Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 9 (2017), pp. 6081–6092

http://www.ripublication.com/gjpam.htm

Interval Valued Anti Fuzzy Ideals of Γ-Near-Rings

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Abstract

In this paper the notion of residue class of Γ-near-rings and interval valued anti fuzzy ideals of Γ-near-rings is introduced. Homomorphism and endomorphism of interval-valued anti fuzzy ideal in Γ-near-rings is discussed. Also union of interval valued Γ-near rings, image and pre-image of interval valued Γ-near-rings with respect to anti fuzzy ideals are studied.

AMS subject classification: 16Y30, 03E72.
Keywords: Γ-near-ring, fuzzy Γ-near-ring, residue class of Γ-near-rings, interval valued anti fuzzy ideal of a Γ-near-ring.

1. Introduction

Zadeh [19] introduced the concept of fuzzy set in 1965 and after that he also introduced the notion of interval valued fuzzy subset [18] (in short i-v fuzzy subset) in 1975, where the values of membership functions are intervals of numbers instead of a single number as in fuzzy set. In [2] Biswas discussed the concept of anti fuzzy subgroups. T. Nagaiah et al. [10, 11] initiated the notion of PO-Gamma semigroups with respect to fuzzy ideals and anti fuzzy ideals. In [3], R. Biswas defined interval-valued fuzzy subgroups of the same nature of Rosenfelds's fuzzy subgroups. A comprehensive review of theory of fuzzy ideals of near-rings and anti fuzzy ideal of near-rings can be found in [5, 6, 8]. K. Murugalingam and K. Arjunan [9] introduced interval valued fuzzy subsemirings of semirings.

Y. B. Jun and K. H. Kim [7] discussed interval-valued R-subgroups in terms of near-rings. Davvaz [4] introduced fuzzy ideals of near-rings with interval-valued membership functions. N. Thillaigovindan et al. [16, 17] have studied interval valued fuzzy ideals and anti fuzzy ideals of near-rings. Abou-Zaid[1] proposed the concept of fuzzy sub near-rings and ideals. T. Srinivas et al. [14, 15] studied the notion of anti fuzzy ideals of Γ-near-rings and anti fuzzy near-algebra over anti fuzzy fields. T. Nagaiah et al. [12, 13] proposed interval-valued fuzzy ideals of Γ-near-rings. In this paper, we have introduced the notion of interval valued anti fuzzy ideals of Γ-near-rings. We have characterized and investigated some related properties of interval valued anti fuzzy ideals of Γ-near-rings (left). Also, we studied homomorphism and endomorphism of interval-valued anti fuzzy ideal in Γ-near-rings (left). Throughout this paper we will use the word "near-ring" to mean "left near-ring".

2. Preliminaries

For the sake of continuity we recall some basic definitions.

Definition 2.1. Let I be an ideal of a Γ-near-ring N. For each $x + I$, $y + I$ in the factor group $\frac{N}{I}$ and $\alpha \in \Gamma$, we define $(x + I) + (y + I) = (x + y) + I$ and $(x + I)\alpha(y + I) = (x\alpha y) + I$. Then $\frac{N}{I}$ is a Γ-near-ring which we call the residue class Γ-near-ring of N with respect to I.

Definition 2.2. Let R and S be Γ-near rings. A map $f : R \to S$ is called a Γ-near-ring homomorphism if $f(x + y) = f(x) + f(y)$ and $f(x\alpha y) = f(x)\alpha f(y)$, for all $x, y \in R; \alpha \in \Gamma$.

Notation 2.3. [16] An interval-valued number $\widetilde{a}$ on [0, 1] is a closed subinterval of [0, 1], that is $\widetilde{a} = [a^-, a^+]$ such that $0 \le a^- \le a^+ \le 1$ where a^- and a^+ are lower and upper limits of $\widetilde{a}$ respectively. The set of all closed sub intervals of [0, 1] is denoted by D[0, 1]. In this notation $\widetilde{0} = [0^-, 0^+]$ and $\widetilde{1} = [1^-, 1^+]$. We also identify the interval $[a, a]$ by the number $a \in [0, 1]$. For any two interval numbers $\widetilde{a} = [a^-, a^+]$

and $\widetilde{b} = [b^-, b^+]$ on [0, 1], we define

(i) $\widetilde{a} \le \widetilde{b} \Leftrightarrow a^- \le b^-$ and $a^+ \le b^+$

(ii) $\widetilde{a} = \widetilde{b} \Leftrightarrow a^- = b^-$ and $a^+ = b^+$.

