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An Efficient Class of Exponential Estimator of Finite Population Mean Under Double Sampling Scheme in Presence of Non-Response

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Abstract

The present study deals with the problem of estimating the population mean in simple random sampling in the presence of non-response. Using an auxiliary variable x an exponential dual to ratio cum dual to product estimator of the population mean of the study variable y in double sampling is defined. The bias and mean square error (MSE) of the proposed estimator has been obtained for both the cases. The asymptotically optimum estimator (AOE) of the proposed estimator has also been obtained along with its bias and MSE. Comparisons have been made with the existing similar estimators theoretically and numerically to demonstrate the superiority of the proposed estimator.

Keywords: Non-response, Bias, Mean squared error (MSE), optimum estimator, Efficiency.

1. INTRODUCTION

All sample surveys on large population are susceptible to a variety of errors that affect parts of the survey process and results. Non response of survey forms is one type of such errors that are caused in selected sample survey as a result of the failure to measure information from some of the units in the selected sample. The result of non-response in the survey makes the sample size to be smaller than the expected quantum and in cases of high non-response rates, the results may show high variances. This situation may cause biased estimates of the population parameters. Hansen and

Hurwitz (1946) were the first to recognize that non-response could lead to biased estimates of population characteristics while conducting mail surveys. They proposed a technique of sub-sampling the non-respondents to deal with the problem of non-response and its adjustments. They developed an unbiased estimator for population mean in the presence of non-response by dividing the population into responding and non-responding group and by taking a sub-sample of the non-responding units in order to avoid bias due to non-response. The problem of estimation of population mean of the study character utilizing auxiliary information have been demonstrated by Srivastava (1971), Reddy (1974), Ray and Sahai (1980), Srivenkataramana (1980), Srivastava and Jhajj (1981), Khare and Srivastava (1993), Khare and Sinha (2004) and Singh and Kumar (2011).

For the estimation of population mean $\bar{X}$ of the auxiliary variable x, a large first phase sample of size n' is selected from a finite population $U:\left(U_1,U_2,...,U_N\right)$ of N units by simple random sampling without replacement (SRSWOR). A smaller second phase sample of size n is selected from n' by SRSWOR. Non-response occurs on the second phase sample of size n in which n_1 units respond and n_2 units do not. From the n_2 non-respondents, by SRSWOR a sample of r $\left(r=n_2k^{-1},k>1\right)$ units is selected where k is the inverse sampling rate at the second phase sample of size n. All the r units respond this time round. The auxiliary information can be used at the estimation stage to compensate for units selected for the sample that fail to provide adequate responses and for population units missing from the sampling frame.

An unbiased estimator for the population mean $\bar{Y}$ of the study variable y proposed by Hansen and Hurwitz (1946) is defined as

$$\bar{y}^* = \frac{n_1}{n}\bar{y}_1 + \frac{n_2}{n}\bar{y}_{2r}$$

where $\bar{y}_1$ and $\bar{y}_{2r}$ denote the sample means of variable y based on n_1 and r units respectively. The estimators $\bar{y}^*$ is unbiased and has variance

$$V(\bar{y}^*) = \lambda S_y^2 + \lambda' S_{2y}^2 \tag{1}$$

where $\lambda = \left(\frac{1}{n}-\frac{1}{N}\right)$, $\lambda' = \frac{W_2\left(k-1\right)}{n}, W_2 = \frac{N_2}{N}$;

$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i-\bar{Y})^2$ and $S_{2y}^2 = \frac{1}{N_2-1}\sum_{i=1}^{N_2}(y_i-\bar{Y}_2)^2$ are the population mean square of y for the entire population and for the non-responding part of the population.

Similarly, the estimator $\bar{x}^*$ for population mean $\bar{X}$ in the presence of non-response based on corresponding (n_1+r) observations is given by

$$\bar{x}^* = \frac{n_1}{n}\bar{x}_1 + \frac{n_2}{n}\bar{x}_{2r}$$

where $\bar{x}_1$ and $\bar{x}_{2r}$ are the sample means of variable x based on n_1 and r units respectively. We have

$$V(\bar{x}^*) = \lambda S_x^2 + \lambda' S_{2x}^2$$

where $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{X})^2$ and $S_{2x}^2 = \frac{1}{N_2-1}\sum_{i=1}^{N}(x_i-\bar{X})^2$ are population mean squares of x for the entire population and non-responding part of the population.

In case, when population mean $\bar{X}$ is not known, then, it is estimated by taking a preliminary sample of size n' $(n' < N)$ from the population of size N by using simple random sampling without replacement (SRSWOR) sampling scheme. Neyman (1938) was the first to give the concept of double sampling in estimating the population parameters and then several authors have used the concept to found more precise estimates of population parameter.

