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On Optimal Solution of a Transportation Problem

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Abstract

In this paper, we develop a new method to find the initial basic feasible solution as well as the optimal solution (or near to the optimal solution) of transportation problem. Also, the new algorithm provided here gives the idea for the optimality in comparison with MODI method as the flow of step by step procedure.

Keywords: Transportation problem, initial basic feasible solution, optimality, the degree of freedom (m-1) for optimality.

INTRODUCTION

Transportation problem is the most useful special class of linear programming problem which can be applied for different sources of supply to different destination of demand in such a way that the total transportation cost should be minimized. Usually, the initial basic feasible solution of any transportation problem is obtained by using well known methods such as North-West corner method (NWCM) or Least-Cost Method (LCM) or Vogel's Approximation Method (VAM), and then finally the optimality of the given transportation problem is checked by MODI.

Transportation problem was first derived by F. L. Hitchcock in [1]. T. C. Koopmans presented the work of F. L. Hitchcock in the paper [2]. These two contributions are most helpful in the development of transportation methods which involve a number of shipping sources and a number of destinations.

In 1990, Bazarra, Jarvis and Sherali [insert ref.] defined the linear programming problems with fuzzy numbers and used simplex method to find its optimal solution. Lai and Huang [insert ref.], in 1992 considered the situations where all parameters are in fuzzy number. They assume that the parameters have a triangular possibility distribution. Recently, in 2006, Swarup, Gupta and Mohan [insert ref.] explained the

method to obtain sensitivity analysis or post optimality analysis of the different parameters in the linear programming problems.
From last few years, many manufacturers used the optimization technique most frequently in linear programming problem to solve the real world problems. For that, it is crucial to develop the new approaches that can give the model to fit into the real world as much as possible.
In this paper, we develop a new method to find the initial basic feasible solution as well as the optimal solution or near to the optimal solution of transportation. Also, the new algorithm provided here gives the idea for the optimality in comparison with MODI method as the flow of step by step procedure. We also give the numerical examples explaining the new algorithm.

MATHEMATICAL BEYGROUND:
Let us consider the standard balanced transportation problem with m sources A_i (with supplies a_i), i ∈ I={1,2,3........,m} and n destinations B_j (with demands b_j), j∈ J={1,2,3,......,n} .
If X_{ij}= the number of load units moving from A_i to B_j , the feasible solution (x) and set of feasible solutions (X) is:
$X=\{x/\sum_{j\in J} X_{ij} = a_i, \forall i \in I; \equiv \sum_{i\in I} X_{ij} = b_j, \forall j \in J; X_{ij} \geq 0\ \forall (i,j); \sum a_i = \sum b_j\}.$
Mathematically the problem can be stated as minimize $z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\, x_{ij}$ subject to
$\sum_{j=1}^{n} x_{ij} = a_i; for\ i = 1,2, \ldots\ldots, m\ (supply\ constraints)$ And
$\sum_{i=1}^{n} x_{ij} = b_j$ For j=1, 2... n (demand constraints) $X_{ij} \geq 0$ for all i & j.

A transportation problem is said to be balanced if the total supply from all sources equals to the total demands in all destinations i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, otherwise it is called the unbalanced transportation problem.

<u>Transportation Problem:</u>

Origins (i)	Destinations (j)				Supply (a_i)
	1	2	....	n	
1	X_{11} C_{11}	X_{12} C_{12}	....	X_{1n} C_{1n}	a_1
2	X_{21} C_{21}	X_{22} C_{22}	....	X_{2n} C_{2n}	a_2
3	X_{31} C_{31}	X_{32} C_{32}		X_{3n} C_{3n}	a_3
....	....	....	....	....	....
M	X_{m1} C_{m1}	X_{m2} C_{m2}	....	X_{mn} C_{mn}	a_m

Demand (b_j)	b_1	b_2	....	b_n	$\sum a_i = \sum b_j$

First, letus convert the constraints of the transportation problem into our standard matrix form for a linear programming problem, Ax=B.

$$X=[x_{11},........,x_{1n},, x_{21}......,x_{2n},......,x_{mn}]$$

$$B= [a_1,........,a_{m},, b_1......,b_n] \quad (1)$$

If the Constraints are written as,

$$x_{11} + x_{12} + ... + x_{1n} = a_1,$$
$$x_{21} + ... + x_{2n} = a_2,$$
$$\ddots \vdots$$
$$x_{m1} + ... + x_{mn} = a_m,$$
$$x_{11} + x_{21} + ... + x_{m1} = b_1,$$
$$\ddots$$
$$\ddots$$
$$x_{1n} + x_{2n} + ... + x_{mn} = b_n, \quad (2)$$