(iii) $\widetilde{a} < \widetilde{b} \Leftrightarrow \widetilde{a} \le \widetilde{b}$ and $\widetilde{a} \ne \widetilde{b}$

(iv) $k\widetilde{a} = [ka^-, ka^+]$, for $0 \le k \le 1$.

Definition 2.4. [16] A mapping $\overset{i}{\min} : D[0, 1] \times D[0, 1] \to D[0, 1]$ defined by $\overset{i}{\min}(\widetilde{a}, \widetilde{b}) = [\min(a^-, b^-), \min(a^+, b^+)]$ for all $\widetilde{a}, \widetilde{b} \in D[0, 1]$ is called an interval min-norm. A mapping $\overset{i}{\max} : D[0, 1] \times D[0, 1] \to D[0, 1]$ defined by $\overset{i}{\max}(\widetilde{a}, \widetilde{b}) = [\max(a^-, b^-), \max(a^+, b^+)]$ for all $\widetilde{a}, \widetilde{b} \in D[0, 1]$ is called interval max-norm.

Let $\overset{i}{\min}$ and $\overset{i}{\max}$ be the interval-valued min-norm and interval-valued max-norm on $D[0, 1]$ respectively. Then the following are true:

(1) $\overset{i}{\min}\{\widetilde{a}, \widetilde{a}\} = \widetilde{a}$ and $\overset{i}{\max}\{\widetilde{a}, \widetilde{a}\} = \widetilde{a}\ \forall\, \widetilde{a} \in D[0, 1]$.

(2) $\overset{i}{\min}\{\widetilde{a}, \widetilde{b}\} = \overset{i}{\min}\{\widetilde{b}, \widetilde{a}\}$ and $\overset{i}{\max}\{\widetilde{a}, \widetilde{b}\} = \overset{i}{\max}\{\widetilde{b}, \widetilde{a}\}\ \forall\, \widetilde{a}, \widetilde{b} \in D[0, 1]$.

(3) If $\forall\, \widetilde{a}, \widetilde{b}, \widetilde{c} \in D[0, 1]$, $\widetilde{a} \ge \widetilde{b}$, then $\overset{i}{\min}\{\widetilde{a}, \widetilde{c}\} \ge \overset{i}{\min}\{\widetilde{b}, \widetilde{c}\}$ and $\overset{i}{\max}\{\widetilde{a}, \widetilde{c}\} \le \overset{i}{\max}\{\widetilde{b}, \widetilde{c}\}$.

Definition 2.5. Let $f : X \to Y$ be a function. For a fuzzy set μ in Y, we define $f^{-1}(\mu)(x) = \mu(f(x))$ for every $x \in X$. For a fuzzy set λ in X, $f(\lambda)$ is defined by

$$(f(\lambda))(y) = \begin{cases} \sup \lambda(x) & \text{if } f(x) = y,\ x \in X \\ 0 & \text{if there is no such } x \end{cases}$$

where $y \in Y$.

3. Interval-valued anti fuzzy ideals

Definition 3.1. An interval valued fuzzy subset $\bar{\mu}$ of a Γ-near-ring N is called Interval valued anti fuzzy sub Γ-near-ring of N if

$$(i)\quad \bar{\mu}(x - y) \le \overset{i}{\max}\{\bar{\mu}(x), \bar{\mu}(y)\}$$
$$(ii)\quad \bar{\mu}(x\alpha y) \le \overset{i}{\max}\{\bar{\mu}(x), \bar{\mu}(y)\} \forall\, x, y \in N$$

Definition 3.2. An interval valued fuzzy subset $\bar{\mu}$ of a Γ-near-ring N is called an interval valued anti fuzzy ideal of N, if $\bar{\mu}$ is an interval valued fuzzy sub Γ-near-ring of N and

$$(iii)\quad \bar{\mu}(y+x-y) \leq \bar{\mu}(x)$$
$$(iv)\quad \bar{\mu}(x\alpha y) \leq \bar{\mu}(y)$$
$$(v)\quad \bar{\mu}((x+z)\alpha y - x\alpha y) \leq \bar{\mu}(z),\ \forall x, y, z \in N, \alpha \in \Gamma$$

Note that $\bar{\mu}$ is an interval valued anti fuzzy left ideal of N if it satisfies (i), (ii), (iii) and (iv), $\bar{\mu}$ is an interval valued anti fuzzy right ideal of N if it satisfies (i), (ii), (iii) and (v).