Khare and Srivastava (1995) proposed conventional T_1 and alternative T_2 double sampling ratio estimators for population mean $\bar{Y}$ in the two different cases of non-response, i.e. when there is non-response on both the study variable y as well as on the auxiliary variable x and when there is non-response in the study variable y only, which are given as

$$\bar{y}_R^{d*} = \bar{y}^* \frac{\bar{x}'}{\bar{x}} \text{ and } \bar{y}_R^{d**} = \bar{y}^* \frac{\bar{x}'}{\bar{x}^*}$$

where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, $\bar{x}' = \frac{1}{n'}\sum_{i=1}^{n'} x_i$.

The MSE of $\bar{y}_R^{d*}$ and $\bar{y}_R^{d**}$ are given respectively as

$$MSE(\bar{y}_R^{d*}) = \bar{Y}^2\left[\lambda C_y^2 + \left(\frac{1}{n}-\frac{1}{n'}\right)C_x^2\left(1-2k_{yx}\right) + \lambda' C_{2y}^2\right] \tag{2}$$

$$MSE(\bar{y}_R^{d**}) = \bar{Y}^2\left[\lambda C_y^2 + \left(\frac{1}{n}-\frac{1}{n'}\right)C_x^2\left(1-2k_{yx}\right) + \lambda'\left\{C_{2y}^2 + C_{2x}^2\left(1-2k_{2yx}\right)\right\}\right] \tag{3}$$

where $C_y^2 = \dfrac{S_y^2}{\overline{Y}^2}$, $C_x^2 = \dfrac{S_x^2}{\overline{X}^2}$, $k_{yx} = \rho_{yx} C_y / C_x$, $k_{2yx} = \rho_{2yx} C_{2y} / C_{2x}$,

C_x, C_y are coefficient of variation;

$\rho_{yx} = S_{xy} / S_x S_y$ is correlation coefficient between the study variable variables y and the auxiliary variable x;

C_x^2, C_y^2 are the variances for the whole population for variables x and y ;

C_{2x}^2, C_{2y}^2 are the population variances for the stratum of non-response for the variables x and y respectively and S_{xy}, S_{2xy} are the covariances for the whole population and the population of non-respondents respectively.

Singh and Vishwakarma (2007) suggested the exponential ratio and product-type estimators for $\overline{Y}$ in double sampling respectively as

$$\overline{y}_{eR}^{d} = \overline{y} \exp\left(\frac{\overline{x}' - \overline{x}}{\overline{x}' + \overline{x}} \right) \text{ and } \overline{y}_{eP}^{d} = \overline{y} \exp\left(\frac{\overline{x} - \overline{x}'}{\overline{x} + \overline{x}'} \right)$$

The MSE of $\overline{y}_{eR}^{d}$ and $\overline{y}_{eP}^{d}$ in the two cases of non-response are given respectively as

$$\operatorname{MSE}\left(\overline{y}_{eR}^{d*}\right) = \overline{Y}^2 \left[\lambda C_y^2 + \frac{1}{4}\left(\frac{1}{n} - \frac{1}{n'}\right) C_x^2 \left(1 - 4k_{yx}\right) + \lambda' C_{2y}^2 \right] \tag{4}$$

$$\operatorname{MSE}\left(\overline{y}_{eR}^{d**}\right) = \overline{Y}^2 \left[\lambda C_y^2 + \frac{1}{4}\left(\frac{1}{n} - \frac{1}{n'}\right) C_x^2 \left(1 - 4k_{yx}\right) + \lambda' \left\{ C_{2y}^2 + \frac{1}{4} C_{2x}^2 \left(1 - 4k_{2yx}\right) \right\} \right] \tag{5}$$

$$\operatorname{MSE}\left(\overline{y}_{eP}^{d*}\right) = \overline{Y}^2 \left[\lambda C_y^2 + \frac{1}{4}\left(\frac{1}{n} - \frac{1}{n'}\right) C_x^2 \left(1 + 4k_{yx}\right) + \lambda' C_{2y}^2 \right] \tag{6}$$

$$\operatorname{MSE}\left(\overline{y}_{eP}^{d**}\right) = \overline{Y}^2 \left[\lambda C_y^2 + \frac{1}{4}\left(\frac{1}{n} - \frac{1}{n'}\right) C_x^2 \left(1 + 4k_{yx}\right) + \lambda' \left\{ C_{2y}^2 + \frac{1}{4} C_{2x}^2 \left(1 + 4k_{2yx}\right) \right\} \right] \tag{7}$$

In the present paper, we have suggested an efficient class of exponential estimator for the population mean under double sampling scheme in the presence of non-response where population mean of auxiliary variable is not known. The optimum bias and MSE of the proposed

estimators are obtained. The properties of the suggested estimator have been studied and. theoretical comparisons of the optimum MSE are made with others existing estimators and illustrated with the help of an empirical study.

