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# Higher Separation Axioms via Semi*regular open sets

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**Abstract**

The purpose of this paper is to introduce new separation axioms semi*r-regular, semi*r-normal, s*r-regular, s**r-normal using semi*regular open sets and investigate their properties. We also study the relationships among themselves and with known axioms regular, normal, semi-regular and semi-normal.

**Keywords and phrases:** s**r-normal, s*r-regular, semi*r-normal, semi*r-regular.

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## I. INTRODUCTION

Separation axioms are useful in classifying topological spaces. Maheswari and Prasad [8, 9] introduced the notion of s-regular and s-normal spaces using semi-open sets. Dorsett [3, 4] introduced the concept of semi-regular and semi-normal spaces and investigate their properties. In this paper, we define semi*r-regular, semi*r-normal, s*r-regular and s**r-normal spaces using semi*regular open sets and investigate their basic properties. We further study the relationships among themselves and with known axioms regular, normal, semi-regular and semi-normal.

## II. PRELIMINARIES

Throughout this paper (X, τ) will always denote a topological space on which no separation axioms are assumed, unless explicitly stated. If A is a subset of the space (X, τ), *Cl*(A) and *Int*(A) respectively denote the closure and the interior of A in X .

**Definition 2.1**[7]**:** A subset A of a topological space (X, τ) is called (i) generalized closed (briefly g-closed) if *Cl*(A)⊆U whenever A⊆U and U is open in X. (ii) generalized open (briefly g-open) if X\A is g-closed in X.

**Definition 2.2:** Let A be a subset of X. Then (i) generalized closure[5] of A is defined as the intersection of all g-closed sets containing A and is denoted by *Cl**(A). (ii) generalized interior of A is defined as the union of all g-open subsets of A and is denoted by *Int**(A).

**Definition 2.3:** A subset A of a topological space (X, τ) is called (i) semi-open [6] (resp. semi*-open[12]) if A⊆*Cl*(*Int*(A)) (resp. A⊆*Cl**(*Int*(A)). (ii) semi-closed [1] (resp. semi*-closed[13]) if *Int*(*Cl*(A))⊆A (resp. *Int**(*Cl*(A))⊆A). The class of all semi*-open (resp. semi*-closed) sets is denoted by S*O(X, τ) (resp. S*C(X,τ)). The semi*r-interior of A is defined as the union of all semi*regular open sets of X contained in A. It is denoted by *s*rInt*(A).The semi*r-closure of A is defined as the intersection of all semi*regular closed sets in X containing A. It is denoted by *s*rCl*(A).

**Theorem 2.4**[11]**:** Let A⊆X and let x∈X. Then x∈*s*rCl*(A) if and only if every semi*regular open set in X containing x intersects A.

**Theorem 2.5**[11]**:** (i) Every semi*regular open set is semi*-open. (ii) Every semi*-open set is semi-open.

**Definition 2.6:** A space X is said to be $T_1$[14] if for every pair of distinct points x and y in X, there is an open set U containing x but not y and an open set V containing y but not x.

**Definition 2.7:** A space X is $R_0$ [13] if every open set contains the closure of each of its points. **Theorem 2.8:** (i) X is $R_0$ if and only if for every closed set F, *Cl*({x})∩F=ϕ, for all x∈X\F.

(ii) X is semi*r-$R_0$ if and only if for every semi*regular closed set F, *s**r*Cl*({x})∩F=ϕ, for all

x∈X\F.

**Definition 2.9:** A topological space X is said to be (i) regular if for every pair consisting of a point x and a closed set B not containing x, there are disjoint open sets

U and V in X containing x and B respectively.[14] (ii) s-regular if for every pair consisting of a point x and a closed set B not containing x, there are disjoint semi-open sets U and V in X containing x and B respectively.[8] (iii) semi-regular if for every pair consisting of a point x and a semi-closed set B not containing x, there are disjoint semi-open sets U and V in X containing x and B respectively.[3]

**Definition 2.10:** A topological space X is said to be (i) normal if for every pair of disjoint closed sets A and B in X, there are disjoint open sets U and V in X containing A and B respectively.[14] (ii) s-normal if for every pair of disjoint closed sets A and B in X, there are disjoint semi-open sets U and V in X containing A and B respectively.[9] (iii) semi-normal if for every pair of disjoint semi-closed sets A and B in X, there are disjoint semi-open sets U and V in X containing A and B respectively.[4]

