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On Regular *– Open Sets

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Abstract

The aim of this paper is to define some new sets namely regular*-border, regular*-frontier and regular*-exterior of a subset in a topological space using the regular*-interior and regular*-closure. Further their properties are investigated. Also various functions associated with regular*-open sets and their contra versions are introduced and their properties are discussed.

Keywords: Regular*-border, regular*- frontier, regular*-exterior, regular*-open function, regular*-closed function, regular*-homeomorphism.

Mathematical Subject Classification: 54A05, 54C05.

1. INTRODUCTION

In 1963 Levine [8] introduced the concept of semi-open sets and semi continuity in topological spaces. Levine also defined and studied generalized closed sets in 1970. Dunham [6] introduced the concept of generalized closure using Levine's [7] generalized closed sets and defined a new topology τ^* and studied its properties. Navalagi[9] defined the frontier of a subset using pre-open sets, α-open sets and semi-pre- open sets. Pasunkili Pandian[12] defined semi*-preborder, semi*-prefrontier and semi*-preexterior of a subset using the semi*-preinterior and semi*-preclosure and investigated their properties. Dontchev [4] introduced contra continuous functions. Crossely and Hildebrand[3] defined pre-semi open functions. Noiri introduced and studied semi-closed functions. In 1997, contra-open and contra-closed functions were introduced by Baker. S.Pasunkili Pandian[12] defined semi*-pre-continuous and semi*-pre-irresolute functions and their contra versions and investigate their properties. The concept of regular continuous function was first introduced by Arya.S.P[1]. Recently, the authors [13] introduced some new concepts, namely regular*-open sets, regular*-closed sets, regular*-closure, regular*-interior and regular*-derived set. The authors[14] also introduced various functions associated with regular*-open sets and their properties are investigated.

In this paper we define some new sets namely regular*-border, regular*-frontier and regular*-exterior of a subset in topological space using the regular*-interior and regular*-closure. Further their properties are investigated. Also various functions associated with regular*-open sets and their contra versions are introduced and their properties are discussed.

2. PRELIMINARIES

Throughout this paper X, Y and Z will always denote topological spaces on which no separation axioms are assumed.

Definition 2.1: [7] A subset A of a topological space (X, τ) is called (i) **generalized closed** (briefly g-closed) if $Cl(A)\subseteq U$ whenever A⊆U and U is open.
(ii) **generalized open** (briefly g-open) if X\A is g-closed in X.

Definition 2.2: [6] Let A be a subset of X. Then (i) **generalized closure** of A is defined as the intersection of all g-closed sets containing A, and is denoted by $Cl^*(A)$.
(ii) **generalized interior** of A is defined as the union of all g-open subsets of A and is denoted by $Int^*(A)$.

Definition 2.3: [13] A subset A of a topological space (X,τ) is (i) **Regular*-open** (resp. pre-open, regular open) if $A=Int(Cl^*(A))$ (resp. $A\subseteq Int(Cl(A))$, $A=Int(Cl(A))$).
(ii) **Reguler*-closed** (resp. pre-closed, regular closed) if $A=Cl(Int^*(A))$ (resp. $Cl(Int(A))\subseteq A$, $A=Cl(Int(A))$)

Definition 2.4: [13] Let A be a subset of X. Then (i) The **regular*-interior** of A is defined as the union of all regular*-open subsets of A and is denoted by $r^*Int(A)$.
(ii) The **regular*-closure** of A is defined as the intersection of all regular*-closed sets containing A and is denoted by $r^*Cl(A)$.

Theorem 2.5: [13] (i) Every regular-open set is regular*-open. (ii) Every regular*-

open set is open. (iii) Every regular*-open set is pre-open.

Remark 2.6: Similar result for regular*-closed sets are also true.
Theorem 2.7: [13] (i) Intersection of any two regular*-open sets is regular*-open. (ii). Union of any two regular*-closed sets is regular*-closed.

Theorem 2.8: If A and B are subsets of a topological space X, then

(i). $rInt(A) \subseteq r^*Int(A) \subseteq Int(A) \subseteq A$.

(ii). $A \subseteq Cl(A) \subseteq r^*Cl(A) \subseteq rCl(A)$.

Definition 2.9: If A is a subset of X, (i) **the border of A** is defined by $Bd(A)=A \setminus Int(A)$

(ii). **the frontier of A** is defined by $Fr(A)=Cl(A) \setminus Int(A)$.

(iii). **the exterior of A** is defined by $Ext(A)=Int(X \setminus A)$.

Definition 2.10: A function $f: X \rightarrow Y$ is said to be **regular-continuous** if $f^{-1}(F)$ is regular closed in X for every closed set F in Y.