2. THE PROPOSED ESTIMATOR

Utilizing information on the auxiliary variable x with unknown population mean $\bar{X}$, we have suggested an efficient class of exponential estimator in double sampling scheme in presence of non-response. The following two cases will be considered separately.

Case I: When non-response occurs only on y.

Case II: When non-response occurs on both y and x.

2.1 Case I: Non-Response only on y

The proposed estimator is

$$\bar{y}_{edRP}^{d*} = \bar{y}^* \left[\alpha_1 \exp\left(\frac{\bar{x}^{\sigma} - \bar{x}'}{\bar{x}^{\sigma} + \bar{x}'} \right) + \beta_1 \exp\left(\frac{\bar{x}' - \bar{x}^{\sigma}}{\bar{x}' + \bar{x}^{\sigma}} \right) \right] \tag{8}$$

where $\bar{x}^{\sigma} = \left(N\bar{x}' - n\bar{x}\right) / \left(N - n\right)$ and α_1, β_1 are unknown constants such that $\alpha_1 + \beta_1 = 1$.

To obtain the bias and MSE of $\bar{y}_{edRP}^{d*}$, we write

$\bar{y} = \bar{Y}\left(1 + e_0^*\right), \bar{x} = \bar{X}\left(1 + e_1\right)$ and $\bar{x}' = \bar{X}\left(1 + e_2\right)$

Expressing $\bar{y}_{edRP}^{d*}$ in terms of e's, we obtain

$$\bar{y}_{edRP}^{d*} = \bar{y}\left[1 + \frac{g}{2}\left(e_1 - e_2 + e_2^2 - e_1 e_2\right) + \frac{3g^2}{8}\left(e_1^2 + e_2^2 - 2e_1 e_2\right) \right.$$
$$\left. + \alpha g \left\{ e_2 - e_1 + e_1 e_2 - e_2^2 + g e_1 e_2 - \frac{1}{2} g e_1^2 - \frac{1}{2} g e_2^2 \right\} \right]$$

where $g = \dfrac{n}{N - n}$.

Expanding the above equation, multiplying out and ignoring terms of e's greater than two, we get

$$\bar{y}_{edRP}^{d*} - \bar{Y} = \bar{Y}\left[e_0^* + \frac{g}{2}\left(e_1 - e_2 + e_0^* e_1 - e_0^* e_2 + e_2^2 - e_1 e_2\right) + \frac{3g^2}{8}\left(e_1^2 + e_2^2 - 2e_1 e_2\right) \right.$$

$$+\alpha g\left\{e_2-e_1+e_0^*e_2-e_0^*e_1+e_1e_2-e_2^2+ge_1e_2-\frac{1}{2}ge_1^2-\frac{1}{2}ge_2^2\right\}\Bigg] \quad (9)$$

2.1.1 Bias, MSE and Optimum Value of $\bar{y}_{edRP}^{d*}$ in Case I

In this case, we have

$$E(e_0^*)=E(e_1)=E(e_2)=E(e_3)=0;\ E(e_0^{*2})=\left(\lambda C_y^2+\lambda' C_{2y}^2\right);$$

$$E(e_1^2)=\lambda C_x^2;\ E(e_2^2)=\left(\frac{1}{n'}-\frac{1}{N}\right)C_x^2;$$

$$E(e_3^2)=\left(\frac{1}{n'}-\frac{1}{N}\right)C_z^2;\ E(e_0^*e_1)=\lambda k_{yx}C_x^2;$$

$$E(e_0^*e_2)=\left(\frac{1}{n'}-\frac{1}{N}\right)k_{yx}C_x^2;\ E(e_1e_2)=\left(\frac{1}{n'}-\frac{1}{N}\right)C_x^2 \quad (10)$$

On taking expectations on both sides of the equation (9) and using the results of (10), the bias of $\bar{y}_{edRP}^{d*}$ to the first order of approximation is given by

$$B(\bar{y}_{edRP}^{d*})=\bar{Y}\left[\frac{3}{8}\left(\frac{1}{n}-\frac{1}{n'}\right)g^2C_x^2+\frac{1}{2}\left(\frac{1}{n}-\frac{1}{n'}\right)gk_{yx}C_x^2-\alpha\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)gk_{yx}C_x^2+\frac{1}{2}\left(\frac{1}{n}-\frac{1}{n'}\right)g^2C_x^2\right\}\right] \quad (11)$$

