Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 9 (2017), pp. 5703-5716

http://www.ripublication.com

Common Fixed Point Theorem on Compatibility and Continuity in Complex Valued Metric Spaces

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Abstract

In this paper, we generalized the result some common fixed point theorems in complex valued metric spaces of Mala Hakwadiya, R K Gujetiya, Dheeraj kumara mali [6].

Keywords: Common fixed point, complex valued metric space, weakly compatible mappings.

I. INTRODUCTION

In 2011, Azam et al. [1] introduced the notion of complex valued metric space, which is a generalized of the classical metric space and established some fixed point result for mappings satisfying a rational inequality. In what follows, we recall some notations and definitions that will be utilized in our subsequent discussion. Let $\mathbb{C}$ be the set of complex numbers and$z_1, z_2 \in \mathbb{C}$: Define a partial order $\precsim$ on $\mathbb{C}$ as follows: $z_1 \precsim z_2$ if and only if $Re(z_1) \leq Re(z_2)$; $Im(z_1) \leq Im(z_2)$ Consequently, one can infer that $z_1 \precsim z_2$ if one of the following conditions is satisfied: (i) $Re(z_1)=Re(z_2)$; $Im(z_1)<Im(z_2)$, (ii) $Re(z_1)<Re(z_2)$; $Im(z_1)=Im(z_2)$, (iii) $Re(z_1)<Re(z_2)$; $Im(z_1)<Im(z_2)$, (iv) $Re(z_1)=Re(z_2)$; $Im(z_1)=Im(z_2)$, In particular, we write $z_1 \precsim z_2$ if $z_1 \neq z_2$ and one of (i), (ii), and (iii) is satisfied and we write $z_1 \precsim z_2$ if only (iii) is satisfied. Notice that $0 \precsim z_1 \precsim z_2 \Rightarrow |z_1| < |z_1|$ and $z_1 \precsim z_2$, $z_2 \precsim z_3 \Rightarrow z_1 \precsim z_3$. Recently, Azam et al. [1] introduced the notion of complex valued metric spaces and established some fixed point results for a pair of mappings for contraction condition satisfying a rational expression. Though complex valued metric spaces form a special class of cone metric space, yet this idea is intended to define rational expressions which are not meaningful in cone metric spaces and thus many results of analysis cannot be generalized to cone metric spaces.

II. PRELIMINARIES:

Definition 2.1. [4] Let X be a nonempty set whereas $\mathbb{C}$ be the set of complex numbers. Suppose that the mapping $d:X\times X\rightarrow \mathbb{C}$ satisfies following conditions:

(1) $0 \precsim d(x, y)$ for all x, y ∈ X and d(x,y)=0 if and only if x=y ;

(2) d(x, y)= d(y, x) for all x, y ∈ X;

(3) $d(x, y) \precsim d(x, z) + d(z, y)$ for all x, y, z ∈ X. Then d is called a complex valued metric on X, and (X, d) is called a complex valued metric space.

Definition 2.2. [9] Let (X, d) be a complex valued metric space and $\{x_n\}$ be a sequence in X.

(1) If for every c ∈ $\mathbb{C}$ with 0< c, there exists N ∈ $\mathbb{N}$ such that $d(x_n, x) < c$ for all n ≥ N then $\{x_n\}$ is said to be convergent to x ∈ X , and we denote this by $x_n \rightarrow x$ as $n \rightarrow \infty$.

(2) If for every c∈ $\mathbb{C}$ with 0< c, there exists N ∈ $\mathbb{N}$ such that $d(x_n, x_{n+m}) < c$ for all n ≥ N, where m∈ $\mathbb{N}$, then { } is said to be Cauchy sequence.

(3) If every Cauchy sequence in X is convergent, then (X, d) is said to be a complete complex valued metric space.

Lemma 2.3. [9] Let (X, d) be a complex valued metric space and $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n,x)|\rightarrow 0$ as $n\rightarrow\infty$.

Lemma 2.4. [9] Let (X, d) be a complex valued metric space and $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n,x_{n+m})|\rightarrow 0$ as $n\rightarrow\infty$, where m ∈ N.

Definition 2.5. [9] Let f and g be two self-mappings of a metric space (X, d). Then a pair (f, g) is said to be weakly compatible if they commute at coincidence points.

Definition 2.6. [3] Let A and S be mappings from a complete metric space X into itself. The mappings A and S are said to be compatible if $\lim_{n\rightarrow\infty} d(ASx_n, SAx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\rightarrow\infty} Ax_n = \lim_{n\rightarrow\infty} Sx_n = t$ for some t ∈X .

