ in the following form

$$f_0(\eta) = 1 - e^{-\eta}, \tag{13}$$

$$\theta_0(\eta) = e^{-\eta}, \tag{14}$$

$$\phi_0(\eta) = e^{-\eta}. \tag{15}$$

The linear operators are selected as

$$L_1(f) = f''' - f', \tag{16}$$

$$L_2(\theta) = \theta'' - \theta, \tag{17}$$

$$L_3(\phi) = \phi'' - \phi, \tag{18}$$

which have the following properties

$$L_1\left(C_1 + C_2 e^{\eta} + C_3 e^{-\eta}\right) = 0, \tag{19}$$

$$L_3\left(C_4 e^{\eta} + C_5 e^{-\eta}\right) = 0, \tag{20}$$

$$L_2\left(C_6 e^{\eta} + C_7 e^{-\eta}\right) = 0, \tag{21}$$

where $C_i\ (i = 1\ to\ 7)$ are arbitrary constants.

If $p \in [0,1]$ is the embedding parameter, $\hbar_1, \hbar_2$ and $\hbar_3$ are the non-zero auxiliary parameters and $H_1(\eta), H_2(\eta)$ and $H_3(\eta)$ are auxiliary functions, then we can construct the following zeroth-order deformation equations

$$(1-p)L_1(\omega_1(\eta;p)-f_0(\eta)) = p\hbar_1 H_1(\eta) N_1[\omega_1(\eta;p),\omega_2(\eta;p),\omega_3(\eta;p)], \tag{22}$$

$$(1-p)L_2(\omega_2(\eta;p)-\theta_0(\eta)) = p\hbar_2 H_2(\eta) N_2[\omega_1(\eta;p),\omega_2(\eta;p)], \tag{23}$$

$$(1-p)L_3(\omega_3(\eta;p)-\phi_0(\eta)) = p\hbar_3 H_3(\eta) N_3[\omega_1(\eta;p),\omega_3(\eta;p)], \tag{24}$$

subject to the boundary conditions

$$\begin{aligned} &\omega_1(0;p)=0 \qquad \omega_1'(0;p)=1, \qquad \omega_1'(\infty;p)=0,\\ &\omega_2(0;p)=1, \qquad \omega_2(\infty;p)=0,\\ &\omega_3(0;p)=1, \qquad \omega_3(\infty;p)=0. \end{aligned} \tag{25}$$

Based on equations (22) to (24), we define the non-linear operators N_1 and N_2 as

$$\begin{aligned} N_1[\omega_1(\eta;p),\omega_2(\eta;p),\omega_3(\eta;p)] &= \frac{\partial^3\omega_1(\eta;p)}{\partial\eta^3} + \omega_1(\eta;p)\frac{\partial^2\omega_1(\eta;p)}{\partial\eta^2} \\ &-2\left(\frac{\partial\omega_1(\eta;p)}{\partial\eta}\right)^2 - k_1\left(3\frac{\partial f}{\partial\eta}\frac{\partial^3 f}{\partial\eta^3} - \frac{1}{2}f\frac{\partial^4 f}{\partial\eta^4} - \frac{3}{2}\left(\frac{\partial^2 f}{\partial\eta^2}\right)^2\right) + Gr\,\omega_2(\eta;p) + Gc\,\omega_3(\eta;p) - k_2\frac{\partial\omega_1(\eta;p)}{\partial\eta}, \end{aligned} \tag{26}$$

$$N_2[\omega_1(\eta;p),\omega_2(\eta;p)] = \frac{\partial^2\omega_2(\eta;p)}{\partial\eta^2} + Pr\,\omega_1(\eta;p)\frac{\partial\omega_2(\eta;p)}{\partial\eta} - Pr\frac{\partial\omega_1(\eta;p)}{\partial\eta}\omega_2(\eta;p), \tag{27}$$

$$N_3[\omega_1(\eta;p),\omega_3(\eta;p)] = \frac{\partial^2\omega_3(\eta;p)}{\partial\eta^2} + Sc\,\omega_1(\eta;p)\frac{\partial\omega_3(\eta;p)}{\partial\eta} - Sc\frac{\partial\omega_1(\eta;p)}{\partial\eta}\omega_3(\eta;p).$$