Squaring both sides of the above equation (9), taking expectations and using the results of (10), we obtain the MSE of $\bar{y}_{edRP}^{d*}$ to the first order of approximation as

$$MSE\left(\bar{y}_{edRP}^{d*}\right)=\bar{Y}^2\Bigg[\lambda C_y^2+\lambda' C_{2y}^2+\frac{1}{4}\left(\frac{1}{n}-\frac{1}{n'}\right)gC_x^2\left(g+4k_{yx}\right)-$$

$$\left(\frac{1}{n}-\frac{1}{n'}\right)\alpha gC_x^2\left(g+2k_{yx}\right)+\left(\frac{1}{n}-\frac{1}{n'}\right)\alpha^2g^2C_x^2\Bigg] \quad (12)$$

Differentiating (12) in terms of α_1 gives its optimum value as

$$\alpha_1=\frac{g+2k_{yx}}{2g}=\alpha_{1opt} \qquad \text{(say)} \quad (13)$$

Substituting the value of (13) in (12), we get the optimum MSE of $\bar{y}_{edRP}^{d*}$ as

$$MSE(\bar{y}_{edRP}^{d*})_{opt}=\bar{Y}^2\left[\lambda C_y^2+\lambda' C_{2y}^2-\left(\frac{1}{n}-\frac{1}{n'}\right)k_{yx}^2C_x^2\right] \quad (14)$$

Remarks

1. When $\alpha_1 = 1$, the proposed estimator reduces to exponential dual to ratio estimator $\bar{y}_{edR}^{d*}$ under double sampling. The bias and MSE of $\bar{y}_{edR}^{d*}$ are obtained by putting $\alpha_1 = 1$ in (11) and (12) as follows

$$B(\bar{y}_{edR}^{d*}) = \bar{Y}\left[-\frac{1}{8}\left(\frac{1}{n}-\frac{1}{n'}\right)g^2C_x^2 - \frac{1}{2}\left(\frac{1}{n}-\frac{1}{n'}\right)gk_{yx}C_x^2\right]$$

$$MSE\left(\bar{y}_{edR}^{d*}\right) = \bar{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 + \frac{1}{4}\left(\frac{1}{n}-\frac{1}{n'}\right)gC_x^2\left(g-4k_{yx}\right)\right] \tag{15}$$

2. When $\alpha_1 = 0$, the proposed estimator reduces to double sampling exponential dual to product estimator $\bar{y}_{edP}^{d*}$. The bias and MSE of $\bar{y}_{edP}^{d*}$ are obtained by putting $\alpha_1 = 0$ in (11) and (12) as follows

$$B(\bar{y}_{edP}^{d*}) = \bar{Y}\left[\frac{3}{8}\left(\frac{1}{n}-\frac{1}{n'}\right)g^2C_x^2 + \frac{1}{2}\left(\frac{1}{n}-\frac{1}{n'}\right)gk_{yx}C_x^2\right]$$

$$MSE\left(\bar{y}_{edP}^{d*}\right) = \bar{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 + \frac{1}{4}\left(\frac{1}{n}-\frac{1}{n'}\right)gC_x^2\left(g+4k_{yx}\right)\right] \tag{16}$$

2.1.2 Efficiency Comparisons of $\left(\bar{y}_{edRP}^{d*}\right)_{opt}$ in Case I:

(i) Comparison with Mean Per Unit Estimator:

From (1) and (14), we have

$$V\left(\bar{y}^*\right) - MSE(\bar{y}_{edRP}^{d*})_{opt} = \bar{Y}^2\left(\frac{1}{n}-\frac{1}{n'}\right)k_{yx}^2C_x^2 > 0, \tag{17}$$

(ii) Comparison with Usual Ratio Estimator in Double Sampling**:**

From (2) and (14), we have

$$MSE(\bar{y}_R^{d*}) - MSE(\bar{y}_{edRP}^{d*})_{opt} = \bar{Y}^2\left[\left(\frac{1}{n}-\frac{1}{n'}\right)C_x\left(1-k_{yx}\right)^2\right] > 0 \tag{18}$$

(iii) Comparison with Exponential Ratio Estimator in Double Sampling**:**

From (4) and (14), we have

$$MSE\left(\bar{y}_{eR}^{d*}\right)-MSE(\bar{y}_{edRP}^{d*})_{opt}=\bar{Y}^2\left\lfloor\frac{1}{4}\left(\frac{1}{n}-\frac{1}{n'}\right)C_x^2\left(1-2k_{yx}\right)^2\right\rfloor>0 \quad (19)$$

(iv) Comparison with Exponential Product Estimator in Double Sampling**:**

From (6) and (14), we have

$$MSE\left(\bar{y}_{eP}^{d*}\right)-MSE(\bar{y}_{edRP}^{d*})_{opt}=\bar{Y}^2\left\lfloor\frac{1}{4}\left(\frac{1}{n}-\frac{1}{n'}\right)C_x^2\left(1+2k_{yx}\right)^2\right\rfloor>0 \quad (20)$$

