Graphical Interpretation of various BSM Formulas

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Abstract

Paul Wilmott has derived BSM option pricing formula for the payoff function $\max\{\ln(S_T/K), 0\}$. Dedania and Ghevariya have derived BSM option pricing formula for the modified log (ML) payoff function $\max\{S_T \ln(S_T/K), 0\}$ (see [1]). In this paper, we compare the three formulas namely above two and the plain vanilla. It turns out that the formula for ML-payoff function is quite close to the plain vanilla option pricing formula.

AMS subject classification:

Keywords: BSM Formulas for three different payoff functions, comparison of BSM formulas, Analysis.

1. Introduction

Options are financial instruments that primarily are used in developed countries and are applied for reducing unnecessary risk, that exist. The recent global financial crisis, had further increased the uncertainties of the financial markets, which significantly had reduced the trading of these instruments. The argument is supported by the fact that the financial derivatives were the fundamental causes for the crisis. The concept of ‘option’ in financial market plays very crucial role. Many people in financial market use Black-Scholes-Merton (BSM) option pricing formulas directly or indirectly. The fundamental
BSM formula was derived for plain vanilla payoff function. After that several other types of option pricing formulas were derived for the various payoff functions. In Section-2, we give formulas for three payoff functions namely, plain vanilla, log and ML-payoff functions. In Section-3, we compare call/put option values and payoff values of the same three formulas through graphs. Finally, in Section-4, we conclude that the option values of ML-payoff function are nearer to the corresponding values for the plain vanilla.

2. BSM formulas for the ML-payoff functions

The explicit formulas for pricing European options on a non-dividend paying asset for ML-payoff functions is derived in [1]. This is a modification of Paul Wilmott’s log payoff function $\max\{\ln\left(\frac{S_T}{K}\right), 0\}$ [6, p 149]. An American option should be early exercised when the maximum option premium of early exercise is not less than the value of its European option. Here we note that the American call option pricing formula for non dividend paying asset for any payoff function is same as the European call option pricing formula for the same payoff function. Thus, we concentrate the American put option pricing formulas for above three payoff functions. In [5], the closed form solution for pricing American options on a non-dividend paying asset for plain vanilla payoff functions have been derived. In this section, the corresponding formulas for log payoff and ML-payoff functions are given without proof. We list the following notations and formulas for our main three payoff functions which will be required later. Note that $C_i^E$, $P_i^E$ and $P_i^A$ denote values of European call, European put, American put respectively for three payoff functions $i = 1, 2, 3$.

$$
P_{OC_1} = C_1(S, T) = \max\{S_T - K, 0\}
$$

$$
P_{OC_2} = C_2(S, T) = \max\{\ln\left(\frac{S_T}{K}\right), 0\}
$$

$$
P_{OC_3} = C_3(S, T) = \max\{S_T\ln\left(\frac{S_T}{K}\right), 0\}
$$

$$
P_{OP_1} = P_1(S, T) = \max\{K - S_T, 0\}
$$

$$
P_{OP_2} = P_2(S, T) = \max\{\ln\left(\frac{K}{S_T}\right), 0\}
$$

$$
P_{OP_3} = P_3(S, T) = \max\{S_T\ln\left(\frac{K}{S_T}\right), 0\}
$$

$$
C_1^E(S, K, r, T, \sigma) = SN(d_1) - Ke^{-rT}N(d_2)
$$

$$
C_2^E(S, K, r, T, \sigma) = e^{-rT}\eta(d_2)\sqrt{T} + e^{-rT}\left[\ln\left(\frac{S}{K}\right) + (r - \frac{1}{2}\sigma^2)T\right]N(d_2)
$$

$$
C_3^E(S, K, r, T, \sigma) = S\left[\ln\left(\frac{S}{K}\right)N(d_1) + \sigma\sqrt{T}\eta(d_1) + (r + \frac{1}{2}\sigma^2)TN(d_1)\right]
$$
Option Pricing Formulas

