Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 9 (2017), pp. 5965-5975

http://www.ripublication.com

Effect of partial slip and convective boundary condition on MHD mixed convection flow of Williamson fluid over an exponentially stretching sheet in the presence of joule heating

K. Sharada[1*] and B. Shankar[2]

Department of mathematics, Osmania University,
Hyderabad, Telangana, India.

Abstract

The present study investigates the effects of partial slip and convective boundary condition on magnetohydrodynamic mixed convection flow in the presence and absence of joule heating. The governing partial differential equations with boundary conditions are converted into the system of ordinary differential equations via similarity transformations. The reduced system of ordinary differential equations solved numerically by using the Keller-box method. The influence of different physical parameters, such as the magnetic parameter, mixed convection parameter, Williamson parameter, the slip parameter, Biot number and Prandtl number is discussed graphically and numerically for the profiles of velocity and Temperature.

Keywords; MHD, Williamson fluid, Slip effect, joule heating

1. INTRODUCTION

The study of MHD boundary layer flow over a stretching sheet has a lot of applications in industrial area and geothermal applications such as metal spinning, hot metal rolling, glass fibre production, paper production, cooling of nuclear reactors, wire drawing and petroleum production etc. Analytical solution for an incompressible boundary layer flow over a stretching sheet was first discussed by Crane [1]. After Crane [1] several researchers extended the work with the different physical conditions. Vajravelu and Rollins [2] explained heat transfer in an electrically conducting fluid over a stretching surface. The boundary layer flow over an exponentially stretching sheet was first discussed by Magyari and Keller [3]. Heat transfer over an exponentially stretching surface by Elbashbeshy [4]. Al-Odat et al [5] investigated the magnetic field effects on thermal boundary layer flow over an exponentially stretching sheet. The study of viscoelastic fluid flow over an exponentially stretching sheet is explained by Khan [6].

Williamson fluid model is one of the non-Newtonian fluids, which describes the flow of shear thinning liquid. This model was developed by Williamson [7]. Dapra and Scarpi [8] obtained perturbation solution for the flow of a Williamson fluid in a rock fracture. Peristaltic flow of Williamson fluid with different conditions was investigated by few authors [9-12]. Effect of heat transfer and chemical reactions on Williamson model for blood flow is explained by Akbar et al [13]. Steady flow of a Williamson fluid over a porous plate is discussed by Hayat et al [14]. Heat and mass transfer effects on peristaltic flow of Williamson fluid in a vertical annulus is studied by Nadeem and Akbar [15]. Effect of chemical reaction on boundary layer flow of Williamson fluid is investigated by N.A Khan et al [16]. Recently the effect of chemical reaction on MHD boundary layer flow and heat transfer of Williamson nanofluid by Krishnamurthy et al [17]. Rama Subba Reddy and Gireesha [18] described dual solutions for stagnation point flow and convective heat transfer flow of Williamson nanofluid over a stretching sheet.

Several investigators explained the effects of slip and convective boundary condition, these include that Swathi Mukhopadhyay [19] studied the effects of slip on MHD boundary layer flow with suction/blowing and thermal radiation over an exponentially stretching sheet. Peristaltic motion of a Williamson fluid through a porous medium with slip effects in a different channel is investigated by the authors [20, 21]. Effect of convective surface boundary condition on Williamson fluid flow in an asymmetric channel is discussed by Akbar [22]. Impact of MHD slip flow with convective boundary condition over a stretching sheet is studied by Gopi Chand et al [23]. Recently effects of boundary layer slip flow and convective boundary condition is explained by the authors [24-25]. Effects of Newtonian and joule heating in two dimensional flow of Williamson fluid is described by Hayat et al [26]. Gnaneswara Reddy and Venugopal Reddy [27] investigated the influence of velocity slip and joule heating on MHD peristaltic flow through a porous medium with chemical reaction.

