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Numerical Approach to Differential Equations of Fractional order Bratu-type Equations by Differential Transform Method

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Abstract

In the present paper, the Differential Transform Method (DTM) is applied to drive its solution (approximate) of the fractional Bratu-type equations. The fractional derivatives are represented in the caputo sense. The convergence and uniqueness of this method are also studied. Three examples are illustrated to prove that the presented techniques efficiency and implementation of the method and the results are compared with exact solutions.

Keywords: Fractional Bratu-type equations, Caputo fractional derivatives, Differential Transform Method, Numerical Solutions.

1. INTRODUCTION:

We know that Fractional calculus is playing vital role in the field of Mathematics. It deals with derivatives and integral of arbitrary orders. Fractional differential equations and its results used in general in many branches of mathematics and deals with Science and Engineering also. Many effective different techniques for solving its numerical and analytical solutions of FDEs have been presented **[6-8]**.

We have used notations $D^{n\alpha}$ for Jumarie type fractional derivative operator here $n \in \Re, \alpha < 1$. There are many types of fractional integral and differential operators.

Since proposed in (Zhou, 1986), there is good interest on the applications of Differential Transform Method (DTM) to solve various scientific problems. The DTM **[1-2,10-12,14-15]** is an approximation to the exact solution of the functions which are differentiable in the form of polynomials. The DTM is an alternative procedure for getting Taylor series solution of the differential equations. This method reduces the size of computational domain and is easily applicable to many problems. Large list of methods, exact, approximate and purely numerical are available for the solution of differential equations. Most of these methods are computationally intensive because they are a trial-and error in nature, or need complicated symbolic computations. The differential transformation technique is one of the numerical methods for ordinary differential equations. This method constructs a semi-analytical numerical technique that uses Taylor series for the solution of differential equations in the form of a polynomial. It is different from the high-order Taylor series method which requires symbolic computation of the necessary derivatives of the data functions. For the calculation of Bratu-type equation with the help of differential transform method with constant value of λ

$$D_t^{2\alpha} u + \lambda \exp u = 0, \quad 0 < \alpha \le 1, \qquad 0 < x < 1, \quad u(0) = 0, u^{\alpha}(0) = 0, \tag{1.1}$$

These days, a massive quantity of kinds of literature advanced concerning with FDEs in nonlinear dynamics **[3-5].** On the grounds that maximum fractional differential equations do now not have genuine analytic answers, approximation, and numerical techniques, consequently, they are used substantially. lately, the Adomian decomposition technique, Homotopy analysis method and new iterative method were used for decision a large range of problems **[9,13,16].**

2. CAPUTO FRACTIONAL DERIVATIVE:

On this paper, there are many definitions of a fractional derivative of order $\alpha > 0$. The two most normally used definitions like as the Riemann–Liouville and Caputo. Each definition makes use of Riemann–Liouville fractional integration and derivatives of the whole order.

Let $f : R \to R, x \to f(x)$, denote a continuous function, and let the partition $h > 0$ in the interval $[0,1]$. Through the fractional Riemann Liouville integral

$$_0 I_x^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_{x_0}^{x} (x-t)^{\alpha-1} f(t) dt, \alpha > 0, \tag{2.1}$$

the modified Riemann-Liouville derivative is defined as

$$ {}_0D_x^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_0^x (x-t)^{n-\alpha} (f(t) - f(0)) dt, \tag{2.2} $$

$$ where, x \in [0,1] \quad n-1 \le \alpha < n, n \ge 1. $$

The difference between the two definitions is in the order of evaluation. Riemann–Liouville fractional integration of order α is outlined as

$$ J_{x_0}{}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_{x_0}^{x} (x-t)^{\alpha-1} f(t) dt, \quad \alpha > 0, x > 0, \tag{2.3} $$