(v) Comparison with Exponential Dual to Ratio Estimator in Double Sampling**:**

From (15) and (14), we have

$$MSE\left(\bar{y}_{edR}^{d*}\right)-MSE(\bar{y}_{edRP}^{d*})_{opt}=\bar{Y}^2\left\lfloor\frac{1}{4}\left(\frac{1}{n}-\frac{1}{n'}\right)C_x^2\left(g-2k_{yx}\right)^2\right\rfloor>0 \quad (21)$$

(vi) Comparison with Exponential Dual to Product Estimator in Double Sampling**:**

From (17) and (14), we have

$$MSE\left(\bar{y}_{edP}^{d*}\right)-MSE(\bar{y}_{edRP}^{d*})_{opt}=\bar{Y}^2\left\lfloor\frac{1}{4}\left(\frac{1}{n}-\frac{1}{n'}\right)C_x^2\left(g+2k_{yx}\right)^2\right\rfloor>0 \quad (22)$$

2.2 Case II: Non-Response on Both y and x

The suggested estimator in case II is given as follows

$$\bar{y}_{edRP}^{d**}=\bar{y}^*\left\lfloor\alpha_2\exp\left(\frac{\bar{x}^{*\sigma}-\bar{x}'}{\bar{x}^{*\sigma}+\bar{x}'}\right)+\beta_2\exp\left(\frac{\bar{x}'-\bar{x}^{*\sigma}}{\bar{x}'+\bar{x}^{*\sigma}}\right)\right\rfloor \quad (23)$$

where $\bar{x}^{*\sigma}=\left(N\bar{x}'-n\bar{x}^*\right)/\left(N-n\right)$ and α_2,β_2 are scalar constants such that $\alpha_2+\beta_2=1$.

2.2.1 Bias, MSE and Optimum Value of $\bar{y}_{edRP}^{d**}$ in Case II

In this case, we have

$E(e_0^*)=E(e_1^*)=E(e_2)=E(e_3)=0$; $E(e_0^{*2})=\left(\lambda C_y^2+\lambda' C_{2y}^2\right)$;

$E(e_1^{*2})=\lambda C_x^2+\lambda' C_{2x}^2$; $E(e_2^2)=\left(\frac{1}{n'}-\frac{1}{N}\right)C_x^2$;

$$E(e_3^2)=\left(\frac{1}{n'}-\frac{1}{N}\right)C_z^2;\ E(e_0^*e_1^*)=\lambda k_{yx}C_x^2+\lambda' k_{2yx}C_{2x}^2;$$

$$E(e_0^*e_2)=\left(\frac{1}{n'}-\frac{1}{N}\right)k_{yx}C_x^2;\ E(e_1^*e_2)=\left(\frac{1}{n'}-\frac{1}{N}\right)C_x^2 \tag{24}$$

where $\bar{x}^*=\bar{X}\left(1+e_1^*\right)$.

Replacing e_1 by e_1^* and taking expectations on both sides of the equation (9) and using the results of (24), we obtained the bias of $\bar{y}_{edRP}^{d**}$ to the first order of approximation is given by

$$B(\bar{y}_{edRP}^{d**})=\bar{Y}\left[\frac{3}{8}\left(\frac{1}{n}-\frac{1}{n'}\right)g^2C_x^2+\frac{3}{8}\lambda' g^2C_{2x}^2+\frac{1}{2}\left(\frac{1}{n}-\frac{1}{n'}\right)gk_{yx}C_x^2+\frac{1}{2}\lambda' gk_{2yx}C_{2x}^2\right.$$
$$\left.-\alpha\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)gk_{yx}C_x^2+\lambda' gk_{2yx}C_{2x}^2+\frac{1}{2}\left(\frac{1}{n}-\frac{1}{n'}\right)g^2C_x^2+\frac{1}{2}\lambda' g^2C_{2x}^2\right\}\right] \tag{25}$$

Replacing e_1 by e_1^*, squaring both the sides of the equation (9) , taking expectations and using the results of (24), we obtain the MSE of the estimator $\bar{y}_{edRP}^{d**}$ to first order of approximation as

$$MSE\left(\bar{y}_{edRP}^{d**}\right)=\bar{Y}^2\left[\lambda C_y^2+\lambda' C_{2y}^2+\frac{1}{4}\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)gC_x^2\left(g+4k_{yx}\right)+\lambda' gC_{2x}^2\left(g+4k_{2yx}\right)\right\}\right.$$
$$-\alpha_2\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)gC_x^2\left(g+2k_{yx}\right)+\lambda' gC_{2x}^2\left(g+2k_{2yx}\right)\right\}$$
$$\left.+\alpha_2^2\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)g^2C_x^2+\lambda' g^2C_{2x}^2\right\}\right] \tag{26}$$