Definition 2.11: [14] A function $f: X \rightarrow Y$ is said to be **regular*-continuous** (contra-regular*-continuous) if $f^{-1}(V)$ is regular*-open (regular*-closed) in X for every open set V in Y.

Definition 2.12: [14] A function $f: X \rightarrow Y$ is said to be **regular*-irresolute** (resp. contra-regular*-irresolute, strongly regular*-irresolute, contra-strongly regular*-irresolute) if $f^{-1}(V)$ is regular*-open (resp. regular*-closed, open, closed) in X for every regular*-open set V in Y.

Theorem 2.13: (i) Composition of two regular*-continuous functions is regular*-continuous.
(ii) Composition of two regular*-irresolute functions is regular*-irresolute.

3. ON REGULAR*-OPEN SETS:

Definition 3.1: A subset A of X is called **regular*-regular** if it is both regular*-open and regular*-closed.

Example 3.2: Let X = {a, b, c, d}, τ = {ϕ, {a}, {b, c, d}, X}. Here {a}, {b, c, d} are both regular*-open and regular*-closed. Therefore {a}, {b, c, d} are regular*-regular.

Theorem 3.3: If A is a subset of X, then (i) $r^*Cl(X\backslash A)=X\backslash r^*Int(A)$
(ii). $r^*Int(X\backslash A)=X\backslash r^*Cl(A)$.
Proof: (i) Let $x\in X\backslash r^*Int(A)$ then $x\notin r^*Int(A)$.This implies that x does not belong to any regular*-open subset of A. Let F be a regular*-closed set containing X\A. Then X\F is a regular*-open set contained in A. Therefore $x\notin X\backslash F$ and so $x\in F$. Hence $x\in r^*Cl(X\backslash A)$. Therefore $X\backslash r^*Int(A)\subseteq r^*Cl(X\backslash A)$. On the otherhand, let $x\in r^*Cl(X\backslash A)$. Then x belongs to every regular*-closed set containing X\A. Hence x does not belong to any regular*-open subset of A. That is $x\notin r^*Int(A)$, then $x\in X\backslash r^*Int(A)$. This proves (i).
(ii). Can be proved by replacing A by X\A in (i) and using set theoretic properties.

Definition 3.4: The **regular*-borde**r of a subset A of X is defined by $r^*Bd(A)=A\backslash r^*Int(A)$.

Theorem 3.5: In any topological space (X, τ), the following statements hold:
(i) $r^*Bd(\phi)=\phi$
(ii) $r^*Bd(X)=X$

For any subset A of X,

(iii) $r^*Bd(A)\subseteq A$
(iv) $A=r^*Int(A)\cup r^*Bd(A)$
(v) $r^*Int(A)\cap r^*Bd(A)=\phi$
(vi) $r^*Int(A)=A\backslash r^*Bd(A)$
(vii) $rBd(A)\subseteq r^*Bd(A)\subseteq Bd(A)\subseteq A$
(viii) $r^*Int(r^*Bd(A))=\phi$
(ix) If A is regular*-open, then $r^*Bd(A)=\phi$
(x) $r^*Bd(r^*Int(A))=\phi$, if A is regular*-open
(xi) $r^*Bd(r^*Bd(A))=r^*Bd(A)$
(xii) $r^*Bd(A)=A\cap r^*Cl(X\backslash A)$

Proof: The statements (i), (ii), (iii), (iv), (v) and (vi) follow from Definition 3.4. The statement (vii) follows from Definition 3.4 and Theorem 2.8. To prove (viii): If possible, let $x\in r^*Int(r^*Bd(A))$. Then $x\in r^*Bd(A)$. Since $r^*Bd(A)\subseteq A$, $x\in r^*Int(r^*Bd(A))\subseteq r^*Int(A)$. This implies $x\in r^*Int(A)\cap r^*Bd(A)$ which contradicts to (v). This proves (viii). Since A is regular*-open, $r^*Int(A)=A$. That is $r^*Bd(A)=\phi$. This proves (ix). The statement (x) follows from (ix). From Definition 3.4, $r^*Bd(r^*Bd(A))=r^*Bd(A)\backslash r^*Int(r^*Bd(A))$. Since by (viii) $r^*Int(r^*Bd(A))=\phi$, we have $r^*Bd(r^*Bd(A))=r^*Bd(A)$. This proves (xi). From Definition 3.4, we have $r^*Bd(A)=A\backslash r^*Int(A)=A\cap(X\backslash r^*Int(A))$. Apply Theorem 3.3 we get $r^*Bd(A)=A\cap r^*Cl(X\backslash A)$. This proves (xii).