$$ D_{x_0}{}^{\alpha} f(x) = \frac{d^m}{dx^m} \left[J^{m-\alpha} f(x) \right] $$

$$ D_{*x_0}{}^{\alpha} f(x) = J^{m-\alpha} \left[\frac{d^m}{dx^m} f(x) \right], \tag{2.4} $$

where $m-1 < \alpha \le m$ and m ∈ N. For now, the Caputo fractional by-product can be denoted by using D_*^{α} to maintain a clear difference with the Riemann–Liouville fractional derivative. The Caputo fractional derivative is also defined as follows

$$ D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{f^n(\beta)}{(t-\beta)^{\alpha-n+1}} d\beta, \quad (n-1 < \alpha \le n), \quad n \in N \tag{2.5} $$

Suppose that in DTM of kth derivative of function f(t).

$$ F_{\alpha}(k) = \frac{1}{\Gamma(\alpha k + 1)} \left[\left(D_{x_0}{}^{\alpha} \right)^k f(t) \right]_{t=t_0} \tag{2.6} $$

and the inverse of DTM as follows:

$$ f(t) = \sum_{i=0}^{\infty} F_{\alpha}(k)(t-t_0)^{\alpha k} \tag{2.7} $$

The following properties of DTM, Let f(t), u(t), v(t) change into $F_{\alpha}(k), U_{\alpha}(k), V_{\alpha}(k)$

$$ f(t) = u(t) \pm v(t) \qquad F_{\alpha}(k) = U_{\alpha}(k) \pm V_{\alpha}(k) $$

$$ f(t) = \beta u(t) \qquad F_{\alpha}(k) = \beta U_{\alpha}(k) $$

$$ f(t) = u(t).v(t) \quad F_{\alpha}(k) = \sum_{n=0}^{k} V_{\alpha}(n) U_{\alpha}(k-n) $$

$f(t)=\frac{d^m u(t)}{dt^m} \quad F_\alpha(k)=(k+1)(k+2)......(k+m)U_\alpha(k+m)$

$f(t)=(1+t)^m \quad F_\alpha(k)=\frac{1}{k!}m(m-1)...(m-k+1)$

$f(t)=\exp(at) \quad F_\alpha(k)=\frac{a^k}{k!}$

$f(t)=Sin(at+\lambda) \quad F_\alpha(k)=\frac{a^k}{k!}Sin\left(\frac{\pi k}{2}+\lambda\right)$

3. MITTAG-LEFFLER FUNCTION:

Mittag-Leffler Function helps to find out the solution of distinct equations like as fractional order differential, integral equations. The Mittag-Leffler function was introduced by Gosta Mittag-Leffler in 1903.

From the Jumarie definition of fractional derivative we have ${}_a{}^J D_x{}^\alpha[C]=0$.with the help of order α ($0\le\alpha<1$) of Jumarie derivative with the initial point $f(x)=x^{n\alpha}$

$${}_0{}^J D_x{}^\alpha\left[(x^{n\alpha})\right]=\frac{\Gamma(n\alpha+1)}{\Gamma(\alpha(n-1)+1)}(x)^{\alpha(n-1)} n\in R$$

$${}_0{}^J D_x{}^\alpha\left[E_a(ax^a)\right]={}^J D_x{}^\alpha\left[E_a(ax^a)\right]=aE_a(ax^a) \tag{3.1}$$

$$E_\alpha(ix^\alpha)=\cos_\alpha(x^\alpha)+i\sin(x^\alpha), E_\alpha(x^\alpha)=\cosh_\alpha(x^\alpha)+i\sinh(x^\alpha)$$

$$\cos_\alpha(x^\alpha)=\sum_{m=0}^{\infty}(-1)^m\frac{x^{2m\alpha}}{\Gamma(1+2\alpha m)},\ \cosh_\alpha(x^\alpha)=\sum_{m=0}^{\infty}\frac{x^{2m\alpha}}{\Gamma(1+2\alpha m)}$$

$$\sin_\alpha(x^\alpha)=\sum_{m=0}^{\infty}(-1)^m\frac{x^{(2m+1)\alpha}}{\Gamma(1+(2m+1)\alpha)},\sinh_\alpha(x^\alpha)=\sum_{m=0}^{\infty}\frac{x^{(2m+1)\alpha}}{\Gamma(1+(2m+1)\alpha)} \tag{3.2}$$