Now the equivalent reduced row echelon form of the above system – (2) is as follows,

$$x_{11} + x_{12} + .x_{13} + ... + x_{1n} = a_1,$$
$$x_{12} + ... + x_{22} + ... + x_{32} + ... + x_{m2} = b_2$$
$$x_{13} + ... + x_{23} + ... + ..x_{33} + ... + x_{m3} = b_3$$
$$x_{14} + ... + x_{24} + ... + x_{34} + ... + x_{m4} = b_4$$
$$\ddots .... \ddots .... \ddots .... \ddots .... = \vdots$$
$$x_{1n} + ... + x_{2n} + ... + x_{3n} + ... + x_{mn} = b_n$$

$$x_{21} + .x_{22} + x_{23} .... + x_{2n} = a_2,$$
$$\ddots \vdots$$
$$x_{m1} + x_{m2} + x_{m3} + ... + x_{mn} = a_m \quad (3)$$

The Degree of Freedom for Optimality:-

1. Construct the Transportation matrix along with supply and demand equations of order
   (m+n) × (mn) corresponding to the system-(2).
2. Find its reduced row echelon form of the matrix which corresponds to system-(3).
3. Now check the pivot elements (row wise) of this matrix with corresponding allocations of the rows in the simple transportation matrix of order (mxn).
4. Now the number of allocations must follow the relation between the pivot elements in the matrix of order (m+n) × (mn) and the corresponding allocations of rows in

simple Transportation matrix of order (mxn) with the degree of freedom for optimality (m-1) as per the simple transportation matrix, such that (m-1) number of allocations must be from

(m+n-1) pivots from the matrix of order (m+n) × (mn).

ALGORITHM:

Step: 1 Construct the transportation matrix from the given transportation problem.

Step: 2 Find an IBFS using any one of the method as NWCM, LCM, VAM.

Step: 3 Then,skip or omit the minimum n-cost cells only from unoccupied cells (non basic variables) from the transportation matrix.

Step: 4 Assign $+\theta$ to the next minimum cost cell from unoccupied cells and start to make a loop with occupied cells if possible, otherwise move to the next to next cell from unoccupied cells. Then find

$\theta = min(-\theta)$ and add that min (-θ) value at $+\theta$ and subtract that min (-θ) value from (- θ).

Step: 5 Continue this process unless and until the loop made contains at least 2 cells from L-shape of the matrix having (m+n-1) pivots of the system-3. Then, find the cost of the matrix.

If we observe that this transportation cost is less than the cost obtained in Step: 2 then apply the Test for Optimality. Otherwise, go to Step-6.

Step: 6 Repeat Step-3, 4 and 5until**The Test for Optimality** is satisfied.

The Test for Optimality:

(1) All minimum consecutive n-cost cells are allocated in the simple transportation matrix.

(2) All minimum consecutive n-cost cells are allocated in the simple transportation matrix with the degree of freedom for optimality (m-1).

(3) At least one of minimum consecutive n-cost cells are allocated in the simple transportation matrix with the degree of freedom for optimality (m-1).

Step: 7 now the total minimum cost is calculated as sum of the product of cost and corresponding allocated value of Supply/demand. i.e. total cost $=\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\, x_{ij}$.

NUMERICAL EXAMPLES (New Method):

1) Consider the following Cost minimizing Transportation problem**:**

	D_1	D_2	D_3	D_4	D_5	**Supply**
S_1	3	4	6	8	9	**20**
S_2	2	10	1	5	8	**30**
S_3	7	11	20	40	3	**15**
S_4	2	1	9	14	16	**13**
Demand	**40**	**6**	**8**	**18**	**6**	**Total=78**

After Applying the Least Cost Method, for Initial Basic Feasible Solution, the Allocations are
as follows.

	D_1	D_2	D_3	D_4	D_5	**Supply**
S_1	**11** 3	4	6	**9** 8	9	**20**
S_2	**22** 2	10	**8** 1	5	8	**30**
S_3	7	11	20	**9** 40	**6** 3	**15**
S_4	**7** 2	**6** 1	9	14	16	**13**
Demand	**40**	**6**	**8**	**18**	**6**	**Total=78**

The minimum cost using LCM is obtained as follows,
Min Cost :**(11*3) + (9*8) + (22*2) + (8*1) + (9*40) + (6*3) + (7*2) + (6*1) = 555**