Theorem 3.3. Let N be a Γ-near-ring and $\{\widetilde{\mu}_i\ i \in I\}$ a non-empty family of subsets of N. If $\{\widetilde{\mu}_i\ i \in I\}$ is an interval valued anti fuzzy ideal of N then $\bigcup_{i\in I} \widetilde{\mu}_i$ is an interval valued anti fuzzy ideal of N.

Proof. Let $\{\widetilde{\mu}_i\ i \in I\}$ be an interval valued anti fuzzy ideal of N. Let $x, y, z \in N$ and $\alpha \in \Gamma$. Then we have

$$\begin{aligned}(\bigcup_{i\in I}\widetilde{\mu}_i)(x-y) &= \overset{i}{\sup}\{\widetilde{\mu}_i(x-y)\}_{i\in I} \\ &\leq \overset{i}{\sup}\{\overset{i}{\max}\{\widetilde{\mu}_i(x), \widetilde{\mu}_i(y)\}\}_{i\in I} \\ &= \overset{i}{\max}\{\overset{i}{\sup}\{\widetilde{\mu}_i(x)\}_{i\in I}, \overset{i}{\sup}\{\widetilde{\mu}_i(y)\}_{i\in I}\} \\ &= \overset{i}{\max}\{(\bigcup_{i\in I}\widetilde{\mu}_i)(x), (\bigcup_{i\in I}\widetilde{\mu}_i)(y)\}\end{aligned}$$

$$\begin{aligned}(\bigcup_{i\in I}\widetilde{\mu}_i)(x\alpha y) &= \overset{i}{\sup}\{\widetilde{\mu}_i(x\alpha y)\}_{i\in I} \\ &\leq \overset{i}{\sup}\{\overset{i}{\max}\{\widetilde{\mu}_i(x), \widetilde{\mu}_i(y)\}\}_{i\in I} \\ &= \overset{i}{\max}\{\overset{i}{\sup}\{\widetilde{\mu}_i(x)\}_{i\in I}, \overset{i}{\sup}\{\widetilde{\mu}_i(y)\}_{i\in I}\} \\ &= \overset{i}{\max}\{(\bigcup_{i\in I}\widetilde{\mu}_i)(x), (\bigcup_{i\in I}\widetilde{\mu}_i)(y))\}.\end{aligned}$$

$$\begin{aligned}(\bigcup_{i\in I}\widetilde{\mu}_i)(y+x-y) &= \overset{i}{\sup}\{\widetilde{\mu}_i(y+x-y)\}_{i\in I} \\ &\leq \overset{i}{\sup}\{\widetilde{\mu}_i(x)\}_{i\in I} = (\bigcup_{i\in I}\widetilde{\mu}_i)(y)\end{aligned}$$

$$\begin{aligned}(\bigcup_{i\in I}\widetilde{\mu}_i)(x\alpha y) &= \overset{i}{\sup}\{\widetilde{\mu}_i(x\alpha y)\}_{i\in I} \\ &\leq \overset{i}{\sup}\{\widetilde{\mu}_i(y)\}_{i\in I} = (\bigcup_{i\in I}\widetilde{\mu}_i)(y)\end{aligned}$$

$$(\bigcup_{i\in I}\widetilde{\mu_i})((x+z)\alpha y - x\alpha y) = \overset{i}{\sup}\{\widetilde{\mu_i}((x+z)\alpha y - x\alpha y)\}_{i\in I}$$
$$\leq \overset{i}{\sup}\{\widetilde{\mu_i}(z)\}_{i\in I} = (\bigcup_{i\in I}\widetilde{\mu_i})(z).$$

Therefore $\bigcup_{i\in I}\widetilde{\mu_i}$ is an interval valued anti fuzzy ideal of a Γ-near-ring N. ■

Theorem 3.4. Let R and S be two Γ-near-rings and $f : R \to S$ be a homomorphism. If $\widetilde{\nu}$ is an interval valued anti fuzzy ideal of a near-ring S then $f^{-1}(\widetilde{\nu})$ is an interval valued anti fuzzy left (resp. right) ideal of R.