4. BRATU EQUATION:

Now we have to discuss that Uniqueness and convergence of Bratu equation as follows:

4.1 Uniqueness of Bratu equation:

If Bratu-type equation has a result, then it is unique whenever, $0<\alpha<1$. here, $\alpha=\frac{L|\lambda|}{\Gamma(1+\beta)}t^\beta$ and L is a Lipschitz constant.

Proof: Bratu-type equation can be written as in the form

$$u(t)=\frac{-\lambda}{\Gamma(1+\beta)}\int_0^t F(x,u(x))(dx)^\beta \quad \text{where } F(t,u(t))=\frac{1}{\Gamma(1+\beta)}\int_0^t e^{u(x)}(dx)^\beta \tag{4.1}$$

such that the nonlinear term $F(t,u(t))$ is *Lipschitz continuous* with
$\|F(t,u)-F(t,w)\| \le L\|u-w\|$.

The Lipschitz constant L can be calculated as follows. Using this maximum norm,

$$\|F\| = \max_{0\le t \le 1} |F(t,u(t))|,$$

$$|e^u - e^w| \le \sum_{n=0}^{\infty} \frac{|u^n - w^n|}{n!}$$

$$=\sum_{n=1}^{\infty} \frac{1}{n!}|u-w||u^{n-1}+u^{n-2}w+...+w^{n-1}|, \tag{4.2}$$

Since the series is convergent.

$$|u^{n-1}+u^{n-2}w+.............+uw^{n-2}+w^{n-1}|,$$

is bounded $\forall n$ and

$$|u^{n-1}+u^{n-2}w+.............+uw^{n-2}+w^{n-1}| \le N, \qquad n=1,2,... \tag{4.3}$$

Therefore

$$|e^u - e^w| \le |u-w| N \sum_{n=0}^{\infty}\frac{1}{n!} = N(e-1)|u-w|. \quad \text{Hence,}$$

$$\|F(t,u)-F(t,w)\| \le \frac{1}{\Gamma(1+\beta)}\int_0^t |e^u - e^w|(dx)^\beta$$

$$\le \frac{N(e-1)}{\Gamma(1+\beta)}\int_0^t |u-w|(dx)^\beta,$$

$$\therefore \quad \|F(t,u)-F(t,w)\| \le \frac{N(e-1)}{\Gamma(1+\beta)}\|u-w\|\int_0^t (dx)^\beta$$

$$=\frac{N(e-1)t^\beta}{\Gamma(1+\beta)}\|u-w\|$$

$$\le \frac{N(e-1)}{\Gamma(1+\beta)}\|u-w\|. \tag{4.4}$$

Then , we can choose L which is $L = \dfrac{N(e-1)}{\Gamma(1+\beta)}$

Now, Let u and v be two different solutions of Bratu-type equation, then

$$\|u-v\| = \left\| \frac{-\lambda}{\Gamma(1+\beta)} \int_0^t F(x,u(x))(dx)^\beta + \frac{\lambda}{\Gamma(1+\beta)} \int_0^t F(x,v(x))(dx)^\beta \right\|$$