This paper investigates the MHD mixed convection boundary layer flow of Williamson fluid with the effects of velocity slip and convective surface boundary condition over an exponentially stretching sheet with the help of Keller-box method.

2. MATHEMATICAL FORMULATION

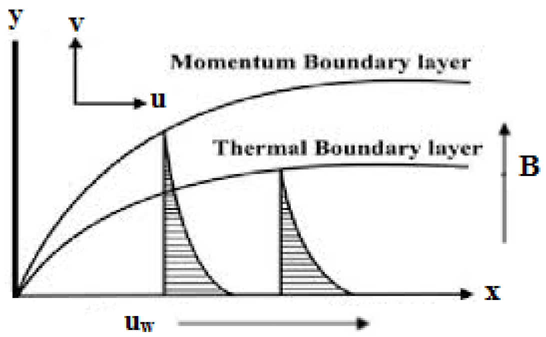

Figure 1: Physical flow model of the problem

Let us consider steady, two-dimensional, laminar, incompressible MHD mixed convection flow and heat transfer of Williamson fluid over an exponentially stretching sheet. Consider a coordinate system, such that x-axis is taken along the stretching sheet in the direction of flow and y-axis is perpendicular to it. The sheet is stretched with velocity $u_w(x) = ae^{\frac{x}{L}}$. A uniform magnetic field is applied in the direction normal to the sheet. The stress tensor of Williamson model is given by [24]

$$S = -pI + \tau \quad (1)$$

$$\tau = [\mu_\infty + \frac{\mu_0 - \mu_\infty}{1 - \Gamma\dot{\gamma}}]\dot{\gamma} \quad (2)$$

Where p is the pressure, I is the Identity vector, τ is extra stress tensor, μ_0 *and* μ_∞ is the limiting viscosity at zero shear rate and at infinite shear rate, Γ is time constant. We consider the equation (2), the case for which $\mu_\infty = 0$ *and* $\Gamma\dot{\gamma}$ and $\dot{\gamma}$ can be defined as

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i\sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\prod} \quad (3)$$

Here $\prod$ is the second invariant strain tensor. Therefore, equation 2 can be written as

$$\tau = \frac{\mu_0}{1 - \Gamma\dot{\gamma}}\dot{\gamma}$$

Or by using binomial expansion

$$\tau = \mu_0[1 + \Gamma\dot{\gamma}]\dot{\gamma} \quad (4)$$

Let u, v are the velocity components along the directions of x and y respectively. The boundary layer equations for the continuity, momentum and energy are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (5)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\sqrt{2}\nu\Gamma\left(\frac{\partial u}{\partial y}\right)\frac{\partial^2 u}{\partial y^2}+g\beta_T\,(T\,-\,T_\infty)-\frac{\sigma B^2 u}{\rho} \tag{6}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{\sigma B^2}{\rho c_p}u^2 \tag{7}$$

Subject to the boundary conditions:

$$u=u_w+N_s\nu\frac{\partial u}{\partial y},v=0,-\mathrm{k}\frac{\partial \mathrm{T}}{\partial \mathrm{y}}=\mathrm{h_f}(T-T_w)\ \ at\ y=0. \tag{8}$$

$$u=0,\ \ T\to T_\infty\ \ as\ \ y\to\infty. \tag{9}$$

where ρ is the density of the fluid, σ is the electrical conductivity, ν is the kinematic viscosity, B is the magnetic induction, α is the thermal diffusivity, β_T is the coefficient of thermal expansion, C_p is the specific heat capacity, $N_s=N_0e^{-\frac{x}{2L}}$ is the velocity slip (N_0 is the initial value of the velocity slip factor).