**Definition 2.11:** A function $f$ :X⟶Y is said to be (i) closed [14] if $f$ (V) is closed in Y for every closed set V in X. (ii) semi*r-continuous [12] if $f^{-1}$(V) is semi*-regular open in X for every open set V in Y. (iii) semi*r-irresolute [12] if $f^{-1}$(V) is semi*regular open in X for every semi*regular open set V in Y. (iv) contra-semi*-irresolute if $f^{-1}$(V) is semi*regular closed in X for every semi*regular open set V in Y. (v) semi*regular open if $f$ (V) is semi*regular open in Y for every open set V in X. (vi) pre-semi*regular open if $f$(V) is semi*regular open in Y for every semi*regular open set V in X (vii) contra-pre-semi*regular open if $f$(V) is semi*regular closed in Y for every semi*regular open set V in X. (viii) pre-semi*regular closed if $f$(V) is semi*regular closed in Y for every semi*regular closed set V in X.

**Lemma 2.12**[11]**:** If A and B are subsets of X such that A∩B=ϕ and A is semi*regular open in X, then A∩*s*rCl*(B)=ϕ.

**Theorem 2.13**[12]: A function $f$ :X⟶Y is semi*r-irresolute if $f^{-1}$(F) is semi*regular closed in X for every semi*regular closed set F in Y.

## III. REGULAR SPACES ASSOCIATED WITH SEMI*REGULAR OPEN SETS.

In this section we introduce the concepts of semi*r-regular and s*r-regular spaces. Also we investigate their basic properties and study their relationship with already existing concepts.

**Definition 3.1:** A space X is said to be ***semi*r-regular*** if for every pair consisting of a point x and a semi*regular closed set B not containing x, there are disjoint semi*regular open sets U and V in X containing x and B respectively.

**Theorem 3.2:** In a topological space X, the following are equivalent:

(i) X is semi*r-regular.

(ii) For every x∈X and every semi*regular open set U containing x, there exists a semi*regular open set V containing x such that *s**rC*l*(V)⊆U.

(iii) For every set A and a semi*regular open set B such that A∩B≠ϕ, there exists a semi*regular open set U such that A∩U≠ϕ and *s**r*Cl*(U)⊆B.

(iv) For every non-empty set A and semi*regular closed set B such that A∩B=ϕ, there exist disjoint semi*regular open sets U and V such that A∩U≠ϕ and B⊆V.

**Proof:** (i)⟹(ii): Let U be a semi*regular open set containing x. Then B=X\U is a semi*regular closed not containing x. Since X is semi*r-regular, there exist disjoint semi*regular open sets V and W containing x and B respectively. If y∈B, W is a semi*regular open set containing y that does not intersect V and hence by Theorem 2.4, y cannot belong to *s**rCl*(V).Therefore *s**rCl*(V) is disjoint from B. Hence *s**rCl*(V)⊆U

(ii)⟹(iii): Let A∩B≠ϕ and B be semi*regular open. Let x∈A∩B. Then by assumption, there exists a semi*regular open set U containing x such that *s**rCl*(U)⊆B. Since x∈A, A∩U≠ϕ. This proves (iii).

(iii)⟹(iv): Suppose A∩B=ϕ, where A is non-empty and B is semi*regular closed. Then X\B is semi*regular open and A∩(X\B)≠ϕ. By (iii), there exists a semi*regular open set U such that A∩U≠ϕ, and U⊆*s**rCl*(U)⊆X\B. Put V=X*s**rCl*(U). Hence V is a semi*regular open set containing B such that U∩V=U∩(X*s**rCl*(U))⊆U∩(X\U)=ϕ. This proves (iv).

(iv)⟹(i). Let B be semi*regular closed and x∉B. Take A={x}. Then A∩B=ϕ. By (iv), there exist disjoint semi*regular open sets U and V such that U∩A≠ϕ and B⊆V. Since U∩A≠ϕ, x∈U. This proves that X is semi*r-regular.

**Theorem 3.3:** Let X be a semi*r-regular space.

(i) Every semi*regular open set in X is a union of semi*regular closed sets.

(ii) Every semi*regular closed set in X is an intersection of semi*regular open sets.