Proof. Let $\widetilde{\nu}$ be an interval valued anti fuzzy ideal of S. Let $x, y, z \in R$ and $\alpha \in \Gamma$. Then

$$f^{-1}(\widetilde{\nu})(x-y) = \widetilde{\nu}(f(x-y)) = \widetilde{\nu}(f(x) - f(y)))$$
$$\leq \overset{i}{\max}\{\widetilde{\nu}(f(x)), \widetilde{\nu}(f(y))\} = \overset{i}{\max}\{f^{-1}(\widetilde{\nu}(x)), f^{-1}(\widetilde{\nu}(y))\}.$$

$$f^{-1}(\widetilde{\nu})(x\alpha y) = \widetilde{\nu}(f(x\alpha y)) = \widetilde{\nu}(f(x)\alpha f(y)))$$
$$\leq \overset{i}{\max}\{\widetilde{\nu}(f(x)), \widetilde{\nu}(f(y))\} = \overset{i}{\max}\{f^{-1}(\widetilde{\nu}(x), f^{-1}(\widetilde{\nu}(y)\}$$

$$f^{-1}(\widetilde{\nu})(y+x-y) = \widetilde{\nu}(f(y+x-y)) = \widetilde{\nu}(f(y) + f(x) - f(y)))$$
$$\leq \widetilde{\nu}(f(x)) = f^{-1}(\widetilde{\nu}(x))$$

$$f^{-1}(\widetilde{\nu})(x\alpha y) = \widetilde{\nu}(f(x\alpha y)) = \widetilde{\nu}(f(x)\alpha f(y))$$
$$\leq \widetilde{\nu}(f(y)) = f^{-1}(\widetilde{\nu}(y)).$$

Now the right ideal case is proved as follows.

$$f^{-1}(\widetilde{\nu})((x+z)\alpha y - x\alpha y) = \widetilde{\nu}(f((x+z)\alpha y - x\alpha y))$$
$$= \widetilde{\nu}((f(x) + f(z))\alpha f(y) - f(x)\alpha f(y)))$$
$$\leq \widetilde{\nu}(f(z)) = f^{-1}(\widetilde{\nu}(z))$$

Hence $f^{-1}(\widetilde{\nu})$ is an interval valued anti fuzzy left (resp. right) ideal of a Γ-near-ring R. ■

The converse of the theorem 3.4 does not hold directly. We can state the converse of this theorem by strengthening the condition on f as follows.

Theorem 3.5. Let $f : R \to S$ be an onto homomorphism of Γ-near-rings R and S. Let $\widetilde{\nu}$ be an interval valued fuzzy subset of S. If $f^{-1}(\widetilde{\nu})$ is an interval valued anti fuzzy left (resp. right) ideal of R, then $\widetilde{\nu}$ is an interval valued anti fuzzy left (resp. right) ideal of S.

Proof. Let $x, y, z \in S$ and $\alpha \in \Gamma$. Then $f(a) = x$, $f(b) = y$ and $f(c) = z$ for some $a, b, c \in R$. We have

$$\begin{aligned}
\widetilde{\nu}(x-y) &= \widetilde{\nu}(f(a)-f(b)) = \widetilde{\nu}(f(a-b)) = f^{-1}(\widetilde{\nu})(a-b)) \\
&\leq \overset{i}{\max}\{f^{-1}(\widetilde{\nu})(a), f^{-1}(\widetilde{\nu})(b)\} \\
&= \overset{i}{\max}\{\widetilde{\nu}(f(a)), (\widetilde{\nu}(f(b))\} = \overset{i}{\max}\{\widetilde{\nu}(x), \widetilde{\nu}(y)\} \\
\widetilde{\nu}(x\alpha y) &= \widetilde{\nu}(f(a)\alpha f(b)) \qquad (3.1) \\
&= f^{-1}(\widetilde{\nu})(a\alpha b)) \leq \overset{i}{\max}\{f^{-1}(\widetilde{\nu})(a), f^{-1}(\widetilde{\nu})(b))\} \\
&= \overset{i}{\max}\{\widetilde{\nu}(f(a)), \widetilde{\nu}(f(b))\} = \overset{i}{\max}\{\widetilde{\nu}(x), \widetilde{\nu}(y)\}
\end{aligned}$$