Introducing the similarity transformations are

$$u=ae^{\frac{x}{L}}f'(\eta),\quad \eta=\sqrt{\frac{a}{2\nu L}}e^{\frac{x}{2L}}y,\quad v=-\sqrt{\frac{a\nu}{2L}}e^{\frac{x}{2L}}[f(\eta)+\eta f'(\eta)],\ \ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}.$$

The reduced governing equations are

$$f'''+ff''-2(f')^2+\lambda_1 f''f'''-Mf'+2\lambda\theta=0 \tag{10}$$

$$(\frac{1}{Pr})\theta''+f\theta'+H(f')^2=0. \tag{11}$$

The corresponding boundary conditions are

$$\eta\to 0: f(\eta)=0, f'(\eta)=1+\lambda_v f''(\eta),\ \theta'(\eta)=-Bi(1-\theta(\eta)) \tag{12}$$

$$\eta\to\infty: f'(\eta)\to 0,\ \theta(\eta)\to 0 \tag{13}$$

Where $M=\frac{2\sigma {B_0}^2 L}{\rho a}$ is the Magnetic parameter, $\lambda=\frac{Gr}{{Re_x}^2}$ is the Mixed convection parameter, $Re_x=\frac{aL}{\nu}e^{\frac{x}{L}}$ is the Local Reynolds number, $Gr=\frac{g\beta_T(T_w-T_\infty)L^3}{\nu^2}$ is the Grashoff number, $Pr=\frac{\nu}{\alpha}$ is the Prandtl number, $\lambda_1=\Gamma\sqrt{\frac{a^3e^{\frac{3x}{L}}}{\nu L}}$ is the dimensionless Williamson parameter, $\lambda_v=N_0\sqrt{\frac{a\nu}{2L}}$ is the velocity slip parameter, $Bi=\frac{h_f}{k}\sqrt{\frac{2\nu L}{ae^{x/L}}}$ is the thermal Biot number.

The coefficient of skin friction and local Nusselt number at the stretching sheet is defined as $C_f=\frac{\tau_w}{\rho {u_w}^2}$, $Nu_x=\frac{xq_w}{k(T_w-T_\infty)}$. Here $\tau_w=\mu(\frac{\partial u}{\partial y}+\frac{\Gamma}{\sqrt{2}}(\frac{\partial u}{\partial y})^2)$ is the wall shear stress and $q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$ is the heat transfer coefficient.

Therefore $\sqrt{2Re_x}C_f = [f''(0) + \frac{\lambda_1}{2}f''^2(0)]$ and $\frac{\sqrt{2}L}{x}Nu_x Re_x^{-\frac{1}{2}} = -\theta'(0)$.

3. METHOD OF SOLUTION:

The governing equations with boundary conditions are solved numerically by using finite difference scheme known as Keller box method which is described by Cebeci and Bradshaw [27]. This method involves the following steps.

Step1: Reducing higher order ODEs (systems of ODES) in to systems of first order ODEs.

Step2: Writing the systems of first order ODEs into difference equations using central difference scheme.

Step3: Linearizing the difference equations using Newton's method and writing it in vector form.

Step4: Solving the system of equations using block elimination method.

4. NUMERICAL DISCUSSIONS:

In this paper, we discussed numerical results of $-f''(0)$ *and* $-\theta'(0)$ for various values of the parameters. Table 1 represents the comparison values of $-\theta'(0)$ by the present method and of Ishak and Rahman. For this, we used Matlab software to find the numerical solution by taking the step size as $\Delta\eta = 0.01$. The value of the parameters throughout the table 2 are M=0.5, H=0.5, λ=0.1, Bi=0.1, Pr=0.71, λ_1=0.5, λ_v=0.1 unless otherwise mentioned.

Table I: Comparison vales of $-\theta'(0)$ with Pr=0.1 and rest of the parameters are zero.

Bi	Ishak[28]	Rahman[29]	Present
0.1	0.0583	0.0584	0.0584
0.2	0.0823	0.0824	0.0826
0.4	0.1037	0.1038	0.1041
0.6	0.1135	0.1136	0.1140
0.8	0.1191	0.1193	0.1197
1.0	0.1228	0.1229	0.1234

Table II : Numerical values of $-f''(0)$ *and* $-\theta'(0)$ for M, λ, λ_1, λ_v, Bi, Pr, H.