**Proof:** (i) Suppose X is semi*r-regular. Let G be a semi*regular open set and x∈G. Then F=X\G is semi*regular closed and x∉ F. Since X is semi*r-regular, there exist disjoint semi*regular open sets Ux and V in X such that x∈Ux and F⊆V.Since Ux∩F ⊆Ux ∩V=ϕ, we have Ux ⊆X\F=G. Take Vx=*s**rCl*(Ux). Then Vx is semi*regular closed and by Lemma 2.12, Vx∩V=ϕ. Now F⊆V implies that Vx∩F ⊆ Vx∩V=ϕ. It follows that $x \in V_x \subseteq X \setminus F = G$. This proves that $G = \cup\{V_x : x \in G\}$. Thus G is a union of semi*regular closed sets.

(ii) Follows from (i) and set theoretic properties.

**Theorem 3.4:** If $f$ is a semi*r-irresolute and pre-semi*regular closed injection of a topological space X into a semi*r-regular space Y, then X is semi*r-regular.

**Proof:** Let $x \in X$ and A be a semi*regular closed set in X not containing x. Since $f$ is pre-semi*regular closed, $f(A)$ is a semi*regular closed set in Y not containing $f(x)$. Since Y is semi*r-regular, there exist disjoint semi*regular open sets $V_1$ and $V_2$ in Y such that $f(x) \in V_1$ and $f(A) \subseteq V_2$. Since $f$ is semi*r-irresolute, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint semi*regular open sets in X containing x and A respectively. Hence X is semi*r-regular.

**Theorem 3.5:** If $f$ is a semi*r-continuous and closed injection of a topological space X into a regular space Y and if every semi*regular closed set in X is closed, then X is semi*r-regular.

**Proof:** Let $x \in X$ and A be a semi*regular closed set in X not containing x. Then by assumption, A is closed in X. Since $f$ is closed, $f(A)$ is a closed set in Y not containing $f(x)$. Since Y is regular, there exist disjoint open sets $V_1$ and $V_2$ in Y such that $f(x) \in V_1$ and $f(A) \subseteq V_2$. Since $f$ is semi*r-continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint semi*regular open sets in X containing x and A respectively. Hence X is semi*r-regular.

**Theorem 3.6:** If $f: X \longrightarrow Y$ is a semi*r-irresolute bijection which is pre-semi*regular open and X is semi*r-regular. Then Y is also semi*r-regular.

**Proof:** Let $f: X \longrightarrow Y$ be a semi*r-irresolute bijection which is semi*regular open and X be semi*r-regular. Let $y \in Y$ and B be a semi*regular closed set in Y not containing y. Since $f$ is semi*r-irresolute, by Theorem 2.13 $f^{-1}(B)$ is a semi*regular closed set in X not containing $f^{-1}(y)$. Since X is semi*r-regular, there exist disjoint semi*regular open sets $U_1$ and $U_2$ containing $f^{-1}(y)$ and $f^{-1}(B)$ respectively. Since $f$ is pre-semi*regular open, $f(U_1)$ and $f(U_2)$ are disjoint semi*regular open sets in Y containing y and B respectively. Hence Y is semi*r-regular.

**Theorem 3.7:** If $f$ is a continuous semi*regular open bijection of a regular space X into a space Y and if every semi*regular closed set in Y is closed, then Y is semi*r-regular.

**Proof:** Let $y \in Y$ and B be a semi*regular closed set in Y not containing y. Then by assumption, B is closed in Y. Since $f$ is a continuous bijection, $f^{-1}(B)$ is a closed set in X not containing the point $f^{-1}(y)$. Since X is regular, there exist disjoint open sets $U_1$

and $U_2$ in X such that $f^{-1}(y)\in U_1$ and $f^{-1}(B)\subseteq U_2$. Since $f$ is semi*regular open, $f(U_1)$ and $f(U_2)$ are disjoint semi*regular open sets in Y containing x and B respectively. Hence Y is semi*r-regular.