(28)

For $p=0$ and $p=1$, we have

$$\omega_1(\eta;0) = f_0(\eta), \quad \omega_2(\eta;0) = \theta_0(\eta), \quad \omega_3(\eta;0) = \phi_0(\eta), \tag{29}$$

$$\omega_1(\eta;1) = f(\eta), \quad \omega_2(\eta;1) = \theta(\eta), \quad \omega_3(\eta;1) = \phi(\eta). \tag{30}$$

Thus, as p increases from 0 to 1, $\omega_1(\eta;p),\omega_2(\eta;p)$ and $\omega_3(\eta;p)$ vary from initial approximations to the exact solutions of the original non-linear differential equations. Now, expanding $\omega_1(\eta;p),\omega_2(\eta;p)$ and $\omega_3(\eta;p)$ in Taylor's series with respect to p,

we have $\omega_1(\eta;p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)p^m,$ (31)

$$\omega_2(\eta;p) = \theta_0(\eta) + \sum_{m=1}^{\infty}\theta_m(\eta)p^m, \tag{32}$$

$$\omega_3(\eta;p) = \phi_0(\eta) + \sum_{m=1}^{\infty}\phi_m(\eta)p^m, \tag{33}$$

where

$$f_m(\eta) = \frac{1}{m!}\left.\frac{\partial^m \omega_1(\eta;p)}{\partial p^m}\right|_{p=0}, \tag{34}$$

$$\theta_m(\eta) = \frac{1}{m!}\left.\frac{\partial^m \omega_2(\eta;p)}{\partial p^m}\right|_{p=0}. \tag{35}$$

$$\phi_m(\eta) = \frac{1}{m!}\left.\frac{\partial^m \omega_3(\eta;p)}{\partial p^m}\right|_{p=0}. \tag{36}$$

If the initial approximations, auxiliary linear operators and non-zero auxiliary parameters are chosen in such a way that the series (31) to (33) are convergent at $p = 1$, then

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \tag{37}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty}\theta_m(\eta), \tag{38}$$

$$\phi(\eta) = \phi_0(\eta) + \sum_{m=1}^{\infty}\phi_m(\eta). \tag{39}$$

We get the mth-order deformation equations by differentiating equations (22) to (24), m times with respect to p and taking $p = 0$ and then dividing with $m!$. The mth-order deformation equations are as follows:

$$L_1\left(f_m(\eta) - \chi_m f_{m-1}(\eta)\right) = \hbar_1 H_1(\eta) R_m^f(\eta), \tag{40}$$

$$L_2\left(\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)\right) = \hbar_2 H_2(\eta) R_m^\theta(\eta), \tag{41}$$

$$L_3\left(\phi_m(\eta) - \chi_m \phi_{m-1}(\eta)\right) = \hbar_3 H_3(\eta) R_m^\phi(\eta), \tag{42}$$

with the following boundary conditions

$$\begin{aligned} &f_m(0) = 0, \quad f_m'(0) = 0, \quad f_m'(\infty) = 0, \\ &\theta_m(0) = 0, \quad \theta_m(\infty) = 0, \\ &\phi_m(0) = 0, \quad \phi_m(\infty) = 0, \end{aligned} \tag{43}$$

where

$$R_m^f = f_{m-1}''' + \sum_{k=0}^{m-1} f_k f_{m-1-k}'' - 2\sum_{k=0}^{m-1} f_k' f_{m-1-k}' - k_1\left(3\sum_{k=0}^{m-1} f'_{m-1-k} f_k''' - \frac{1}{2}\sum_{k=0}^{m-1} f_{m-1-k} f_k'''' - \frac{3}{2}\sum_{k=0}^{m-1} f''_{m-1-k} f_k''\right) - k_2 f'_{m-1}. \quad (44)$$

$$R_m^\theta(\eta) = \theta_{m-1}'' + Pr\sum_{i=0}^{m-1} f_{m-1-i}\,\theta_i' - Pr\sum_{i=0}^{m-1} f_{m-1-i}'\,\theta_i \quad (45)$$

$$R_m^\phi(\eta) = \phi_{m-1}'' + Sc\sum_{i=0}^{m-1} f_{m-1-i}\,\phi_i' - Sc\sum_{i=0}^{m-1} f_{m-1-i}'\,\phi_i \quad (46)$$

and $$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1. \end{cases} \quad (47)$$