M	λ	λ_1	λ_v	Bi	Pr	H	−f"(0)	−θ′(0)
0.1							1.3517	0.0594
0.5							1.6233	0.0626
1							1.9552	0.0607
	0.3						1.4189	0.0597
	0.6						1.1978	0.058
	1						0.9322	0.0549
		0.1					1.2243	0.0589
		0.4					1.4342	0.0597
		0.5					1.5933	0.0602
			0.2				1.223	0.0613
			0.4				0.8936	0.0626
			0.5				0.796	0.0631
				0.5			1.5325	0.1629
				1			1.5084	0.2084
				1.5			1.4975	0.23
					0.71		1.5933	0.0602
					1		1.6027	0.0618
					3		1.6233	0.0626
						0.1	1.6312	0.0742
						1	1.5464	0.0416
						2.1	1.4432	-0.0039

RESULTS AND DISCUSSION:

Figure 2-5 demonstrates the velocity profile for the various parameters in the presence & absence of the Joule heating parameter. Fig 2 depicts the velocity profile for various values of the magnetic parameter M. Due to the Lorentz force the boundary layer becomes thinner when increasing the values of M. Fig 3 presents the effect of mixed convection parameter λ on the velocity field. The fluid velocity rises when we increase the values of λ because of the buoyancy effect. Fig 4 plots the effect of Williamson parameter λ_1 for the profile of velocity. When we increase the Williamson fluid parameter, the change in boundary layer thickness is very less. Fig 5 shows the influence of λ_v on the field of velocity. We observed that the velocity profile reduced under the slip condition. Therefore, the boundary layer thickness decreases. In the presence/absence of joule heating parameter, we observed there is no much variation in the boundary layer thickness for the parameters M, λ_1 and λv. But there is some reduction in the boundary layer thickness for the mixed convection parameter λ. Similarly, fig 6 displays the temperature profile for various values of thermal Biot number Bi, which increases the temperature field. We noticed that there exist a good variation in the boundary layer region and thickness with joule heating parameter when compared with H=0. The temperature profile for various values of the Prandtl number is explained in fig7. In general Prandtl number represents momentum diffusivity to thermal diffusivity. It reduces the thermal boundary layer thickness. Influence of Prandtl number on temperature field in the presence/absence of joule heating is significant. There exists much difference in the boundary layer thickness as well as the region. Fig 8 depicts the impact of joule heating parameter H on the temperature file. The temperature found to be increased. The joule heating parameter enhances the temperature of the fluid. Thus, the boundary layer thickness increased.