$$\begin{aligned}
\widetilde{\nu}(y+x-y) &= \widetilde{\nu}(f(b)+f(a)-f(b))) = \widetilde{\nu}(f(b+a-b)) = f^{-1}(\widetilde{\nu})(b+a-b)) \\
&\leq f^{-1}(\widetilde{\nu})(a)) = \widetilde{\nu}(f(a)) = \widetilde{\nu}(x) \\
\widetilde{\nu}(x\alpha y) &= \widetilde{\nu}(f(a)\alpha f(b)) = \widetilde{\nu}(f(a\alpha b)) \\
&= f^{-1}(\widetilde{\nu})(a\alpha b) \leq f^{-1}(\widetilde{\nu})(b) = \widetilde{\nu}(f(b)) = \widetilde{\nu}(y)
\end{aligned}$$

Now the right ideal case is proved as follows.

$$\begin{aligned}
\widetilde{\nu}((x+z)\alpha y - x\alpha y) &= \widetilde{\nu}((f(a)+f(c))\alpha f(b) - f(a)\alpha f(b))) \\
&= \widetilde{\nu}(f((a+c)\alpha b - a\alpha b)) = f^{-1}(\widetilde{\nu})((a+c)\alpha b - a\alpha b)) \\
&\leq f^{-1}(\widetilde{\nu})(c) = \widetilde{\nu}(f(c)) = \widetilde{\nu}(z).
\end{aligned}$$

Hence $\widetilde{\nu}$ is an interval valued anti fuzzy left (resp. right.) ideal of a Γ-near-ring S. ■

Theorem 3.6. Let $\widetilde{\mu}$ be an interval valued fuzzy subset of a Γ-near ring N. Then $\widetilde{\mu} = [\mu^-, \mu^+]$ is an interval valued anti fuzzy left (resp. right) ideal of a Γ-near-ring N if and only if μ^-, μ^+ are anti fuzzy left (resp. right) ideals of N.

Proof. Let $\widetilde{\mu}$ be an interval valued anti fuzzy left (resp. right) ideal of a Γ-near-ring N. For any $x, y, z \in N$ and $\alpha \in \Gamma$, then we have

$$\begin{aligned}
[\mu^-(x-y), \mu^+(x-y)] &= \widetilde{\mu}(x-y) \\
&\leq \overset{i}{\max}\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \\
&= [\max\{\mu^-(x), \mu^-(y)\}, \max\{\mu^+(x), \mu^+(y)\}].
\end{aligned}$$

It follows that $\mu^-(x-y) \leq \max\{\mu^-(x), \mu^-(y)\}$ and $\mu^+(x-y) \leq \max\{\mu^+(x), \mu^+(y)\}$.

$$\begin{aligned}
[\mu^-(x\alpha y), \mu^+(x\alpha y)] &= \widetilde{\mu}(x\alpha y) \\
&\leq \overset{i}{\max}\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \\
&= [\max\{\mu^-(x), \mu^-(y)\}, \max\{\mu^+(x), \mu^+(y)\}].
\end{aligned}$$

It follows that $\mu^-(x\alpha y) \leq \max\{\mu^-(x), \mu^-(y)\}$ and $\mu^+(x\alpha y) \leq \max\{\mu^+(x), \mu^+(y)\}$.

$$\begin{aligned}[\mu^-(y+x-y), \mu^+(y+x-y)] &= \widetilde{\mu}(y+x-y) \\ &\leq \widetilde{\mu}(x) \\ &= [\mu^-(x), \mu^+(x)].\end{aligned}$$

It follows that $\mu^-(y+x-y) \leq \mu^-(x)$ and $\mu^+(y+x-y) \leq \mu^+(x)$. Also

$$\begin{aligned}[\mu^-(x\alpha y), \mu^+(x\alpha y)] &= \widetilde{\mu}(x\alpha y) \\ &\leq \widetilde{\mu}(y) \\ &= [\mu^-(y), \mu^+(y)].\end{aligned}$$