We choose the auxiliary functions as follows:

$$H_1(\eta) = 1,$$
$$H_2(\eta) = 1, \quad (48)$$
$$H_3(\eta) = 1.$$

If we let $f_m^*(\eta)$, $\theta_m^*(\eta)$ and $\phi_m^*(\eta)$ as the special solutions of the mth-order deformation equations, the general solutions are given by

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2 e^{\eta} + C_3 e^{-\eta}, \quad (49)$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_4 e^{\eta} + C_5 e^{-\eta}, \quad (50)$$

$$\phi_m(\eta) = \phi_m^*(\eta) + C_6 e^{\eta} + C_7 e^{-\eta}, \quad (51)$$

where the integral constants $C_i\,(i = 1\,\text{to}\,7)$ are determined using the boundary conditions (43).Now it is easy to solve the linear non-homogeneous equations (40) and (42) using MATHEMATICA software one after the other by considering $m = 1, 2, \ldots$

Convergence of HAM solution:

Liao [16] showed that for an analytic solution obtained by HAM , its convergence and rate approximation strongly depending upon the auxiliary parameters $\hbar_1$, $\hbar_2$ and $\hbar_3$ .If these parameters are chosen properly , then the solution is effective . Hence , $\hbar$ -curves are plotted at 25th order approximation in order to obtain the suitable ranges for -1.2 ≤ $\hbar_1$ ≤ -0.1 , -1.2 ≤ $\hbar_2$ ≤ 0.1, -1.29 ≤ $\hbar_3$ ≤ -0.1.

RESULTS AND DISCUSSION:

The analytical computations were carried out for various values of visco – elastic parameter (k_1), porosity parameter (k_2), Grashof number (G_r), modified Grashof number (G_c),Prandtl number (Pr) and Schmidt number (Sc) using HAM. For this study, we have taken Prandtl number *Pr* as 0.72 which corresponds to air and Schmidt number *Sc* is chosen to be 0.24, 0.62, 0.78 representing diffusing chemical species of most common interest in air like H_2, H_2O and NH_3 and Propyl Benzene, respectively. In order to illustrate the results graphically, the convergence of velocity, temperature and concentration profiles are shown in Fig: 1- 3.

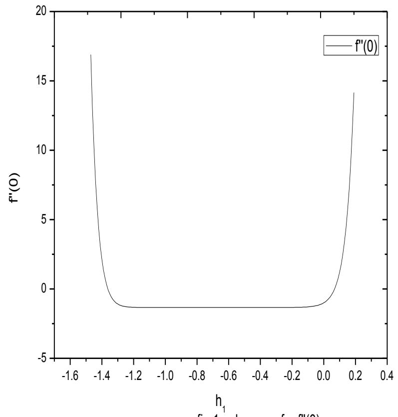

fig 1 : h_1 curve for f''(0)

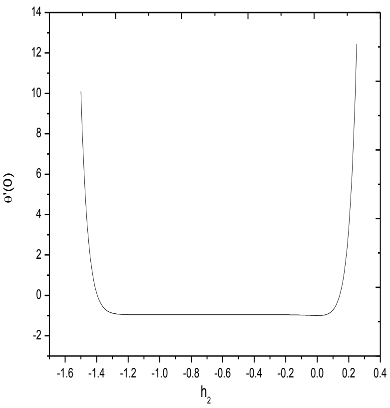

fig:2 h2 curve for θ'(0)

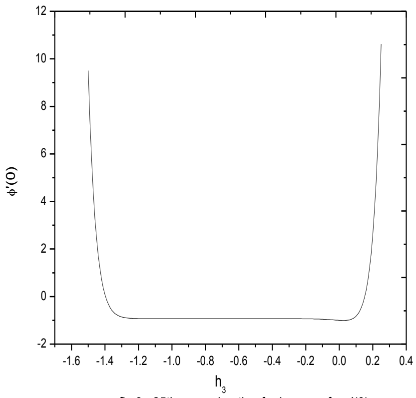

fig 3: 25th approximation for h_3 curve for ϕ'(0)

The effect of visco - elastic parameter on velocity, temperature and concentration fields is shown in Figs. 4 to 6. It is noticed that decrease in velocity with increases in visco- elastic parameter, and increase in visco – elastic parameter with increase in temperature and concentration. This is because of the fact that the introduction of tensile stress due to visco-elasticity causes traverse contraction of the boundary layer.