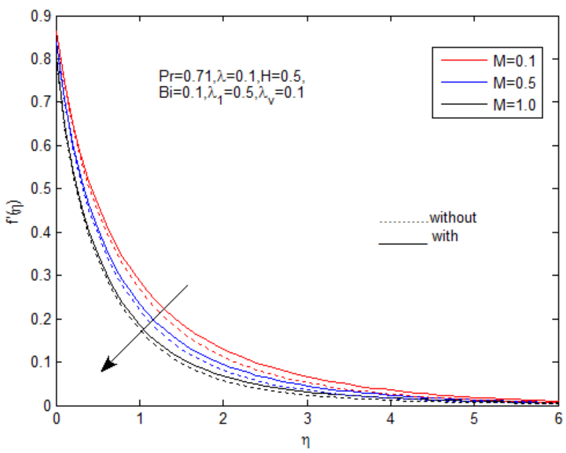

Fig2: Velocity profile for various values of M

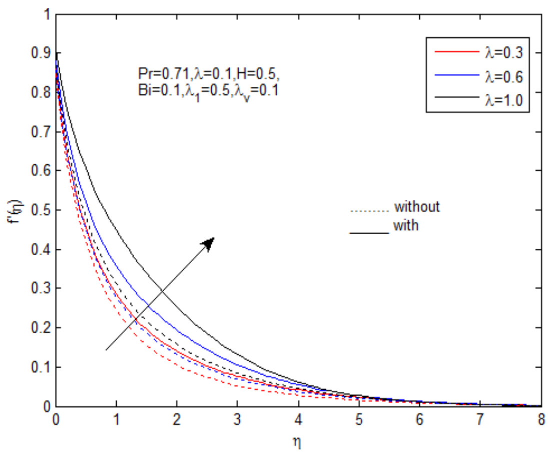

Fig3: Velocity profile for various values of λ

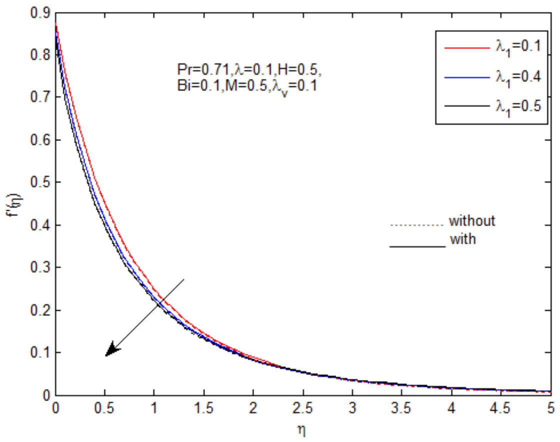

Fig4: Velocity profiles for various values of λ_1

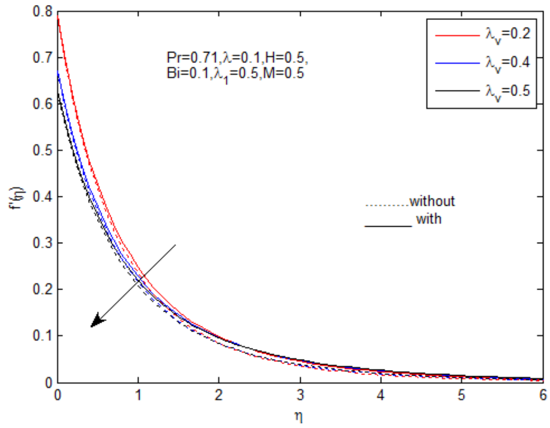

Fig5: Velocity profiles for various values of λ_v

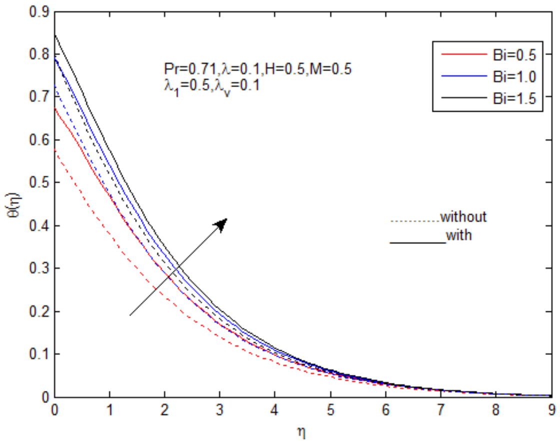

Fig 6: Temperature profile for various values of Bi

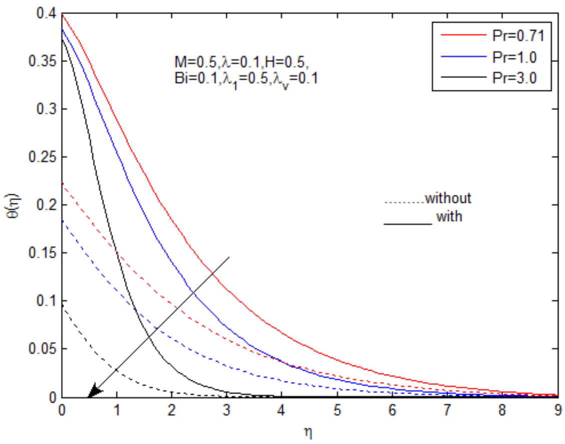

Fig7: Temperature profile for various values of Pr

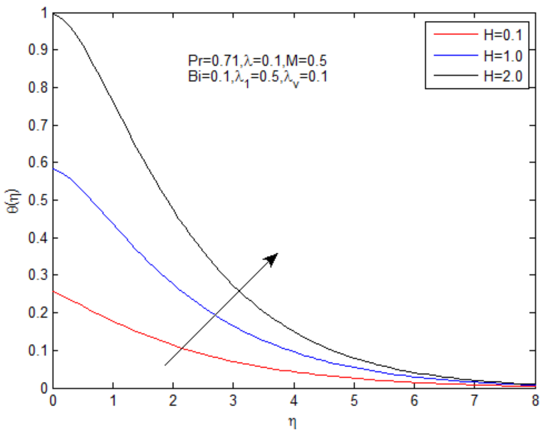

Fig 8: Temperature profile for various values of H

5. CONCLUSIONS

Following are the results of the graphical solution for MHD mixed convection flow of Williamson fluid with the effects of partial slip and convective boundary condition in the presence of joule heating.

- Velocity profile decreases for the Williamson fluid parameter λ_1.
- The velocity profile decreases for larger values of M and λ_v.
- For increasing values of λ, the velocity of the fluid increases.
- Temperature profile increases for larger values of Bi and for joule heating parameter.
- An increase in the Prandtl number Pr reduces the temperature profile.

REFERENCES

[1] L J Crane, Flow past a stretching plate, ZAMP, 21(4), 1970, pp 590–595.

[2] K Vajravelu and D Rollins, Heat transfer in an electrically conducting fluid over a stretching surface, International Journal of Non-linear Mechanics, 27(2), 1992, pp 265-277.

[3] E Magyari, B Keller, Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface, Journal of Physics D, 32(5), 1999, pp 577-585.

[4] E M A Elbashbeshy, Heat transfer over an exponentially stretching continuous surface with suction, Arch. Mech., 53(6), 2001, pp 643-651.