Now by applying the New Method, allocations are obtained as follows,

	D_1	D_2	D_3	D_4	D_5	**Supply**
S_1	**20** 3	4	6	8	9	**20**
S_2	**4** 2	10	**8** 1	**18** 5	8	**30**
S_3	**9** 7	11	20	40	**6** 3	**15**
S_4	**7** 2	**6** 1	9	14	16	**13**
Demand	**40**	**6**	**8**	**18**	**6**	**Total=78**

Total Cost obtained by new method is as follows,
Total Minimum Cost = (20*3)+(4*2)+(8*1)+(18*5)+(9*7)+(6*3)+(7*2)+(6*1)
= 267

2) Consider the following Cost minimizing Transportation problem**:**

	D_1	D_2	D_3	**Supply**
S_1	6	4	1	**50**
S_2	3	8	7	**40**
S_3	4	4	2	**60**
Demand	**20**	**95**	**35**	**Total=150**

After Applying the North West Corner Rule, for Initial Basic Feasible Solution, the Allocations are as follows.

	D_1	D_2	D_3	Supply
S_1	**20** 6	**30** 4	1	**50**
S_2	3	**40** 8	7	**40**
S_3	4	**25** 4	**35** 2	**60**
Demand	**20**	**95**	**35**	**Total=150**

The minimum cost using NWCM is obtained as follows,

Min Cost:(20*6) + (30*4) + (40*8) + (25*4) + (35*2)

= 730

Now by applying the New Method, allocations are obtained as follows,

	D_1	D_2	D_3	Supply
S_1	6	**15** 4	**35** 1	**50**
S_2	**20** 3	**20** 8	7	**40**
S_3	4	**60** 4	2	**60**
Demand	**20**	**95**	**35**	**Total=150**

Total Cost obtained by new method is as follows,

Total Minimum Cost = (15*4) + (35*1) + (20*3) + (20*8) + (60*4)

= 555

3) Consider the following Cost minimizing Transportation problem**:**

	D_1	D_2	D_3	Supply
S_1	16	20	12	**200**
S_2	14	8	18	**160**
S_3	26	24	16	**90**
Demand	**180**	**120**	**150**	**Total=450**

After Applying the North West Corner Rule, for Initial Basic Feasible Solution, the Allocations are as follows.

	D_1	D_2	D_3	Supply
S_1	**180** 16	**20** 20	12	**200**
S_2	14	**100** 8	**60** 18	**160**
S_3	26	24	**90** 16	**90**
Demand	**180**	**120**	**150**	**Total=450**

The minimum cost using NWCM is obtained as follows,

Min Cost = (180*16) + (20*20) + (100*8) + (60*18) + (90*16)
=6600

Now by applying the New Method, allocations are obtained as follows,

	D_1	D_2	D_3	Supply
S_1	**140** 16	20	**60** 12	**200**
S_2	**40** 14	**120** 8	18	**160**
S_3	26	24	**90** 16	**90**
Demand	**180**	**120**	**150**	**Total=450**

Total Cost obtained by New method is as follows,

Total Minimum Cost = (140*16) + (60*12) + (40*14) + (120*8) + (90*16)
= 5920

4) Consider the following Cost minimizing Transportation problem:

	D_1	D_2	D_3	Supply
S_1	5	4	3	**100**
S_2	8	4	3	**300**
S_3	9	7	5	**300**
Demand	**300**	**200**	**200**	**Total=700**

After Applying the North West Corner Rule, for Initial Basic Feasible Solution, the Allocations are as follows.

	D_1	D_2	D_3	Supply
S_1	**100** 5	4	3	**100**
S_2	**200** 8	**100** 4	3	**300**
S_3	9	**100** 7	**200** 5	**300**
Demand	**300**	**200**	**200**	**Total=700**

The minimum cost using NWCM is obtained as follows,

Min Cost =(100*5) + (200*8) + (100*4) + (100*7) + (200*5)
=4200

Now by applying the New Method, allocations are obtained as follows,

	D_1	D_2	D_3	Supply
S_1	**100** 5	4	3	**100**

S_2	8	**200** 4	**100** 3	**300**
S_3	**200** 9	7	**100** 5	**300**
Demand	**300**	**200**	**200**	**Total=700**

Total Cost obtained by New method is as follows,

Total Minimum Cost =(100*5) + (200*4) + (100*3) + (200*9) + (100*5)
= 3900

CONCLUSION

The main aim of this paper is to achieve the optimal transportation cost by using the new method with the required number of steps and it's very easy to understand. Based on the optimal solution it allows us to taking a decision effectively. Thus, there are possible extensions to improve our algorithm of the method. The decision maker goes through all the steps of algorithm which makes our approach very useful to be applied in a lot of real world problems.

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