Differentiating (26) in terms of α_2 gives its optimum value as

$$\alpha_2=\frac{\left(\frac{1}{n}-\frac{1}{n'}\right)gC_x^2(g+2k_{yx})+\lambda' gC_{2x}^2(g+2k_{2yx})}{2\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)g^2C_x^2+\lambda' g^2C_{2x}^2\right\}}$$
$$=\frac{1}{2}+\frac{A}{B}=\alpha_{2opt}\qquad \text{(say)} \tag{27}$$

where $A=\left(\frac{1}{n}-\frac{1}{n'}\right)k_{yx}C_x^2+\lambda' k_{2yx}C_{2x}^2$ and $B=\left(\frac{1}{n}-\frac{1}{n'}\right)gC_x^2+\lambda' gC_{2x}^2$.

Substituting the value of (27) in (26), we get the optimum MSE of $\bar{y}_{edRP}^{d**}$ as

$$MSE(\bar{y}_{edRP}^{d**})_{opt}=\bar{Y}^2\left[\lambda C_y^2+\lambda' C_{2y}^2-\frac{gA^2}{B}\right] \tag{28}$$

Remarks

1. When $\alpha_2=1$, the proposed estimator reduces to exponential dual to ratio estimator $\bar{y}_{edR}^{d**}$ under double sampling. The bias and MSE of $\bar{y}_{edR}^{d**}$ are obtained by putting $\alpha_2=1$ in (25) and (26) as follows

$$B(\bar{y}_{edR}^{d**})=\bar{Y}\left[-\frac{1}{8}\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)g^2C_x^2+\lambda' g^2C_x^2\right\}-\frac{1}{2}\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)gk_{yx}C_x^2+\lambda' gk_{2yx}C_{2x}^2\right\}\right]$$

$$MSE\left(\bar{y}_{edR}^{d**}\right)=\bar{Y}^2\left[\lambda C_y^2+\lambda' C_{2y}^2+\frac{1}{4}\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)gC_x^2\left(g-4k_{yx}\right)+\lambda' gC_{2x}^2\left(g-4k_{2yx}\right)\right\}\right] \tag{29}$$

2. When $\alpha_2=0$, the proposed estimator reduces to double sampling exponential dual to product estimator $\bar{y}_{edP}^{d**}$. The bias and MSE of $\bar{y}_{edP}^{d**}$ are obtained by putting $\alpha_2=0$ in (25) and (26) as follows

$$B(\bar{y}_{edP}^{d**})=\bar{Y}\left[\frac{3}{8}\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)g^2C_x^2+\lambda' g^2C_{2x}^2\right\}+\frac{1}{2}\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)gk_{yx}C_x^2+\lambda' gk_{2yx}C_{2x}^2\right\}\right]$$

$$MSE\left(\bar{y}_{edP}^{d**}\right)=\bar{Y}^2\left[\lambda C_y^2+\lambda' C_{2y}^2+\frac{1}{4}\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)gC_x^2\left(g+4k_{yx}\right)+\lambda' gC_{2x}^2\left(g+4k_{2yx}\right)\right\}\right] \tag{30}$$

2.2.2 Efficiency Comparisons of $\left(\bar{y}_{edRP}^{d**}\right)_{opt}$ in Case II:

(i) Comparison with Mean per Unit Estimator:

From (1) and (28), we have

$$V\left(\bar{y}^*\right)-MSE(\bar{y}_{edRP}^{d**})_{opt}=\bar{Y}^2\left[\frac{gA^2}{B}\right]>0 \tag{31}$$

(ii) Comparison with Usual Ratio Estimator in Double Sampling**:**

From (3) and (28), we have

$$MSE(\bar{y}_R^{d**}) - MSE(\bar{y}_{edRP}^{d**})_{opt} = \bar{Y}^2\left[\sqrt{\frac{B}{g}} + \sqrt{\frac{g}{B}}A\right]^2 > 0 \tag{32}$$

(iii) Comparison with Exponential Ratio Estimator in Double Sampling**:**

From (5) and (28), we have

$$MSE\left(\bar{y}_{eR}^{d**}\right) - MSE(\bar{y}_{edRP}^{d**})_{opt} = \bar{Y}^2\left[\frac{1}{2}\sqrt{\frac{B}{g}} - A\sqrt{\frac{g}{B}}\right]^2 > 0 \tag{33}$$