**Theorem 3.8:** If X is semi*r-regular, then it is semi*r-$R_0$.

**Proof:** Suppose X is semi*-regular. Let U be a semi*-open set and $x\in U$. Take F = X\U. Then F is a semi*regular closed set not containing x. By semi*r-regularity of X, there are disjoint semi*regular open sets V and W such that $x\in V$, $F\subseteq W$. If $y\in F$, then W is a semi*regular open set containing y that does not intersect V. Therefore $y\notin s^*rCl(\{V\}) \Rightarrow y\notin s^*rCl(\{x\})$. That is $s^*rCl(\{x\})\cap F=\phi$ and hence $s^*rCl(\{x\})\subseteq X\setminus F= U$. Hence X is semi*r-$R_0$.

**Definition 3.9:** A space X is said to be ***s*r-regular*** if for every pair consisting of a point x and a closed set B not containing x, there are disjoint semi*regular open sets U and V in X containing x and B respectively.

**Theorem 3.10:** (i) Every s*r-regular space is s*-regular.

(ii) Every s*r-regular space is s-regular.

**Proof:** Suppose X is s*r-regular . Let F be a closed set and $x\notin F$. Since X is s*r-regular, there exist disjoint semi*regular open sets U and V containing x and F respectively. Then by Theorem 2.5(i) U and V are semi*-open in X. This implies that X is s*-regular. This proves (i). Suppose X is s*r-regular. Let F be a closed set and $x\notin F$. Since X is s*r-regular, there exist disjoint semi*regular open sets U and V containing x and F respectively. Then by Theorem 2.5(ii) U and V are semi-open in X. This implies that X is s-regular. This proves (ii).

**Remark 3.11:** The reverse implications of the statements in the above theorem are not true as shown in the following examples.

**Example 3.12:** Let X={a, b, c, d} with topology $\boldsymbol{\tau}$={ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}, X}. Clearly (X, $\boldsymbol{\tau}$) is s*-regular but not s*r- regular.

**Example 3.13:** Let X={a, b, c, d} with topology $\boldsymbol{\tau}$ ={ϕ, {a}, {c}, {a,c}, X} Clearly (X, $\boldsymbol{\tau}$) is s-regular not s*r-regular.

**Theorem 3.14:** For a topological space X, the following are equivalent:

(i) X is s*r-regular.

(ii) For every $x\in X$ and every open set U containing x, there exists a semi*regular open set V containing x such that $s^*rCl(V)\subseteq U$.

(iii) For every set A and an open set B such that $A\cap B\neq\phi$, there exists a semi*regular open set U such that $A\cap U\neq\phi$ and $s^*rCl(U)\subseteq B$.

(iv) For every non-empty set A and closed set B such that $A\cap B=\phi$, there exist disjoint semi*regular open sets U and V such that $A\cap U\neq\phi$ and $B\subseteq V$.

**Proof:** (i)⟹(ii): Let U be an open set containing x. Then B=X\U is a closed set not containing x. Since X is s*r-regular, there exist disjoint semi*regular open sets V and W containing x and B respectively. If $y\in B$, W is a semi*regular open set containing y that does not intersect V and hence by Theorem 2.4, y cannot belong to $s^*rCl(V)$. Therefore $s^*rCl(V)$ is disjoint from B. Hence $s^*rCl(V)\subseteq U$.

(ii)⟹(iii): Let $A\cap B\neq\phi$ and B be open. Let $x\in A\cap B$. Then by assumption, there exists a semi*regular open set U containing x such that $s^*rCl(U)\subseteq B$. Since $x\in A$, $A\cap U\neq\phi$.

This proves (iii).

(iii)⟹(iv): Suppose $A\cap B=\phi$, where A is non-empty and B is closed. Then X\B is open and $A\cap(X\setminus B)\neq\phi$. By (iii), there exists a semi*regular open set U such that $A\cap U\neq\phi$, and $U\subseteq s^*rCl(U)\subseteq X\setminus B$. Put $V=X\setminus s^*rCl(U)$. Hence V is a semi*regular open set containing B such that $U\cap V=U\cap(X\setminus s^*rCl(U))\subseteq U(X\setminus U)=\phi$. This proves (iv).

(iv)⟹(i). Let B be closed and $x\notin B$. Take A={x}. Then $A\cap B=\phi$. By (iv), there exist disjoint semi*regular open sets U and V such that $U\cap A\neq\phi$ and $B\subseteq V$. Since $U\cap A\neq\phi$, $x\in U$. This proves that X is s*r-regular.

**Theorem 3.15:** (i) Every s*r-regular $T_1$ space is semi*r-$T_2$.

(ii) Every semi*r-regular semi*r-$T_1$ space is semi*r-$T_2$.