\[
P_1^E(S, K, r, T, \sigma) = C_1^E(S, K, r, T, \sigma) - S + Ke^{-rT} \\
P_2^E(S, K, r, T, \sigma) = C_2^E(S, K, r, T, \sigma) - e^{-rT} \left[ \ln \left( \frac{S}{K} \right) + (r - \frac{1}{2}\sigma^2)T \right] \\
P_3^E(S, K, r, T, \sigma) = C_3^E(S, K, r, T, \sigma) - S \ln \left( \frac{S}{K} \right) + (r - \frac{1}{2}\sigma^2)T \\
P_1^A(S, K, r, T, \sigma) = P_1^E(S, Ke^{rT}, r, T, \sigma)N(-d_3) + \max\{(K - S), P_1^E(S, K, r, T, \sigma)\}N(d_3) \\
P_2^A(S, K, r, T, \sigma) = P_2^E(S, Ke^{rT}, r, T, \sigma)N(-d_3) + \max \left\{ \ln \left( \frac{K}{S} \right), P_2^E(S, K, r, T, \sigma) \right\} N(d_3) \\
P_3^A(S, K, r, T, \sigma) = P_3^E(S, Ke^{rT}, r, T, \sigma)N(-d_3) + \max \left\{ S \ln \left( \frac{K}{S} \right), P_3^E(S, K, r, T, \sigma) \right\} N(d_3)
\]

where

\[
d_1 = \frac{\ln \left( \frac{S}{K} \right) + (r + \frac{1}{2}\sigma^2)T}{\sigma \sqrt{T}}, \quad d_2 = \frac{\ln \left( \frac{S}{K} \right) + (r - \frac{1}{2}\sigma^2)T}{\sigma \sqrt{T}}, \quad d_3 = \frac{\ln \left( \frac{S}{K} \right) - \frac{1}{2}\sigma^2T}{\sigma \sqrt{T}}, \\
\eta(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2} \text{ and } N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}} du.
\]

3. Comparisons of Various BSM Formulas

On almost all stock exchanges, the prices and settlement of various options are as per the plain vanilla payoff function. This is in the center of all other types of options. The options with different payoff functions are normally used in OTC market. So it is very much logical to compare any option formula with the plain vanilla option. Therefore, in this section, we compare Paul Wilmott’s option formula for log payoff function and option formula for ML-payoff function with the option formula for the plain vanilla payoff function. Throughout this section, we fix the current asset price \( S_0 = 100 \), the maturity time \( T = 0.5 \) and the risk free interest rate \( r = 0.08 \); using these, we draw the graphs of call/put option values verses volatility and striking price as well as payoff versus striking prices and asset prices.
Plain Vanilla Call Option ($C_1^E$)
Log Call Option ($C_2^E$)
Modified Log Call Option ($C_3^E$)

Plain Vanilla Put Option ($P_1^E$)
Log European Put Option ($P_2^E$)
Modified Log Put Option ($P_3^E$)

Plain Vanilla Put Option ($P_1^A$)
Log Put Option ($P_2^A$)
Modified Log Put Option ($P_3^A$)
The following table is based on the analysis of the above graphs.

<table>
<thead>
<tr>
<th>Graph No.</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 &amp; 2</td>
<td>modified log call option is much closer to the plain vanilla call option compare to the log call option. Moreover, the log call option values are near to zero.</td>
</tr>
<tr>
<td>3 &amp; 4</td>
<td>modified log put option is much closer to the plain vanilla put option compare to the log put option. Moreover, the log put option values are near to zero.</td>
</tr>
<tr>
<td>5 &amp; 6</td>
<td>modified log American put option is closer to the plain vanilla American put option compare to the American log put option. Moreover, the American log put option values are near to zero.</td>
</tr>
<tr>
<td>7 &amp; 8</td>
<td>modified log call payoff function is much closer to plain vanilla call payoff function compare to the log call payoff function. Moreover, the log call payoff function values are near to zero.</td>
</tr>
<tr>
<td>9 &amp; 10</td>
<td>modified log put payoff function is much closer to plain vanilla put payoff function compare to the log put payoff function. Moreover, the log put payoff function values are near to zero.</td>
</tr>
</tbody>
</table>
4. Conclusion

The plain vanilla option is the most used one in financial market. The very first BSM formula was derived for this option. Then there are many exotic options also. As explained earlier, all of the other options are compared directly or indirectly with plain vanilla. We also compare our modified log option with the plain vanilla option.

Our first conclusion is the following. From the ten graphs above, we can see that the option values and payoff values for the log option are strictly less than unity. Sometimes, they are even less than the transaction costs (see [2, Table 4.4]). So this BSM formula does not seem to be practically useful in the financial market. On the other hand, our modified log option contract is very close to the plain vanilla.

Our second important conclusion is that as compared to the European plain vanilla, the writer is more beneficial to enter into a call option using the modified log payoff whereas the holder is more beneficial to enter into a put option using the same. Moreover, American modified log put option is beneficial for both the traders as compared to American put option.

References