(iv) Comparison with Exponential Product Estimator in Double Sampling**:**

From (7) and (28), we have

$$MSE\left(\bar{y}_{eP}^{d**}\right) - MSE(\bar{y}_{edRP}^{d**})_{opt} = \bar{Y}^2\left[\frac{1}{2}\sqrt{\frac{B}{g}} + A\sqrt{\frac{g}{B}}\right]^2 > 0 \tag{34}$$

(v) Comparison with Exponential Dual to Ratio Estimator in Double Sampling**:**

From (29) and (28), we have

$$MSE\left(\bar{y}_{edR}^{d**}\right) - MSE(\bar{y}_{edRP}^{d**})_{opt} = \bar{Y}^2\left[\frac{1}{2}\sqrt{gB} - A\sqrt{\frac{g}{B}}\right]^2 > 0 \tag{35}$$

(vi) Comparison with Exponential Dual to Product Estimator in Double Sampling**:**

From (30) and (28), we have

$$MSE\left(\bar{y}_{edP}^{d**}\right) - MSE(\bar{y}_{edRP}^{d**})_{opt} = \bar{Y}^2\left[\frac{1}{2}\sqrt{gB} + A\sqrt{\frac{g}{B}}\right]^2 > 0 \tag{36}$$

3. EMPIRICAL STUDY

In this section, we have illustrated the relative efficiency of the estimators with respect to. For this purpose, we have used the data considered by Khare and Sinha (2007). The data are based on the physical growth of upper-socio-economic group of 95 school children of Varanasi district under an ICMR study, Department of Paediatrics, BHU, India during 1983-84. The description of the population is given below**:**

x : Chest circumference of the children (in cm)

y : Weights of the children (in kg)

$N = 95,\ n' = 70,\ n = 35,\ N_1 = 71,\ N_2 = 24,\ \bar{Y} = 19.5,\ \bar{X} = 55.86,$

$S_y^2 = 9.2416, S_{2y}^2 = 5.547,\ S_x^2 = 10.7158,\ S_{2x}^2 = 6.3001,$

$S_{xy} = 8.4587,\ S_{2xy} = 4.3095,\ \rho = 0.85, \rho_2 = 0.729,$

$\lambda = 0.018,\ \lambda' = 0.014,\ S_d^2 = 4.6418,\ \ S_{2d}^2 = 3.3059$

Table 1: PRE of the different estimators of $\bar{Y}$ with respect to $\bar{y}^*$

W_2	k	$\bar{y}^*$	$\bar{y}_R^{d*}$	$\bar{y}_{eR}^{d*}$	$\bar{y}_{eP}^{d*}$	$\bar{y}_{edR}^{d*}$	$\bar{y}_{edP}^{d*}$	$\bar{y}_{edRP}^{d*}$
0.1	1.5	100	159.62	127.12	78.94	115.06	87.02	217.86
	2.0	100	155.61	125.65	79.67	114.32	87.51	207.29
	2.5	100	152.10	124.33	80.35	113.64	87.97	198.46
	3.0	100	149.01	123.13	80.98	113.03	88.39	190.97
0.2	1.5	100	155.61	125.65	79.67	114.32	87.51	207.29
	2.0	100	149.01	123.13	80.98	113.03	88.39	190.97
	2.5	100	143.81	121.07	82.13	111.95	89.15	178.96
	3.0	100	139.61	119.34	83.15	111.04	89.82	169.76
0.3	1.5	100	152.10	124.33	80.35	113.64	87.97	198.46
	2.0	100	143.81	121.07	82.13	111.95	89.15	178.96
	2.5	100	137.80	118.58	83.62	110.63	90.13	165.91
	3.0	100	133.23	116.62	84.88	109.58	90.94	156.56

Table 2: PRE of the different estimators of $\overline{Y}$ with respect to $\overline{y}^*$

W_2	k	$\overline{y}^*$	$\overline{y}_R^{d**}$	$\overline{y}_{eR}^{d**}$	$\overline{y}_{eP}^{d**}$	$\overline{y}_{edR}^{d**}$	$\overline{y}_{edP}^{d**}$	$\overline{y}_{edRP}^{d**}$
0.1	1.5	100	165.01	129.17	77.90	116.11	86.32	228.18
	2.0	100	165.69	129.54	77.66	116.31	86.16	226.13
	2.5	100	166.33	129.87	77.44	116.50	86.02	224.46
	3.0	100	166.92	130.18	77.23	116.67	85.89	223.08
0.2	1.5	100	165.69	129.54	77.66	116.31	86.16	226.13
	2.0	100	166.92	130.18	77.23	116.67	85.89	223.08
	2.5	100	167.98	130.74	76.88	116.97	85.65	220.94
	3.0	100	168.90	131.22	76.57	117.24	85.45	219.38
0.3	1.5	100	166.33	129.87	77.44	116.50	86.02	224.46
	2.0	100	167.98	130.74	76.88	116.97	85.65	220.94
	2.5	100	169.32	131.44	76.43	117.36	85.36	218.76
	3.0	100	170.44	132.02	76.08	117.67	85.13	217.32