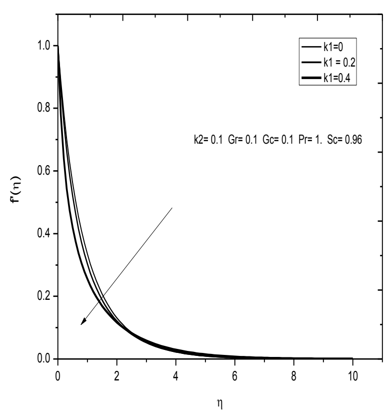

fig4: the effect of k1 on velocity

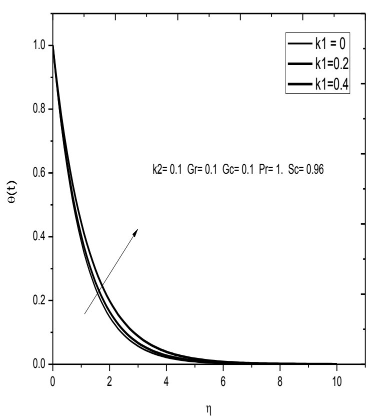

fig5: The effect of visco-elastic parameter on temperature

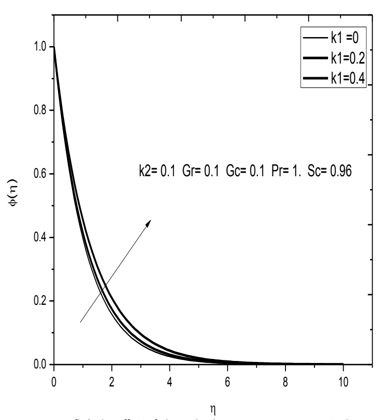

fig6: the effect of visco-elastic parameter on concentration

The effect of porosity parameter on velocity, temperature and concentration fields is shown in Figs. 7 to 9. It is noticed that the increase in porosity parameter with decrease in velocity and increase in temperature, concentration.

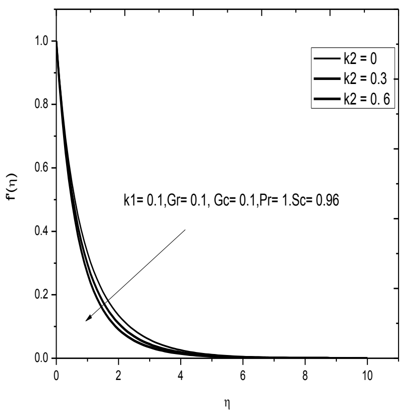

fig 7 : the effect of porosity parameter on dimensionless velocity

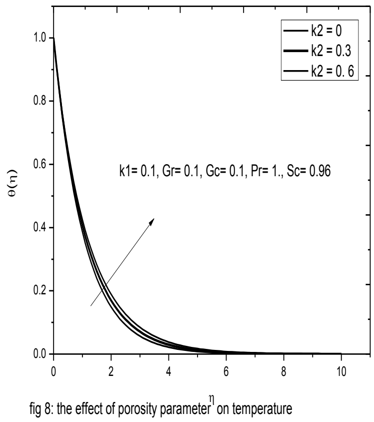

fig 8: the effect of porosity parameter on temperature

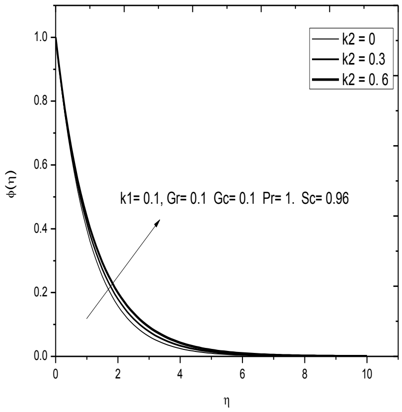

fig 9 : the effect of porosity parameter on concentration

The effect of Grashof number on velocity, temperature and concentration fields is shown in Figs. 10 to 12. It is observed that the increase in Grashof number with is increase in velocity and decrease in temperature and concentration.