It follows that $\mu^-(x\alpha y) \leq \mu^-(y)$ and $\mu^+(x\alpha y) \leq \mu^+(y)$.

$$\begin{aligned}[\mu^-((x+z)\alpha y - x\alpha y), \mu^+((x+z)\alpha y - x\alpha y)] &= \widetilde{\mu}((x+z)\alpha y - x\alpha y)) \\ &\leq \widetilde{\mu}(z) \\ &= [\mu^-(z), \mu^+(z)].\end{aligned}$$

It follows that $\mu^-((x+z)\alpha y - x\alpha y) \leq \mu^-(z)$ and $\mu^+((x+z)\alpha y - x\alpha y) \leq \mu^+(z)$.
Hence μ^- and μ^+ are anti fuzzy left (resp. right) ideals of N.
Conversely suppose that μ^-, μ^+ are anti fuzzy left(resp. right) ideals of N. Let $x, y, z \in N$ and $\alpha \in \Gamma$. Then
$\widetilde{\mu}(x-y) = [\mu^-(x-y), \mu^+(x-y)] \leq [\max\{\mu^-(x), \mu^-(y)\}, \max\{\mu^+(x), \mu^+(y)\}]$
$= \overset{i}{\max}\{[\mu^-(x), \mu^+(x)], [\mu^-(y), \mu^+(y)]\} = \overset{i}{\max}\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}$,

$$\begin{aligned}\widetilde{\mu}(x\alpha y) &= [\mu^-(x\alpha y), \mu^+(x\alpha y)] \\ &\leq [\max\{\mu^-(x), \mu^-(y)\}, \max\{\mu^+(x), \mu^+(y)\}] \\ &= \overset{i}{\max}\{[\mu^-(x), \mu^+(x)], [\mu^-(y), \mu^+(y)]\} = \overset{i}{\max}\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}\end{aligned}$$

$$\begin{aligned}\widetilde{\mu}(y+x-y) &= [\mu^-(y+x-y), \mu^+(y+x-y)] \\ &\leq [\mu^-(x), \mu^+(x)] = \widetilde{\mu}(x)\end{aligned}$$

$\widetilde{\mu}(x\alpha y) = [\mu^-(x\alpha y), \mu^+(x\alpha y)] \leq [\mu^-(y), \mu^+(y)] = \widetilde{\mu}(y)$ and
$\widetilde{\mu}((x+z)\alpha y - x\alpha y) = [\mu^-((x+z)\alpha y - x\alpha y), \mu^+((x+z)\alpha y - x\alpha y)] \leq [\mu^-(z), \mu^+(z)] = \widetilde{\mu}(z)$.
Hence $\widetilde{\mu}$ is an interval valued anti fuzzy left (resp. right) ideal of a Γ-near-ring N. ■

Theorem 3.7. Let N be a Γ-near-ring. Let f be an endomorphism of N. If $\widetilde{\mu}$ is an interval valued anti fuzzy ideal of N, then so is $\widetilde{\mu}^f$.

Proof. Let $\widetilde{\mu}$ be an interval valued anti fuzzy ideal of N. Let $x, y, z \in N$ and $\alpha \in \Gamma$.

Then

$$\begin{aligned}\widetilde{\mu}^f(x-y) &= \widetilde{\mu}(f(x-y))\\ &= \widetilde{\mu}(f(x)-f(y))\\ &\leq \overset{i}{\max}\{\widetilde{\mu}(f(x)),\widetilde{\mu}(f(y))\}\\ &= \overset{i}{\max}\{\widetilde{\mu}^f(x),\widetilde{\mu}^f(y)\}\end{aligned}$$

$$\begin{aligned}\widetilde{\mu}^f(x\alpha y) &= \widetilde{\mu}(f(x\alpha y))\\ &= \widetilde{\mu}(f(x)\alpha f(y))\\ &\leq \overset{i}{\max}\{\widetilde{\mu}(f(x)),\widetilde{\mu}(f(y))\} = \overset{i}{\max}\{\widetilde{\mu}^f(x),\widetilde{\mu}^f(y)\}\end{aligned}$$

$$\begin{aligned}\widetilde{\mu}^f(y+x-y) &= \widetilde{\mu}(f(y+x-y))\\ &= \widetilde{\mu}(f(y)+f(x)-f(y)) \leq \widetilde{\mu}(f(x)) = \widetilde{\mu}^f(x)\end{aligned}$$

$$\begin{aligned}\widetilde{\mu}^f(x\alpha y) &= \widetilde{\mu}(f(x\alpha y))\\ &= \widetilde{\mu}(f(x)\alpha f(y)) \leq \widetilde{\mu}(f(y)) = \widetilde{\mu}^f(y)\end{aligned}$$