$$\leq \frac{|\lambda|}{\Gamma(1+\beta)} \int_0^t \|F(u)-F(v)\|(dx)^\beta$$

$$\leq \frac{L|\lambda|}{\Gamma(1+\beta)} \|u-v\| t^\beta .$$

$$\|u-v\| \left(1 - \frac{L|\lambda|}{\Gamma(1+\beta)} t^\beta \right) \leq 0,$$

$$\|u-v\| \ (1-\alpha) \leq 0,$$

$$\alpha = \frac{L|\lambda|}{\Gamma(1+\beta)} t^\beta \ \|u-v\| = 0,$$

$$\therefore u = v. \tag{4.5}$$

One important outcome (*) will be there if $f(u(t)) = \exp u(t),\ t_0 = 0$ and F(k) is coefficient of power series in Fractional form $f(u(x))$, then

$$F(k) = \begin{cases} \dfrac{\Gamma(\alpha(k-1)+1)}{\Gamma(\alpha k+1)} \displaystyle\sum_{i=1}^{k} \frac{\Gamma(\alpha i-1)}{\Gamma(\alpha(i-1)+1)} u(i) F(k-i) & when \quad k \geq 1. \\ \exp u(0) & when \quad k = 0 \end{cases}$$

Where $U(i) = \dfrac{1}{\Gamma(\alpha i+1)} D_x^{\alpha i} u(t), \quad at \quad t = 0, \qquad i = 1,2,3,........k.$

For the calculation of Bratu-type equation with the help of differential transform method

$$F(u) = D_t^{2\alpha} u + \lambda \exp u$$

$$y(t) = \sum_{u=0}^{\infty} F(u) t^u$$

We get, $u_0(t) = 0, u_1(t) = \dfrac{(-\lambda) t^{2\alpha}}{\Gamma(2\alpha+1)},\ u(t) = u_0(t) + u_1(t) + ... = \displaystyle\sum_{k=0}^{\infty} u_k(t).$

4.2 Convergence of Bratu equation:

If the series $\sum_{k=0}^{\infty} u_k(t)$ is convergent to s(t), then it must be the exact solution of equation (1.1).

Proof: By using above result (*), equation (1.1) can be written as

$$u_k(t) = \begin{cases} U(2k)t^{2k\alpha}, & when \quad k \geq 1, \\ 0 & when \quad k = 0 \end{cases}$$

$$U(2k) = \begin{cases} \dfrac{-\lambda\Gamma(2\alpha(k-1)+1)}{\Gamma(2\alpha k+1)} F(2(k-1)), & k \geq 1, \\ 0 & k = 0 \end{cases}$$

Using above result(*),

$$F(2(k-1)) = \begin{cases} \dfrac{\Gamma(\alpha(2k-3)+1)}{\Gamma(2\alpha(k-1)+1)} \sum_{i=1}^{k-1} \dfrac{\Gamma(2\alpha i-1)}{\Gamma(\alpha(2i-1)+1)} u(2i)F(2(k-i-1)) \\ \quad when \;\; k \geq 2. \\ 1 \quad when \quad k = 1 \end{cases}$$

$\therefore \;\; S(t) = \sum_{i=1}^{\infty} U(2k)t^{2k\alpha} \;\; \& \;\; e^{s(t)} = \sum_{i=1}^{\infty} F(2k)t^{2k\alpha}$. Now, S(t) satisfied the bratu-type equation, therefore we get,

$$D_t^{2\alpha} s(t) + \lambda \exp s(t) = D_t^{2\alpha}\left(\sum_{i=1}^{\infty} U(2k)t^{2k\alpha} \right) + \lambda\left(\sum_{i=1}^{\infty} F(2k)t^{2k\alpha} \right).$$

$$= \sum_{i=1}^{\infty} U(2k) \frac{-\lambda\Gamma(2\alpha k+1)}{\Gamma(2\alpha(k-1)+1)} t^{2(k-1)\alpha} + \lambda\left(\sum_{i=1}^{\infty} F(2k)t^{2k\alpha} \right) = 0.$$

5. ILLUSTRATIVE EXAMPLES

In this section, Differential transform method is applied on Bratu equations. Then, we get its approximate solutions and exact solutions with different values of α. we solve three examples by DTM as follows:

Example1. Suppose the following type of Fractional Bratu-type Equation

$$D_t^{2\alpha} u(t) - 2\exp u(t) = 0, \tag{5.1}$$

with initial conditions $0 < \alpha \le 1,\ 0 < t < 1,\ u(0) = u^{\alpha}(0) = 0$ (5.2)

Solution: The exact solution of equation (5.1) when value of α =1 is

$u(t) = -2\ln(\cos t)$

Apply DTM with $t_0 = 0$ and $\alpha = 1$, to both sides of BTE and making use of properties of DTM. Let $D_t^{\beta}u(t) - 2\exp u(t) = 0,$ (5.3)

$$\frac{\Gamma(\alpha k+\beta+1)}{\Gamma(\alpha k+1)}Y_{\alpha}\left(K+\frac{\beta}{\alpha}\right)+\frac{\lambda}{k!}=0$$

$$Y_{\alpha}\left(K+2\right)=-\left(\frac{\lambda}{k!}\right)\frac{\Gamma(\alpha k+1)}{\Gamma(\alpha k+\beta+1)} \tag{5.4}$$

Put the values of $\alpha = 1$, $\lambda = -2$, in (5.4), and get the result by putting the values of k = 0, 1, 2, 3, 4, 5, 6…..,n as follows:

$$Y_1(0)=0, Y_1(1)=0, Y_1(2)=1, Y_1(3)=\frac{1}{3},$$
$$Y_1(4)=\frac{1}{6}, Y_1(5)=\frac{1}{10}, Y_1(6)=\frac{1}{15}, Y_1(7)=\frac{1}{21},$$
$$Y_1(8)=\frac{1}{28}, Y_1(9)=\frac{1}{36}, Y_1(10)=\frac{1}{45}, \ldots\ldots.$$

Then takes inverse transformation, we get results with different values of α. Now, four different cases are here, as follows: $y(t)=\sum_{0}^{\infty}Y_{\alpha}(t)t^{\alpha}$

Case I: Evaluate y(t) ,when value of α is 0.25.

$$u(t)=2(1.1284t^{\frac{1}{2}}+0.9862t^{\frac{3}{4}}+0.8862t+0.8111t^{\frac{5}{4}}+0.7522t^{\frac{6}{4}}+0.7044t^{\frac{7}{4}}+0.6646t^{2}+\ldots\ldots)$$

Case II: Evaluate y(t) ,when value of α is 0.5.

$$u(t)=2(1.t+0.6668t^{\frac{3}{2}}+0.5t^{2}+0.3999t^{\frac{5}{2}}+0.3333t^{3}+0.2857t^{\frac{7}{2}}+0.25t^{4}+0.2222t^{\frac{9}{2}}+\ldots\ldots\ldots)$$

Case III: Evaluate y(t) ,when value of α is 0.75.

$$u(t)=2(0.7522t^{\frac{3}{2}}+0.93605t^{\frac{9}{4}}+0.8862t^{3}+0.1536t^{\frac{15}{4}}+0.1146t^{\frac{9}{2}}+0.0897t^{\frac{21}{4}}+0.60726t^{6}+\ldots\ldots)$$

Case IV: Evaluate y(t) ,when value of α is 1.

$$u(t) = (t^2 + 0.3333t^3 + 0.1666t^4 + 0.1t^5 + 0.0666t^6 + 0.0476t^7 + 0.0357t^8 + 0.0277t^9 + ...)$$