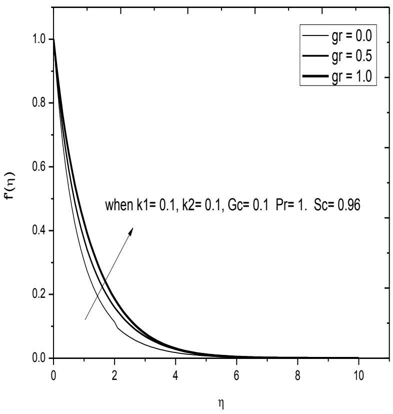

fig 10: the effect of Gc on non-dimensional velocity parameter

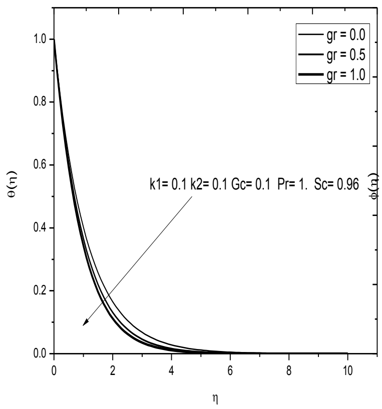

fig 11 the effect of grassif number on dimensionless temperature

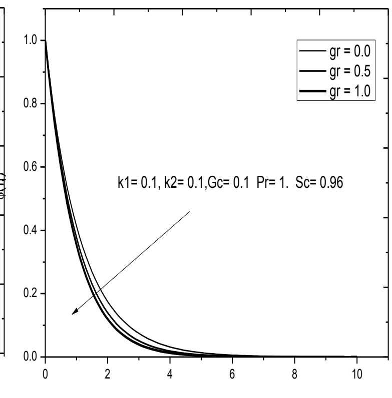

fig 12:the effect of grasiff number on dimensionless concentration

The effect of modified Grashof number on velocity, temperature and concentration fields is shown in Figs. 13 to 15. It is observed that the increase in modified Grashof number with increase in velocity and decrease in temperature and concentration.

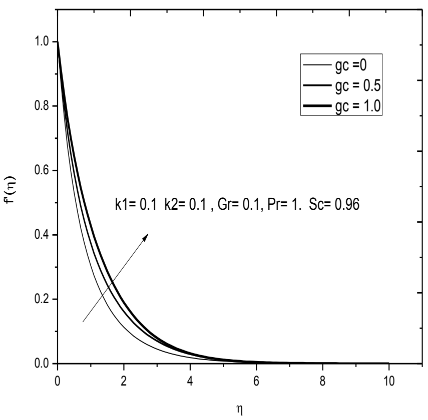

fig 13: the effect of modified grashof number on dimensional velocity

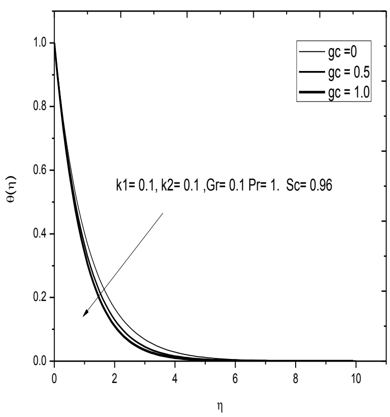

fig 14:the effect of gc on the dimensionless temperature

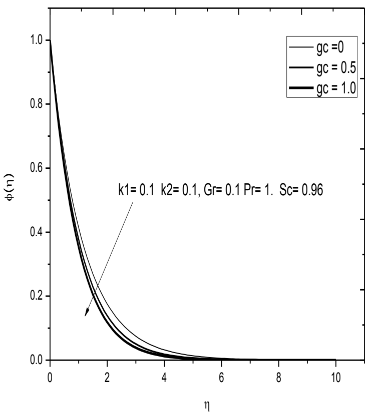

fig15: the effect of gc on the dimensionless concentration

The effect of Prandtl number and Schmidt number on temperature is shown in fig 16 – 17. It is observed that the increase in Prandtl and Schmidt number with decrease in temperature.