Remark 3.6: It can be easily seen that the inclusion in (vii) of Theorem 3.5 involving $r^*Bd(A)$ may be strict and equality may hold as well.

Definition 3.7: If A is subset of X, then **the regular*-frontier** of A is defined by $r^*Fr(A)=r^*Cl(A)\backslash r^*Int(A)$.

Theorem 3.8: If A is a subset of X, the following hold:

(i) $r^*Fr(\phi)=\phi$
(ii) $r^*Fr(X)=\phi$
(iii) $r^*Cl(A)=r^*Int(A)\cup r^*Fr(A)$
(iv) $r^*Int(A)\cap r^*Fr(A)=\phi$
(v) $r^*Bd(A)\subseteq r^*Fr(A)\subseteq r^*Cl(A)$
(vi) If A is regular*-closed then, $A=r^*Int(A)\cup r^*Fr(A)$
(vii) $rFr(A)\subseteq r^*Fr(A)\subseteq Fr(A)\subseteq A$
(viii) $r^*Fr(A)=r^*Cl(A)\cap r^*Cl(X\backslash A)$
(ix) $r^*Fr(A)=r^*Fr(X\backslash A)$
(x) If A is regular*-closed, then $r^*Fr(A)=r^*Bd(A)$
(xi) If A is regular*-regular, then $r^*Fr(A)=\phi$
(xii) $r^*Int(A)=A\backslash r^*Fr(A)$
(xiii) $X=r^*Int(A)\cup r^*Int(X\backslash A)\cup r^*Fr(A)$

Proof: (i), (ii), (iii), (iv) and (v) follow from Definition 3.7. Clearly (vi) follows from (iii) above. The statement (vii) follows from Theorem 2.8. (viii) follows from Definition 3.7 and Theorem 3.3. (ix) follows from (viii). To prove (x): If A is regular*-closed, then $r^*Cl(A)=A$. Hence $r^*Fr(A)=r^*Cl(A)\backslash r^*Int(A)=A\backslash r^*Int(A)=r^*Bd(A)$. (xi) follows from Definition 3.1, Definition 2.3. (xii) follows from Definition 3.7 and set theoretic properties. (xiii) follows from the fact $X=r^*Cl(A)\cup(X\backslash r^*Cl(A))$ and from (iii) and Theorem 3.3.

Remark 3.9: It can be easily seen that the inclusion in (vii) of Theorem 3.8 involving $r^*Fr(A)$ may be strict and equality may hold as well.

Theorem 3.10: If A and B are subsets of X such that A⊆B and B is regular*-closed in X, then $r^*Fr(A)\subseteq B$.

Proof: $r^*Fr(A)=r^*Cl(A)\backslash r^*Int(A)\subseteq r^*Cl(B)\backslash r^*Int(A)=B\backslash r^*Int(A)\subseteq B$.

Definition 3.11: If A is a subset of X, the **regular*-exterior** of A is defined by $r^*Ext(A)=r^*Int(A)$

Theorem 3.12: In any topological space (X, τ), the following statement hold:

(i) $r^*Ext(\phi)=X$
(ii) $r^*Ext(X)=\phi$

If A and B are subsets of X,

(iii) $A\subseteq B\Rightarrow r^*Ext(B)\subseteq r^*Ext(A)$
(iv) $rExt(A)\subseteq r^*Ext(A)\subseteq Ext(A)\subseteq X\backslash A$
(v) If A is regular*-closed then $r^*Ext(A)=X\backslash A$
(vi) $r^*Ext(A)=X\backslash r^*Cl(A)$
(vii) $r^*Ext(r^*Ext(A))=r^*Int(r^*Cl(A))$
(viii) If A is regular*-regular then $r^*Ext(r^*Ext(A))=A$
(ix) $r^*Ext(A)=r^*Ext(X\backslash r^*Ext(A))$
(x) $r^*Int(A)\subseteq r^*Ext(r^*Ext(A))$
(xi) $X=r^*Int(A)\cup r^*Ext(A)\cup r^*Fr(A)$
(xii) $r^*Ext(A\cup B)\subseteq r^*Ext(A)\cap r^*Ext(B)$
(xiii) $r^*Ext(A\cap B)\supseteq r^*Ext(A)\cup r^*Ext(B)$