**Proof:** Suppose X is s*-regular and $T_1$. Let x and y be two distinct points in X. Since X is $T_1$, {x} is closed and $y\notin\{x\}$. Since X is s*r-regular, there exist disjoint semi*regular open sets U and V in X containing {x} and y respectively. It follows that X is semi*r-$T_2$. This proves (i).

Suppose X is semi*-regular and semi*-$T_1$. Let x and y be two distinct points in X. Since X is semi*r-$T_1$, {x} is semi*regular closed and $y\notin\{x\}$. Since X is semi*r-regular, there exist disjoint semi*regular open sets U and V in X containing {x} and y respectively. It follows that X is semi*r-$T_2$. This proves (ii).

**Theorem 3.16:** Let X be an s*r-regular space.

i) Every open set in X is a union of semi*regular closed sets.

ii) Every closed set in X is an intersection of semi*regular open sets.

**Proof:** (i) Suppose X is s*r-regular. Let G be an open set and x∈G. Then F=X\G is closed and

x∉ F. Since X is s*r-regular, there exist disjoint semi*regular open sets Ux and U in X such that x∈Ux and F⊆ U.Since Ux∩F ⊆Ux ∩U=ϕ, we have Ux ⊆X\F=G. Take Vx=*s*rCl*(Ux). Then Vx is semi*regular closed. Now F⊆U implies that Vx∩F ⊆ Vx∩U=ϕ.

It follows that x∈Vx⊆ X\F = G. This proves that G =∪{Vx : x∈G}. Thus G is a union of semi*regular closed sets.

(ii) Follows from (i) and set theoretic properties.

## IV. NORMAL SPACES ASSOCIATED WITH SEMI*REGULAR OPEN SETS.

In this section we introduce variants of normal spaces namely semi*r-normal spaces and s**r-normal spaces and investigate their basic properties. We also give characterizations for these spaces.

**Definition 4.1:** A space X is said to be ***semi*r-normal*** if for every pair of disjoint semi*regular closed sets A and B in X, there are disjoint semi*regular open sets U and V in X containing A and B respectively.

**Theorem 4.2:** In a topological space X, the following are equivalent:

(i) X is semi*r-normal.

(ii) For every semi*regular closed set A in X and every semi*regular open set U containing A, there exists a semi*regular open set V containing A such that *s**r*Cl*(V)⊆U.

(iii) For each pair of disjoint semi*regular closed sets A and B in X, there exists a semi*regular open set U containing A such that *s**r*Cl*(U)∩B=ϕ.

(iv) For each pair of disjoint semi*regular closed sets A and B in X, there exist semi*regular open sets U and Vcontaining A and B respectively such that *s**r*Cl*(U)∩*s**r*Cl*(V)=ϕ.

**Proof:** (i)⟹(ii): Let U be a semi*regular open set containing the semi*regular closed set A. Then B=X\U is a semi*regular closed set disjoint from A. Since X is semi*r-normal, there exist disjoint semi*regular open sets V and W containing A and B respectively. Then *s*rCl*(V) is disjoint from B, since if y∈B, the set W is a semi*regular open set containing y disjoint from V. Hence *s*rCl*(V)⊆U.

(ii)⟹(iii): Let A and B be disjoint semi*regular closed sets in X. Then X\B is a semi*regular open set containing A. By (ii), there exists a semi*regular open set U containing A such that $s^*rCl(U) \subseteq X\setminus B$. Hence $s^*rCl(U) \cap B = \phi$. This proves (iii).

(iii)⟹(iv): Let A and B be disjoint semi*regular closed sets in X. Then, by (iii), there exists a semi*regular open set U containing A such that $s^*rCl(U) \cap B = \phi$. Since $s^*rCl(U)$ is semi*regular closed**,** B and $s^*rCl(U)$ are disjoint semi*regular closed sets in X. Again by (iii), there exists a semi*regular open set V containing B such that $s^*rCl(U) \cap s^*rCl(V) = \phi$. This proves (iv).

(iv)⟹(i): Let A and B be the disjoint semi*regular closed sets in X. By (iv), there exist semi*regular open sets U and V containing A and B respectively such that $s^*rCl(U) \cap s^*rCl(V) = \phi$. Since $U \cap V \subseteq s^*rCl(U) \cap s^*rCl(V)$, U and V are disjoint semi*regular open sets containing A and B respectively. Thus X is semi*r-normal.