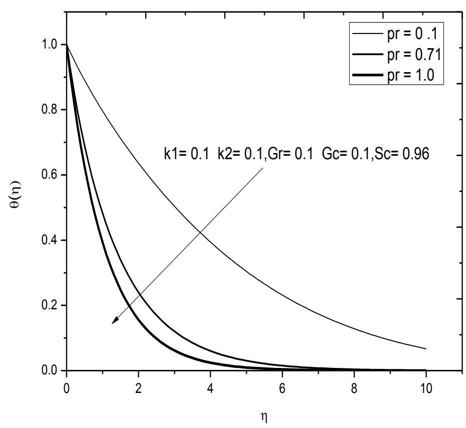

fig16: the effect of prandtl number on dimensionless temperature

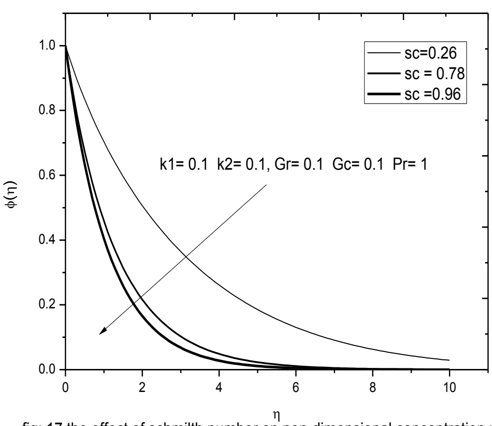

fig: 17 the effect of schmilth number on non-dimensional concentration parameter

In Table, we compared the convergence results for skin friction , temperature and concentration parameters, and In Table 2, the values of surface temperature gradient –$\theta'(0)$ are compared with the previous results. From this comparison, it is noticed that our analytic solutions agree well with the previous results reported by Bidin and Nazar [23].

Table 1. Convergence of HAM solution for different orders of approximations when $k_1 = 0.1, k_2 = 0.1,\ Gr = 0.1, Gc = 0.1, Pr = 1.,\ Sc = 0.96.$

Order	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
5	1.333340	0.956394	0.932642
10	1.334280	0.955004	0.930247
15	1.334300	0.955102	0.930213
20	1.334303	0.955119	0.930212
25	1.334303	0.955120	0.930211
30	1.334303	0.955120	0.930211
35	1.334303	0.955120	0.930211
40	1.334303	0.955120	0.930211

Table 2. Comparison of $-\theta'(0)$ for different values of Pr when $k_1 = k_2 = Gr = Gc = 0.0.$

Pr	Bidin and Nazar [22]	HAM
1.0	0.9547	0.954783
2.0	1.4714	1.471460
3.0	1.8691	1.869067

CONCLUSION:

In this paper, the analysis was made on the heat and mass transfer of two- dimensional steady incompressible laminar flow of Walter's liquid B through a porous medium over exponentially stretching sheet. Analytical solutions are obtained through HAM.

The obtained results supported the results existing in the literature. The effects of various parameters on velocity, temperature and concentration profiles were analysed and are shown graphically.

- The increase in Grashof number (G_r) leads to the increase of horizontal velocity profile.
- The combined effect of increasing the values of Prandtl number , Grashof number and modified Grashof number is to reduce the temperature profile significantly on the boundary sheet.
- The effects of free convective parameters (G_r),(G_c) and (S_c) numbers are to decrease the concentration distribution in the boundary layer.

REFERENCES

[1] Sakiadis, B. C., Boundary Layer Behaviour on Continuous Solid Surface: II – Boundary Layer on a Continuous Flat Surface*, AIChE.J., 7 (1961), 2, pp. 221-225.*

[2] L.J. Crane, Flow past a stretching plate, *Z. Angew. Math. Phys. 21 (1970) 645–647.*

[3] P.S. Gupta, A.S. Gupta, Heat and mass transfer on a stretching sheet with suction or blowing, *Can.J. Chem. Eng. 55 (1977) 744–746*.