4. RESULTS AND DISCUSSIONS

In the present study, the study proposes an exponential dual to ratio cum dual to product estimators $\overline{y}_{edRP}^{d*}$ and $\overline{y}_{edRP}^{d**}$ in two phase sampling in the presence of non-response. The bias and MSE of the proposed estimators have been obtained in the two different cases. The MSE equations have been compared with the MSEs of the estimators $\overline{y}^*$, $\overline{y}_R^{d*}$, $\overline{y}_{eR}^{d*}$, $\overline{y}_{eP}^{d*}$, $\overline{y}_{edR}^{d*}$, $\overline{y}_{edP}^{d*}$, $\overline{y}_R^{d**}$, $\overline{y}_{eR}^{d**}$, $\overline{y}_{eP}^{d**}$, $\overline{y}_{edR}^{d**}$, $\overline{y}_{edP}^{d**}$ on a theoretical basis and the situations under which the proposed estimator at its optimum is more efficient than the estimators under considerations have been obtained. The same is seen and discussed in relation to other estimators in terms of percent relative efficiency (PRE) from Table 1 and Table 2.

REFERENCES

[1] Cochran, W.G., 1977, Sampling techniques, 3rd ed., John Wiley and sons, New York.

[2] Hansen, M. H. and Hurwitz, W. N., 1946, "The problem of non-response in sample surveys," Journal of the American Statistical Association, 41, pp. 517-529.

http://dx.doi.org/10.1080/01621459.1946.10501894

[3] Khare, B. B., and Srivastava, S., 1993, "Estimation of population mean using auxiliary character in presence of non-response, "National Academy Science Letters-India, 16, pp. 111-114.

[4] Khare, B. B., and Srivastava, S., 1995, "Study of conventional and alternative two phase sampling ratio, product and regression estimators in presence of nonresponse, "Proceedings-National Academy Of Sciences India Section A, 65, pp. 195-204.

[5] Khare, B. B., and Srivastava, S., 1997, "Transformed ratio type estimators for the population mean in the presence of non-response," Comm. Statist. Theory Methods, 26, pp. 1779-1791.

http://dx.doi.org/10.1080/03610929708832012

[6] Khare, B.B., and Sinha, R.R., 2002, "General class of two phase sampling estimators for the population means using an auxiliary character in the presence of non-response," Proc. of Vth International Symposium on Optimization and Statistics, AMU, Aligarh, pp. 233-245.

[7] Khare, B.B., and Sinha, R.R., 2004, "Estimations of finite population ratio using two phase sampling scheme in the presence of non-response," Aligarh Journal of Statistics, 24, pp. 43-56.

[8] Khare, B.B., and Sinha, R.R., 2007, "Estimation of the ratio of the two population means using multi auxiliary characters in the presence of non-response," Statistical Technique in Life testing, Reliability, Sampling Theory and Quality Control, Edited by B.N. Pandey, Narosa publishing house, New Delhi, pp. 163-171.

[9] Neyman, J., 1938, "Contribution to the theory of sampling human populations," Journal of the American Statistical Association, 33(201), pp. 101-116.

[10] Rao, P. S. R. S., 1986, "Ratio estimation with sub sampling the non-respondents," Survey Methodology, 12, pp. 217-230.

[11] Ray, S.K., and Sahai, A., 1980, "Efficient families of ratio and product type

estimators," Biometrika, 67, pp. 215-217.

[12] Reddy, V.N., 1974, "On a transformed ratio method of estimation," Sankhya, 36C, pp. 59-70.

[13] Singh, H.P., and Vishwakarma, G.K., 2007, "Modified exponential ratio and product estimators for finite population mean in double sampling," Austrian Journal of Statistics, 36(3), pp. 217-225.

[14] Singh, R., and Kumar, M., 2011, "A note on transformation on auxiliary variable in survey sampling," Model Assisted Statistics and Applications, 6(1), pp. 17-19.

doi 10.3233/MASA-2011-0154.

[15] Srivastava, S.K., 1971, "A generalized estimator for the mean of a finite population using multi-auxiliary information," Journal of the American Statistical Association, 66 (334), pp. 404-407.

[16] Srivastava, S. K., and Jhajj, H.S., 1981, "A class of estimators of the population mean in survey sampling using auxiliary information," Biometrika, 68(1), pp. 341-343.

[17] Srivenkataramana, T., 1980, "A dual to ratio estimator in sample surveys," Biometrika, 67(1), pp. 199-204.