Proof: (i), (ii) and (iii) follow from Definition 3.11. (iv) follows from Theorem 2.8. (v) follows from Definition 3.11. (vi) follows from Definition 3.11 and Theorem 3.3. (vii) follows from Definition 3.11 and Theorem 3.3. (viii) If A is regular*-regular, from (vi) we have $r^*Ext(A)=X\backslash A$. Hence $r^*Ext(r^*Ext(A))=A$. Now from Definition 3.11, we have $r^*Ext(X\backslash r^*Ext(A))=r^*Ext(X\backslash r^*Int(X\backslash A))=r^*Int(X\backslash (X\backslash r^*Int(X\backslash A)))=r^*Int(r^*Int(X\backslash A))= r^*Int(X\backslash A)=r^*Ext(A)$. This proves (ix). (x) follows from (vi) and the fact that $A\subseteq r^*Cl(A)$. (xi) follows from Definition 3.11 and Theorem 3.8(xiii). (xii) and (xiii) follow from (iii) and using set theoretic results.

Remark 3.13: It can be easily seen that the inclusion in (iv), (xii) and (xiii) of Theorem 3.12 may be strict and equality may hold as well.

4. MORE FUNCTIONS ASSOCIATED WITH REGULAR*-OPEN SETS

Definition 4.1: A function $f: X\rightarrow Y$ is said to be **regular*-open** if $f(U)$ is regular*-open in Y for every open set U in X.

Definition 4.2: A function $f: X\rightarrow Y$ is said to be **contra-regular*-open** if $f(U)$ is regular*-closed in Y for every open set U in X.

Definition 4.3: A function $f: X\rightarrow Y$ is said to be **pre-regular*-open** if $f(U)$ is regular*-open in Y for every regular*-open set U in X.

Definition 4.4: A function $f: X \to Y$ is said to be **contra-pre-regular*-open** if $f(U)$ is regular*-closed in Y for every regular*-open set U in X.

Definition 4.5: A function $f: X \to Y$ is said to be **regular*-closed** if $f(F)$ is regular*-closed in Y for every closed set F in X.

Definition 4.6: A function $f: X \to Y$ is said to be **contra-regular*-closed** if $f(F)$ is regular*-open in Y for every closed set F in X.

Definition 4.7: A function $f: X \to Y$ is said to be **pre-regular*-closed** if $f(F)$ is regular*-closed in Y for every regular*-closed set F in X.

Definition 4.8: A function $f: X \to Y$ is said to be **contra-pre-regular*-closed** if $f(F)$ is regular*-open in Y for every regular*-closed set F in X.

Definition 4.9: A bijection $f: X \to Y$ is called **regular*-homeomorphism** if f is both regular*-irresolute and pre-regular*-open.

The set of all regular*-homeomorphisms of (X, τ) into itself is denoted by $r^*H(X, \tau)$.

Definition 4.10: A function $f: X \to Y$ is said to be **regular*-totally continuous** if $f^{-1}(V)$ is clopen in X for every regular*-open set V in Y.

Definition 4.11: A function $f: X \to Y$ is said to be **totally regular*-continuous** if $f^{-1}(V)$ is regular*-regular in X for every open set V in Y.

Theorem 4.12: (i) Every pre-regular*-open function is regular*-open.

(ii) Every regular-open function is regular*-open.

(iii) Every contra-pre-regular*-open function is contra-regular*-open.

(iv) Every pre-regular*-closed function is regular*-closed.

(v) Every contra-pre-regular*-closed function is contra-regular*-closed.

Proof: Follow from Definitions, Theorem 2.5 and Remark 2.6.

Theorem 4.13: Let $f: X \to Y$ and $g: Y \to Z$ be functions. Then

(i) $g \circ f$ is pre-regular*-open if both f and g are pre-regular*-open.

(ii) $g \circ f$ is regular*-open if f is pre-regular*-open and g is regular*-open.

(iii) $g \circ f$ is pre-regular*-closed if both f and g are pre-regular*-closed.

(iv) $g \circ f$ is regular*-closed if f is pre-regular*-closed and g is regular*-closed.

Proof: Follow from Definitions.

Theorem 4.14: Let $f{:}X\rightarrow Y$ be a function where X is an Alexandroff space and Y is topological space. Then the following are equivalent:

(i) f is regular*-totally continuous.

(ii) For each $x\in X$ and each regular*-open set V inY with $f(x)\in V$, there exists a clopen set U in X such that $x\in U$ and $f(U)\subseteq V$.

Proof: (i)⇒(ii): Suppose $f{:}X\rightarrow Y$ is regular*-totally continuous. Let $x\in X$ and let V be a regular*-open set containing $f(x)$. then $U=f^{-1}(V)$ is a clopen set in X containing x and hence $f(U)\subseteq V$.