[5] M Q Al-Odat, R Damesh, Azab Taa, Thermal boundary layer on an exponentially stretching continuous surface in the presence of magnetic field effect, International Journal of Applied Mechanics and Engineering, 11(2), 2006, pp 289-299.

[6] Khan S K, Boundary layer viscoelastic fluid flow over an exponentially stretching sheet, International Journal of Applied Mechanics and Engineering, 11(2), 2006, pp321-335.

[7] Williamson R V, The flow of Pseudo plastic materials, Industrial and Engineering Chemistry, 21(11), 1929, pp 1108-1111.

[8] Dapra I, G Scarpi, perturbation solution for pulsatile flow of a non-Newtonian Williamson fluid in a rock fracture, International Journal of Rock Mechanics and Mining Sciences, 44, 2007, pp 271-278.

[9] C Vasudev, U R Rao, M V S Reddy and G P Rao, Peristaltic pumping of Williamson fluid through a porous medium in a horizontal channel with heat transfer, American Journal of Scientific and Industrial Research, 1(3), 2010,

pp 656-666.

[10] S Nadeem, N. S. Akbar, Numerical solutions of peristaltic flow of Williamson fluid with radially varying MHD in an endoscope, International for Numerical methods in Fluids, 66(2), pp 212-220.

[11] K Vajravelu, S Sreenadh, K Rajinikanth, Changhoon Lee, Peristaltic transport of a Williamson fluid in asymmetric channels with permeable walls, Nonlinear Analysis: Real World Applications, 13(6), 2012, pp 2804-2822.

[12] N S Akbar, T Hayat, S Nadeem, S Obaidat, Peristaltic flow of a Williamson fluid in an inclined asymmetric channel with partial slip and heat transfer, International Journal of Heat and Mass Transfer, 55, 2012, pp 1855-1862.