**Theorem 4.3:** For a space X, then the following are equivalent:

(i) X is semi*r-normal.

(ii) For any two semi*regular open sets U and V whose union is X, there exist semi*regular closed subsets A of U and B of V whose union is also X.

**Proof: (i)⟹(ii)**: Let U and V be two semi*regular open sets in a semi*r-normal space X such that $X = U \cup V$. Then X\U, X\V are disjoint semi*regular closed sets. Since X is semi*r-normal, there exist disjoint semi*regular open sets $G_1$ and $G_2$ such that $X\setminus U \subseteq G_1$ and $X\setminus V \subseteq G_2$. Let A=X\G1 and B=X\G2. Then A and B are semi*regular closed subsets of U and V respectively such that $A \cup B = X$. This proves (ii).

**(ii)⟹(i)**: Let A and B be disjoint semi*regular closed sets in X. Then X\A and X\B are semi*regular open sets whose union is X. By (ii), there exists semi*regular closed sets $F_1$ and $F_2$ such that $F_1 \subseteq X\setminus A$, $F_2 \subseteq X\setminus B$ and $F_1 \cup F_2 = X$. Then $X\setminus F_1$ and $X\setminus F_2$ are disjoint semi*regular open sets containing A and B respectively. Therefore X is semi*r-normal.

**Definition 4.4:** A space X is said to be ***s**r-normal*** if for every pair of disjoint closed sets A and B in X, there are disjoint semi*regular open sets U and V in X containing A and B respectively.

**Theorem 4.5:** In a topological space X, the following are equivalent:

(i) X is s**r-normal.

(ii) For every closed set F in X and every open set U containing F, there exists a semi*regular open set Vcontaining F such that $s^*rCl(V) \subseteq U$.

(iii) For each pair of disjoint closed sets A and B in X, there exists a semi*regular open set U containing A such that *s**r*Cl*(U)∩B=ϕ.

**Proof:** (i)⟹(ii): Let U be a open set containing the closed set F. Then H=X\U is a closed set disjoint from F.Since X is s**r-normal, there exist disjoint semi*regular open sets V and W containing F and H respectively. Then *s*rCl*(V) is disjoint from H, since if y∈H, the set W is a semi*regular open set containing y disjoint from V. Hence *s*rCl*(V)⊆U.

(ii)⟹(iii): Let A and B be disjoint closed sets in X. Then X\B is an open set containing A. By (ii), there exists a semi*regular open set U containing A such that *s*rCl*(U)⊆X\B. Hence *s*rCl*(U)∩B=ϕ. This proves (iii).

(iii)⟹(i): Let A and B be the disjoint semi*regular closed sets in X. By (iii), there exists a semi*regular open set U containing A such that *s*rCl*(U)∩B=ϕ. Take V=X*s*rCl*(U). Then U and V are disjoint semi*regular open sets containing A and B respectively.

Thus X is s**r-normal.

**Theorem 4.6:** For a space X, then the following are equivalent:

(i) X is s**r-normal.

(ii) For any two open sets U and V whose union is X, there exist semi*regular closed subsets A of U and B of V whose union is also X.

**Proof: (i)⟹(ii)**: Let U and V be two open sets in an s**r-normal space X such that X=U∪V. Then X\U, X\V are disjoint closed sets. Since X is s**r-normal, there exist disjoint semi*regular open sets $G_1$ and $G_2$ such that X\U⊆$G_1$ and X\V⊆$G_2$. Let A=X\$G_1$ and B=X\$G_2$. Then A and B are semi*regular closed subsets of U and V respectively such that A∪B=X. This proves (ii). **(ii)⟹(i)**: Let A and B be disjoint closed sets in X. Then X\A and X\B are open sets whose union is X. By (ii), there exists semi*regular closed sets $F_1$ and $F_2$ such that $F_1$⊆X\A, $F_2$⊆X\B and $F_1 \cup F_2$ =X. Then X\$F_1$ and X\$F_2$ are disjoint semi*regular open sets containing A and B respectively. Therefore X is s**r-normal.