(ii)⇒(i): Let V be a regular*-open set in Y. Let $x\in f^{-1}(V)$. then V is a regular*-open set containing $f(x)$. By hypothesis, there exists a clopen set U_x containing x such that $f(U_x)\subseteq V$ which implies that $U_x\subseteq f^{-1}(V)$. therefore we have $f^{-1}(V)=\cup\{U_x{:}x\in f^{-1}(V)\}$. Since U_x is open, $f^{-1}(V)$ is open. Since each U_x is closed in the Alexandroff space X, $f^{-1}(V)$ is closed in X. Hence $f^{-1}(V)$ is clopen in X.

Theorem 4.15: A function $f{:}X\rightarrow Y$ is regular*-totally continuous if and only if $f^{-1}(F)$ is clopen in X for every regular*-closed set F in Y.

Proof: Follows from Definition.

Theorem 4.16: A function $f{:}X\rightarrow Y$ is totally regular*-continuous if and only if f is both regular*-continuous and contra-regular*-continuous.

Proof: Follows from Definition.

Theorem 4.17: A function $f{:}X\rightarrow Y$ is regular*- totally continuous if and only if f is both strongly regular*-irresolute and contra-strongly regular*-irresolute.

Proof: Follows from Definition.

Theorem 4.18: Let $f{:}\ X\rightarrow Y$ be regular*-totally continuous and A is a subset of Y. then the restriction $f\backslash_{A}{:}A\rightarrow Y$ is regular*-totally continuous.

Proof: Let V be a regular*-open set in Y, then $f^{-1}(V)$ is clopen in X and hence $(f\backslash_{A})^{-1}(V)=A\cap f^{-1}(V)$ is clopen in A. Hence the theorem follows.

Theorem 4.19: Inverse of a bijective regular*-irresolute function is also regular*-irresolute.

Proof: Follows from definition and set theoretic results.

Theorem 4.20: Let $f{:}\ X\rightarrow Y$ be a function. Then f is not regular*-irresolute at a point x in X if and only if x belongs to the regular*-frontier of the inverse image of some regular*-open set in Y containing $f(x)$.

Proof: Suppose f is not regular*-irresolute at x, then by definition there is a regular*-open set V in Y containing $f(x)$ such that $f(U)$ is not a subset of V for every regular*-open set U in X containing x. Hence $U\cap(X\backslash f^{-1}(V))=\phi$ for every regular*-open set U containing x. Thus $x \in r^*Cl(X \setminus f^{-1}(V))$. Since $x \in f^{-1}(V) \subseteq r^*Cl(X \setminus f^{-1}(V))$, we have $x \in r^*Cl(f^{-1}(V)) \cap r^*Cl(X \setminus f^{-1}(V))$. Hence by Theorem 3.8 $x \in r^*Fr(f^{-1}(V))$. On the other hand, let f be regular*-irresolute at x. Let V be a regular*-open set in Y containing $f(x)$, then $f(x) \in f(U) \subseteq V$. Therefore $U \subseteq f^{-1}(V)$ and hence $x \in r^*Int(f^{-1}(V))$. Therefore by Definition, $x \notin r^*Fr(f^{-1}(V))$ for every open set V containing $f(x)$.

Theorem 4.21: Let $f: X \to Y$ be a bijection. Then the following are equivalent:

(i) f is regular*-irresolute

(ii) f^{-1} is pre-regular*-open

(iii) f^{-1} pre-regular*-closed

Proof: Follows from Definitions.

Theorem 4.22: A bijection $f: X \to Y$ is a regular*-homeomorphism if and only if f and f^{-1} are regular*-irresolute

Proof: Follows from Definition.

Theorem 4.23: (i) The composition of two regular*-homeomorphism is a regular*-homeomorphism.

(ii) The inverse of a regular*-homeomorphism is also a regular*-homeomorphism.

Proof: (i) Let $f: X \to Y$ and $g: Y \to Z$ be regular*-homeomorphisms. By Theorem 2.13(i) And Theorem 4.13(i) $g \circ f$ is a regular*-homeomorphism.

(ii). Let $f: X \to Y$ be a regular*-homeomorphism. Then by Theorem 4.19 and Theorem 4.23, $f^{-1}: Y \to X$ is also regular*-homeomorphism.

Theorem 4.24: If (X, τ) is a topological space, then the set $r^*H(X, \tau)$ of all regular*-homeomorphisms of (X, τ) into itself forms a group.

Proof: Since the identity mapping I on X is regular*-homeomorphism, $I \in r^*H(X, \tau)$ and hence $r^*H(X, \tau)$ is non-empty and the theorem follows from Theorem 4.23.

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