Now the right ideal case is proved as follows.

$$\begin{aligned}\widetilde{\mu}^f((x+z)\alpha y - x\alpha y) &= \widetilde{\mu}(f(x+z)\alpha y - x\alpha y))\\ &= \widetilde{\mu}((f(x)+f(z))\alpha f(y) - f(x)\alpha f(y)))\\ &\leq \widetilde{\mu}(f(z)) = \widetilde{\mu}^f(z).\end{aligned}$$

Hence $\widetilde{\mu}^f$ is an interval valued anti fuzzy ideal of a Γ-near-ring. ■

Theorem 3.8. Let I be an ideal of a Γ-near-ring N and $\widetilde{\mu}$ be an interval valued anti fuzzy left (resp. right) ideal of N. Then the fuzzy subset $\widetilde{\mu}^\star$ of $\frac{N}{I}$ defined by

$$\widetilde{\mu}^\star(a+I) = \overset{i}{\sup_{x\in I}}\widetilde{\mu}(a+x)$$

is an interval valued anti fuzzy left (resp. right) ideal of the residue class Γ-near-ring $\frac{N}{I}$ of N with respect to I.

Proof. Clearly $\widetilde{\mu}^{\star}$ is well defined. Let $x+I, y+I \in \dfrac{N}{I}$ and $\alpha \in \Gamma$. Then we have

$$\begin{aligned}
\widetilde{\mu}^{\star}\{(x+I)-(y+I)\} &= \widetilde{\mu}^{\star}\{(x-y)+I\} \\
&= \sup_{z\in I}^{i} \widetilde{\mu}[(x-y)+z] \\
&= \sup_{z=u-v\in I}^{i} \widetilde{\mu}[(x-y)+(u-v)] \\
&= \sup_{u,v\in I}^{i} \widetilde{\mu}[(x+u)-(y+v)] \\
&\leq \max^{i}\{\sup_{u,v\in I}^{i} \{\widetilde{\mu}(x+u), \widetilde{\mu}(y+v)\}\} \\
&= \max^{i}\{\sup_{u\in I}^{i} \widetilde{\mu}(x+u), \sup_{v\in I}^{i} \widetilde{\mu}(y+v)\} \\
&= \max^{i}\{\widetilde{\mu}(x+I), \widetilde{\mu}(y+I)\}
\end{aligned}$$

$$\begin{aligned}
\widetilde{\mu}^{\star}[(y+x-y)+I] &= \widetilde{\mu}[((y+x)-y)+I] \\
&= \sup_{z\in I}^{i} \widetilde{\mu}[((y+x)-y)+z] \\
&= \sup_{z=u+v-w\in I}^{i} \widetilde{\mu}[(y+x)-y+(u+v-w)] \\
&= \sup_{u,v,w\in I}^{i} \widetilde{\mu}[(y+u)+(x+v)-(y+w)] \\
&\leq \sup_{v\in I}^{i} \widetilde{\mu}(x+v) \\
&= \widetilde{\mu}^{\star}(x+I). \\
\widetilde{\mu}^{\star}(x\alpha y+I) &= \widetilde{\mu}((x+I)\alpha(y+I)) \\
&= \sup_{t\in I}^{i}\{\widetilde{\mu}[(x+t)\alpha(y+t)]\} \\
&\leq \sup_{t\in I}^{i}\{\max^{i}\{\widetilde{\mu}(x+t), \widetilde{\mu}(y+t)\}\} \\
&= \max^{i}\{\sup_{t\in I}^{i} \widetilde{\mu}(x+t), \sup_{t\in I}^{i} \widetilde{\mu}(y+t)\} \\
&= \max^{i}\{\sup_{t\in I}^{i} \widetilde{\mu}(x+t), \sup_{t\in I}^{i} \widetilde{\mu}(y+t)\} \\
&= \max^{i}\{\widetilde{\mu}^{\star}(x+I), \widetilde{\mu}^{\star}(y+I)\}
\end{aligned}$$