**Remark 4.7:** It is not always true that an s**r-normal space X is s*r-regular as shown in the following example. However it is true if X is $R_0$ as seen in Theorem 4.9

**Example 4.8:** Let X={a, b, c, d} with topology **τ**={ϕ, {a}, {b}, {a, b}, { b, c, d}, X}. Clearly (X,) is s**r-normal but not s*r-regular. ?

**Theorem 4.9:** Every s**r-normal $R_0$ space is s*r-regular.

**Proof:** Suppose X is s**r-normal and $R_0$. Let F be a closed set and x∉F. Since X is $R_0$, by Theorem 2.8(i), *Cl*({x})∩F=ϕ. Since X is s**r-normal, there exist disjoint semi*regular open sets U and V in X containing *Cl*({x}) and F respectively. It follows that X is s*r-regular.

**Corollary 4.10:** Every s**r-normal $T_1$ space is s*r-regular.

**Proof:** Follows from the fact that every $T_1$ space is $R_0$ and Theorem 4.9.

**Theorem 4.11:** If *f* is an injective and semi*r-irresolute and pre-semi*regular closed mapping of a topological space X into a semi*r-normal space Y, then X is semi*r-normal.

**Proof:** Let *f* be an injective and semi*r-irresolute and pre-semi*regular closed mapping of a topological space X into a semi*r-normal space Y. Let A and B be disjoint semi*regular closed sets in X. Since *f* is a pre-semi*regular closed function, *f*(A) and *f*(B) are disjoint semi*regular closed sets in Y. Since Y is semi*r-normal, there exist disjoint semi*regular open sets $V_1$ and $V_2$ in Y containing *f*(A) and *f*(B) respectively. Since *f* is semi*r-irresolute, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint semi*regular open sets in X containing A and B respectively. Hence X is semi*r-normal.

**Theorem 4.12:** If *f* is an injective and semi*r-continuous and closed mapping of a topological space X into a normal space Y and if every semi*regular closed set in X is closed, then X is semi*r-normal.

**Proof:** Let A and B be disjoint semi*regular closed sets in X. By assumption, A and B are closed in X. Then *f*(A) and *f*(B) are disjoint closed sets in Y. Since Y is normal, there exist disjoint open sets $V_1$ and $V_2$ in Y such that $f(A)\subseteq V_1$ and $f(B)\subseteq V_2$.Then $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint semi*regular open sets in X containing A and B respectively.Hence X is semi*r- normal.

**Theorem 4.13:** If $f:X\longrightarrow Y$ is a semi*r-irresolute injection which is pre-semi*regular open and X is semi*r-normal, then Y is also semi*r-normal.

**Proof:** Let $f:X\longrightarrow Y$ be a semi*r-irresolute surjection which is semi*regular open and X be semi*r-normal. Let A and B be disjoint semi*regular closed sets in Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint semi*regular closed sets in X. Since X is semi*r-normal, there exist disjoint semi*regular open sets $U_1$ and $U_2$ containing $f^{-1}(A)$ and $f^{-1}(B)$ respectively. Since *f* is pre-semi*regular open, $f(U_1)$ and $f(U_2)$ are disjoint semi*regular open sets in Y containing A and B respectively. Hence Y is semi*r-normal.

**Remark 4.14:** It is not always true that a semi*r-normal space X is semi*r-regular as shown in the following example. However it is true if X is semi*r-$R_0$ as seen in Theorem 4.16.

**Example 4.15:** Let X={a, b, c, d} with topology □={ϕ, {a, b}, X}. Clearly (X,□) is semi*-normal but not semi*-regular.

**Theorem 4.16:** Every semi*r-normal space that is semi*r-$R_0$ is semi*r-regular.

**Proof:** Suppose X is semi*r-normal that is semi*r-$R_0$. Let F be a semi*regular closed set and x∉F. Since X is semi*r-$R_0$, by Theorem 2.8(ii), *s**r*Cl*({x})∩F=ϕ. Since X is semi*r-normal, there exist disjoint semi*regular open sets U and V in X containing *s**r*Cl*({x}) and F respectively. It follows that X is semi*r-regular.

**Corollary 4.17:** Every semi*r-normal semi*r-$T_1$ space is semi*r-regular.

**Proof:** Follows from the fact that every semi*r-$T_1$ space is semi*r-$R_0$ and Theorem 4.13.

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