[13] Noreen Sher Akbar, S Nadeem, Changhoon Lee, Influence of heat transfer and chemical reactions on Williamson fluid model for blood flow through a tapered artery with a stenosis, Asian Journal of Chemistry, 24(6), 2012, pp 2433-2441.

[14] T Hayat, Umbreen Khalid, M Qasim, Steady flow of a Williamson fluid past a porous plate, Asia-Pacific Journal of Chemical Engineering, 7(2), 2012, pp 302-306.

[15] S Nadeem, Nooren Sher Akbar, Effects of heat and mass transfer peristaltic flow of Williamson fluid in a vertical annulus, Meccanica, 47(1), 2012, pp 141-151.

[16] Najeeb Alam Khan, Sidra Khan, Fatima Riaz, Boundary layer flow of Williamson fluid with chemically reactive species using scaling transformation and homotopy analysis method, Mathematical Sciences Letters, 3(3), 2014, pp 199-205.

[17] M R Krishnamurthy, B C Prasannakumara, B J Gireesha, Rama Subba Reddy Gorla, Effect of chemical reaction on MHD boundary layer flow and melting heat transfer of Williamson nanofluid in porous medium, Engineering Science and Technology, an International Journal, 19, 2016, pp 53-61.

[18] R S R Gorla, B J Gireesha, Dual solutions for stagnation point flow and convective heat transfer of a Williamson nanofluid past a stretching sheet, Heat and Mass Transfer, 52(6), pp 1-10.

[19] Swati Mukhopadhyay, Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation, Ain Shams Engineering Journal, 4, 2013, pp 485-491.

[20] S Navaneeswara Reddy, G Viswanatha Reddy, Slip effects on peristaltic motion of a Williamson fluid through a porous medium in a planar channel, Asian Journal of Current Engineering and Maths, 2, 2013, pp 53-58.

[21] B Jyothi, P Koteswara Rao, Slip effects on MHD peristaltic transport of a Williamson fluid through a porous medium in a symmetric channel, Journal of Mathematical and Computational Science, 3(5), 2013, pp 1306-1324.

[22] Noreen Sher Akbar, Non-Newtonian fluid flow in an asymmetric channel with convective surface boundary condition: A note, Journal of Power Technologies, 94(1), 2014, pp 34-41.

[23] Gopi Chand, R N Jat, Dinesh Rajotia, MHD slip flow over a stretching sheet with convective boundary condition, Acta Technica, 60(2), 2015, pp 179-191.

[24] Nor Ashikin Abu Bakar, Wan Mohd Khairy Adly Wan Zaimi, Rohana Abdul Hamid, Biliana Bidin and Anur Ishak, Boundary layer flow over a stretching sheet with a convective boundary condition and slip effect, World Applied Sciences Journal, 17, 2012, pp 49-53.

[25] Mona D, Aljoufi, Abdelhalim Ebaid, Effect of a convective boundary condition on boundary layer slip flow and heat transfer over a stretching sheet in view of the exact solution, Journal of Theoretical and Applied Mechanics, 46(4), 2016, pp 85-95.

[26] Gnaneswara Reddy Machireddy, Venugopal Reddy Kattamreddy, Impact of velocity slip and joule heating on MHD peristaltic flow through a porous medium with chemical reaction, Journal of the Nigerian Mathematical society, 35(1), 2016, pp 227-244.

[27] T Cebeci, P Bradshaw, Physical and computational aspects of convective heat transfer, Springer, 1984.

[28] A. Ishak, similarity solutions for flow and heat transfer over a permeable surface with convective boundary condition, Appl. Math. Comp, 217, 2010, pp 837-842.

[29] M. M. Rahman, Locally similar solutions for hydromagnetic and thermal slip flow boundary layers over a flat plate with variable fluid properties and convective surface boundary condition, Meccanica, 46, 2011, 1127-1143.