$$\begin{aligned}\widetilde{\mu}^\star(x\alpha y + I) &= \widetilde{\mu}((x+I)\alpha(y+I))\\ &= \overset{i}{\sup_{t\in I}}\{\widetilde{\mu}\{(x+t)\alpha(y+t)\}\}\\ &\leq \overset{i}{\sup_{t\in I}}\{\overset{i}{\max}\{\widetilde{\mu}(y+t)\}\} = \widetilde{\mu}^\star(y+I)\end{aligned}$$

Now the right ideal case is proved as follows.

$$\begin{aligned}\widetilde{\mu}^\star(((x+z)\alpha y - x\alpha y) + I) &= \overset{i}{\sup_{t\in I}}\widetilde{\mu}\{((x+z)\alpha y - x\alpha y) + t)\}\\ &= \overset{i}{\sup_{t\in I}}\widetilde{\mu}\{((x+t)+(z+t))\alpha(y+t) - (x+t)\alpha(y+t)\}\\ &\leq \overset{i}{\sup_{t\in I}}\widetilde{\mu}(z+t) = \widetilde{\mu}(z+I)\end{aligned}$$

Hence $\widetilde{\mu}^\star$ is an interval valued anti fuzzy ideal of N. ■

Theorem 3.9. Let I be an ideal of a Γ-near-ring N. If $\widetilde{\mu}$ with $\widetilde{\mu}(a+I) = \mu(a)$, where $a \in N$, is an interval valued anti fuzzy left (resp. right) ideal of $\frac{N}{I}$, then μ is an interval valued anti fuzzy left (resp. right) ideal of N.

Proof. Let I be an ideal of a Γ-near-ring N and $\widetilde{\mu}$ be an interval valued anti fuzzy left ideal of $\frac{N}{I}$. Let $x, y, z \in N$ and $\alpha \in \Gamma$, then we have

$$\begin{aligned}\mu(x-y) &= \widetilde{\mu}((x-y)+I) = \widetilde{\mu}((x+I)-(y+I))\\ &\leq \overset{i}{\max}\{\widetilde{\mu}(x+I), \widetilde{\mu}(y+I)\} = \overset{i}{\max}\{\mu(x), \mu(y)\}\end{aligned}$$

$$\begin{aligned}\mu(x\alpha y) &= \widetilde{\mu}(x\alpha y + I)\\ &= \widetilde{\mu}((x+I)\alpha(y+I))\\ &\leq \overset{i}{\max}\{\widetilde{\mu}(x+I), \widetilde{\mu}(y+I)\} = \overset{i}{\max}\{\mu(x), \mu(y)\}\end{aligned}$$

$$\begin{aligned}\mu(y+x-y) &= \widetilde{\mu}((y+x-y)+I)\\ &= \widetilde{\mu}[(y+I)+(x+I)-(y+I)] \leq \widetilde{\mu}(x+I) = \mu(x)\end{aligned}$$

$$\begin{aligned}\mu(x\alpha y) &= \widetilde{\mu}(x\alpha y + I)\\ &= \widetilde{\mu}((x+I)\alpha(y+I)) \leq \widetilde{\mu}(y+I) = \mu(y)\end{aligned}$$

Next the right ideal case is proved as follows.

$$\begin{aligned}\mu[(x+z)\alpha y - x\alpha y] &= \widetilde{\mu}[(x+z)\alpha y - x\alpha y + I]\\ &= \widetilde{\mu}[((x+z)\alpha y + I) - (x\alpha y + I)]\\ &= \widetilde{\mu}[\{(x+I)+(z+I)\}\alpha(y+I) - (x+I)\alpha(y+I)]\\ &\leq \widetilde{\mu}(z+I) = \mu(z).\end{aligned}$$

Hence μ is an interval valued anti fuzzy ideal of N. ■

Acknowledgements

The authors are deeply grateful to the referee and reviewers for their valuable comments and suggestions for improving this paper.

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