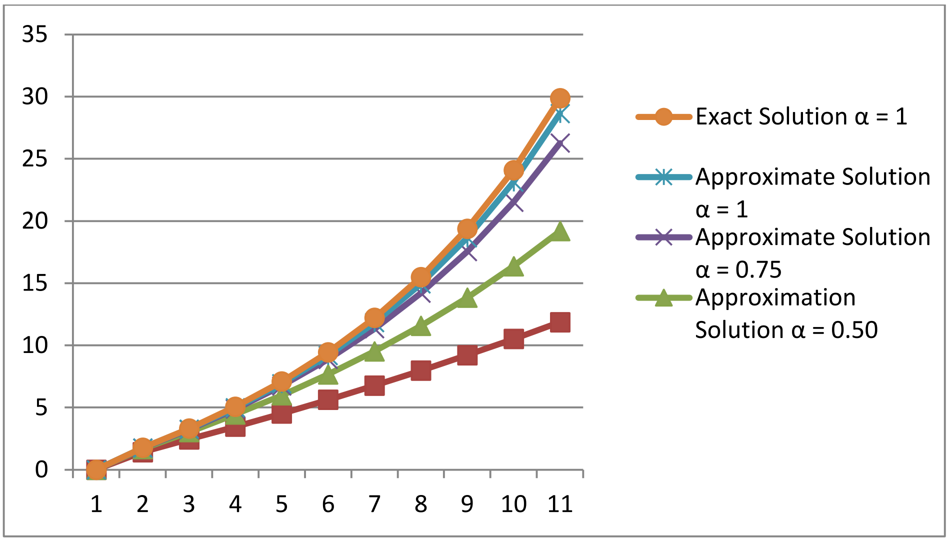

Figure 1.1

This is the approximate solutions of Differential Transformation Method with various values of alphas like 0.25, 0.50, 0.75 and 1.0. in figure (1.1) and included Exact solution at alpha is unity.

Example 2. Suppose the following type of Fractional Bratu-type Equation

$$D_t^{2\alpha} u(t) + \pi^2 \exp u(t) = 0, \tag{5.5}$$

with initial conditions $0 < \alpha \le 1,\ 0 < t < 1,\ u(0) = u^{\alpha}(0) = \pi$ (5.6)

Solution: The exact solution of equation (5.5) when value of α =1 is

$u(t) = \ln(1 + \sin(\pi t)).$

Apply DTM with $t_0 = 0$ and $\alpha = 1$, to both sides of Bratu-type Equation and making use of properties of DTM.

Let $D_t^{\beta} u(t) + \pi^2 \exp u(t) = 0,$ (5.7)

$$\frac{\Gamma(\alpha k + \beta + 1)}{\Gamma(\alpha k + 1)} Y_{\alpha}\left(K + \frac{\beta}{\alpha}\right) + \frac{\lambda}{k!} = 0$$

$$Y_{\alpha}(K + 2) = -\left(\frac{\lambda}{k!}\right)\frac{\Gamma(\alpha k + 1)}{\Gamma(\alpha k + \beta + 1)} \tag{5.8}$$

Put the values of $\alpha = 1$, $\lambda = \pi^2$, in (5.8), and get the result by putting the values of k = 0, 1, 2, ...n similar as in example 1.Then takes inverse transformation; we get results with different values of α. Now, four different cases are here, as follows: $y(t) = \sum_{0}^{\infty} Y_\alpha(t) t^\alpha$

Case I: Evaluate y(t) ,when value of α is 0.25.

$$u(t) = -9.86(1.1284t^{\frac{1}{2}} + 0.9862t^{\frac{3}{4}} + 0.8862t + 0.8111t^{\frac{5}{4}} + 0.7522t^{\frac{6}{4}} + 0.7044t^{\frac{7}{4}} + 0.6646t^2 + ...)$$

Case II: Evaluate y(t) ,when value of α is 0.5.

$$u(t) = -9.86(1.t + 0.6668t^{\frac{3}{2}} + 0.5t^2 + 0.3999t^{\frac{5}{2}} + 0.3333t^3 + 0.2857t^{\frac{7}{2}} + 0.25t^4 + 0.2222t^{\frac{9}{2}} + ...)$$

Case III: Evaluate y(t) ,when value of α is 0.75.