[4] V. Kumaran, G. Ramanaiah, A note on the flow over a stretching sheet, *Acta Mech. 116 (1996) 229–233.*

[5] M.E. Ali, On thermal boundary layer on a power law stretched surface with suction or injection,*Int. J. Heat Fluid Flow 16(1995) 280–290*.

[6] K. Vajravelu, J.R. Cannon, Fluid flow over a nonlinear stretching sheet, *Appl. Math. Comput. 181 (2006) 609–618.*

[7] T.Hymavathi, B.Sankar, Heat transfer in MHD visco-elastic fluid flow over non-isothermal stretching sheet, *Journal of Appl.Mathematics and Computation 215(2009)2045-2054.*

[8] Abel, S., and Veena, P. H. (1998). Visco-elastic fluid flow and heat transfer in a porous medium over a stretching sheet, *Int. J. Non-Linear Mech., 33, 531–540*.

[9] Chauhan, D. S., and Agarwal.R, MHD flow and heat transfer in a channel bounded by shrinking sheet and porous medium bed, *ISRN thermodynamics*, *Volume 2013, Article ID. 291270, Hindawi Publishing Corporation.*

[10] Prasad, K. V., Abel, M. S., and Datti, P. S. (2003). Diffusion of chemically reactive species of a non-Newtonian fluid immersed in a porous medium over a stretching sheet*, Int. J.Non- Linear Mech., 38, 651–657.*

[11] Krishnambal, S., and Anuradha, P. (2006). Effect of radiation on the flow of a visco-elastic fluid and heat transfer in a porous medium over a stretching sheet, *J. Appl. Sci., 6(14), 2901–2906.*

[12] M.Subhas, Abel ,S.K.Khan and K.v.Prasad, Convective heat and mass transfer in a visco-elastic fluid flow through a porous medium over a stretching sheet*, Int. Journal of Numerical Methods for Heat & Fluid flow, Vol.11 No.8,2001,pp.779-792.*

[13] W. C.Troy, E. A. Overmanii and G. B. Ermentrout, Uniqueness of flow of a second order fluid past a stretching sheet, *Quarterly of Applied Mathematics, Volume XLIV, Number4, January 1987.*

[14] Elbashbeshy, E.M. A., Heat Transfer Over an Exponentially Stretching Continuous Surface with Suction, *Arch. Mech., 53 (2001), 6, pp. 643-651.*

[15] E.Magyari and B.Keller, Heat and mass transfer in boundary layers on an exponentially stretching surface, *J. Phys. D: Appl. Phys.* ***32*** *(1999) 577–585.*

[16] Khan, S. K., Sanjayanand, E., Viscoelastic Boundary Layer Flow and Heat Transfer over an Exponentially Stretching Sheet*, Int. J. Heat Mass Transfer, 48 (2005), 8, pp. 1534-1542*.

[17] Sanjayanand, E., Khan, S. K., On Heat and Mass Transfer in a Viscoelastic Boundary Layer Flow over an Exponentially Stretching Sheet, *Int. J. Therm. Sci., 45 (2006), 8, pp. 819-828.*

[18] Liao SJ. The proposed homotopy analysis technique for the solution of nonlinear problems (Ph.D. thesis), *Shanghai Jiao Tong University; 1992.*

[19] Liao SJ. Beyond perturbation: introduction to the homotopy analysis method. *Boca Raton: Chapman and Hall/CRC Press; 2003.*

[20] Wu W, Liao SJ. Solving solitary waves with discontinuity by means of the homotopy analysis method. *Chaos Solitons Fractals 2005;26:177–85.*

[21] Sajid, M., Hayat,T., Influence of Thermal Radiation on the Boundary Layer Flow Due to an Exponentially Stretching Sheet*, Int. Comm. Heat Mass Transfer, 35 (2008), 3, pp. 347-356.*

[22] T.Hymavathi and B.Akkayya, Homotopy Analysis method to flow and heat transfer flow in visco-elastic fluid flow in porous medium over exponentially

stretching sheet, *Int.Journal of Computational and Appl.Mathematics,Vol.11No.2(2016),pp.119-128.*

[23] B. Bidin, R. Nazar, Numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation*, Eur. J. Sci. Res. 33 (4) (2009) 710–717*.