$$u(t) = -9.86(0.7522t^{\frac{3}{2}} + 0.93605t^{\frac{9}{4}} + 0.8862t^3 + 0.1536t^{\frac{15}{4}} + 0.1146t^{\frac{9}{2}} + 0.0897t^{\frac{21}{4}} + 0.60726t^6 + ...)$$

Case IV: Evaluate y(t) ,when value of α is 1.

$$u(t) = -4.93(t^2 + 0.3333t^3 + 0.1666t^4 + 0.1t^5 + 0.0666t^6 + 0.0476t^7 + 0.0357t^8 + 0.0277t^9 + ...)$$

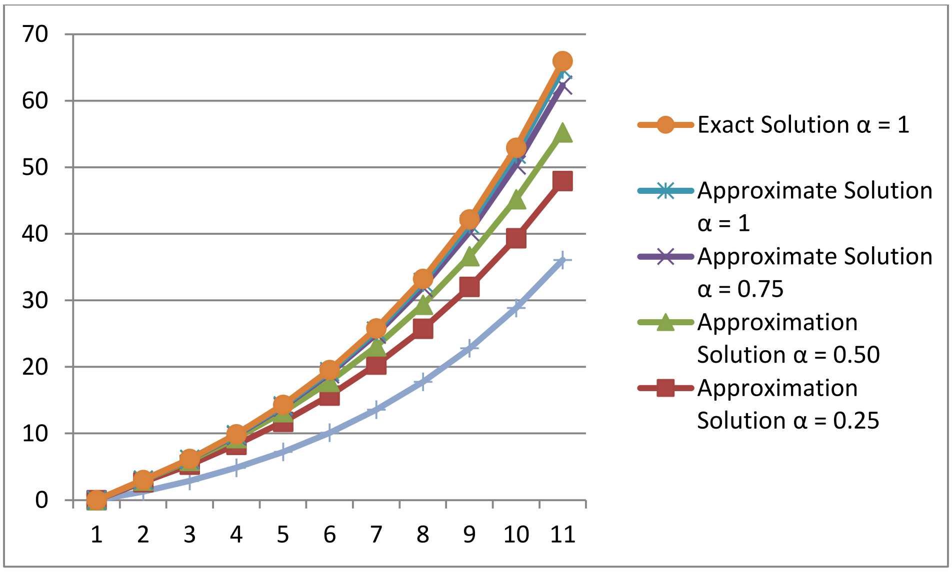

Figure 1.2

An Exact and approximate solution of Differential Transformation Method with various values of alphas like 0.25, 0.50, 0.75 and 1.0. We have seen in the figure

(1.2.) to reduce the value of alpha that is close to the exact solution with minimum error.

Example 3. Suppose the following type of Fractional Bratu-type Equation

$$D_t^{2\alpha} u(t) - \pi^2 \exp u(t) = 0, \qquad (5.9)$$

with initial conditions $0 < \alpha \le 1,\ 0 < t < 1,\ u(0) = u^{\alpha}(0) = \pi$ (5.10)

Solution: The exact solution of equation (5.9) when value of α =1 is

$u(t) = -\ln(1 - \sin(\pi t))$.

Apply DTM with $t_0 = 0$ and $\alpha = 1$, to both sides of Bratu-type Equation and making use of properties of DTM.

$$\text{Let } D_t^{\beta} u(t) - \pi^2 \exp u(t) = 0, \qquad (5.11)$$

$$\frac{\Gamma(\alpha k + \beta + 1)}{\Gamma(\alpha k + 1)} Y_\alpha\left(K + \frac{\beta}{\alpha}\right) + \frac{\lambda}{k!} = 0$$

$$Y_\alpha(K+2) = -\left(\frac{\lambda}{k!}\right)\frac{\Gamma(\alpha k + 1)}{\Gamma(\alpha k + \beta + 1)} \qquad (5.12)$$

Put the values of $\alpha = 1,\ \lambda = -\pi^2$, in (5.12), and get the result by putting the values of k = 0, 1, 2, ...n similar as in example 1.Then takes inverse transformation; we get results with different values of α. Now, four different cases are here, as follows:

$$y(t) = \sum_{0}^{\infty} Y_\alpha(t) t^{\alpha}$$

Case I: Evaluate y(t) ,when value of α is 0.25.

$$u(t) = 9.86(1.1284t^{\frac{1}{2}} + 0.9862t^{\frac{3}{4}} + 0.8862t + 0.8111t^{\frac{5}{4}} + 0.7522t^{\frac{6}{4}} + 0.7044t^{\frac{7}{4}} + 0.6646t^2 + ...)$$

Case II: Evaluate y(t) ,when value of α is 0.5.

$$u(t) = 9.86(1.t + 0.6668t^{\frac{3}{2}} + 0.5t^2 + 0.3999t^{\frac{5}{2}} + 0.3333t^3 + 0.2857t^{\frac{7}{2}} + 0.25t^4 + 0.2222t^{\frac{9}{2}} + ...)$$

Case III: Evaluate y(t) ,when value of α is 0.75.

$$u(t) = 9.86(0.7522t^{\frac{3}{2}} + 0.93605t^{\frac{9}{4}} + 0.8862t^3 + 0.1536t^{\frac{15}{4}} + 0.1146t^{\frac{9}{2}} + 0.0897t^{\frac{21}{4}} + 0.60726t^6 + ...)$$

Case IV: Evaluate y(t) ,when value of α is 1.

$$u(t) = 4.93(t^2 + 0.3333t^3 + 0.1666t^4 + 0.1t^5 + 0.0666t^6 + 0.0476t^7 + 0.0357t^8 + 0.0277t^9 + ...)$$

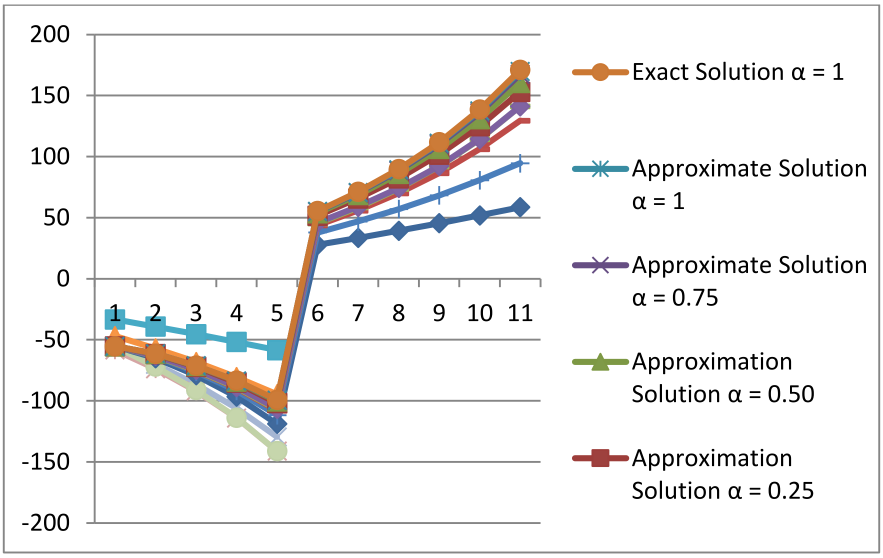

Figure 1.3

Similarly as above, in this figure (1.3.) to show that the exact and approximate solutions with various values of alphas like 0.25, 0.50, 0.75 and 1.0.

CONCLUSION:

The Differential Transformation Method for fractional differential equations has been extensively puzzled out for several years. From the obtained results it's clear that the Differential Transformation technique steered during this paper give the solutions in terms of Taylor series with simply calculable parts, therefore it's associate economical and applicable technique for finding fractional Bratu-type equations. All effective outcomes are shown as above figures from 1.1. to 1.3.

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