III. MAIN RESULT:

Theorem 3.1 : Let (X, d) be a complex valued metric space and A, B, D, M, S and T be six self mappings in X satisfying the condition:

1. S(X) ⊂ BD(X) and T(X) ⊂ AM(X).

2. For each x, y ∈ X, such that x ≠ y, d(Ty, AMx) + d(Sx, BDy)+ d(BDy, AMx) ≠ 0, where α, β, γ, η, and ξ are non negative real number with α+ β+2 γ+ η+ ξ < 1, or d(Sx, Ty) = 0 if d(Ty, AMx) + d(Sx, BDy)+ d(BDy, AMx) = 0, such that

$$d(Sx, Ty) \le \alpha \left[\frac{d(AMx, Sx)+ d(BDy, Ty).d(AMx, Sx)}{1+d(Sx, Ty)}\right] + \beta \max\left\{\frac{d(AMx, BDy)}{1+d(AMx, BDy)}, \frac{d(AMx, Sx)}{1+d(AMx, Sx)}, \frac{d(BDy, Sx)}{1+d(BDy, Sx)}\right\} + \gamma \left[\frac{d(BDy, Ty)+d(Ty, AMx)+d(Sx, BDy)}{1+d(BDy, Ty)+d(Ty, AMx)+d(Sx, BDy)}\right] + \eta \left[\frac{d(Ty, BDy).d(Sx, AMx)}{d(Ty, AMx)+d(Sx, BDy)+d(BDy, AMx)}\right] + \xi \left[\frac{d(Sx, BDy)+d(BDy, AMx)}{1+d(Sx, BDy).d(Ty, BDy).d(Sx, AMx)}\right]$$

3. The pair (AM, S) and (BD, T) are commute.

4. The pair (AM, S) and (BD, T) are weakly compatible.

Then A, B, D, M, S and T have a unique common fixed point.

Proof : Let $x_0 \in X$. Since S(X) ⊂ BD(X) and T(X) ⊂ AM(X), define for each n ≥ 0, the sequence $\{y_n\}$ in X by

$$y_{2n+1} = Sx_{2n} = BDx_{2n+1} \text{ and } y_{2n+2} = Tx_{2n+1} = AMx_{2n+2}, \quad n = 0, 1, 2, \ldots\ldots$$

$$d(y_{2n+1}, y_{2n+2}) = d(Sx_{2n},Tx_{2n+1}) \le \alpha \left[\frac{d(AMx_{2n}, Sx_{2n})+ d(BDx_{2n+1}, Tx_{2n+1}).d(AMx_{2n}, Sx_{2n})}{1+d(Sx_{2n}, Tx_{2n+1})}\right]$$

$$+ \beta \max\left\{\frac{d(AMx_{2n}, BDx_{2n+1})}{1+d(AMx_{2n}, BDx_{2n+1})}, \frac{d(AMx_{2n}, Sx_{2n})}{1+d(AMx_{2n}, Sx_{2n})}, \frac{d(BDx_{2n+1}, Sx_{2n})}{1+d(BDx_{2n+1}, Sx_{2n})}\right\} + \gamma \left[\frac{d(BDx_{2n+1}, Tx_{2n+1})+d(Tx_{2n+1}, AMx_{2n})+d(Sx_{2n}, BDx_{2n+1})}{1+d(BDx_{2n+1}, Tx_{2n+1})+d(Tx_{2n+1}, AMx_{2n})+d(Sx_{2n}, BDx_{2n+1})}\right] + \eta \left[\frac{d(Tx_{2n+1}, BDx_{2n+1}).d(Sx_{2n}, AMx_{2n})}{d(Tx_{2n+1}, AMx_{2n})+d(Sx_{2n}, BDx_{2n+1})+d(BDx_{2n+1}, AMx_{2n})}\right] + \xi \left[\frac{d(Sx_{2n}, BDx_{2n+1})+d(BDx_{2n+1}, AMx_{2n})}{1+d(Sx_{2n}, BDx_{2n+1}).d(Tx_{2n+1}, BDx_{2n+1}).d(Sx_{2n}, AMx_{2n})}\right]$$

$$d(y_{2n+1}, y_{2n+2}) \le \alpha \left[\frac{d(y_{2n}, y_{2n+1})+ d(y_{2n+1}, y_{2n+2}).d(y_{2n}, y_{2n+1})}{1+d(y_{2n+1}, y_{2n+2})}\right] + \beta \max\left\{\frac{d(y_{2n}, y_{2n+1})}{1+d(y_{2n}, y_{2n+1})}, \frac{d(y_{2n}, y_{2n+1})}{1+d(y_{2n}, y_{2n+1})}, \frac{d(y_{2n+1}, y_{2n+1})}{1+d(y_{2n+1}, y_{2n+1})}\right\} + \gamma \left[\frac{d(y_{2n+1}, y_{2n+2})+d(y_{2n+2}, y_{2n})+d(y_{2n+1}, y_{2n+1})}{1+d(y_{2n+1}, y_{2n+2})+d(y_{2n+2}, y_{2n})+d(y_{2n+1}, y_{2n+1})}\right] + \eta \left[\frac{d(y_{2n+2}, y_{2n+1}).d(y_{2n+1}, y_{2n})}{d(y_{2n+2}, y_{2n})+d(y_{2n+1}, y_{2n+1})+d(y_{2n+1}, y_{2n})}\right] + \xi \left[\frac{d(y_{2n+1}, y_{2n+1})+d(y_{2n+1}, y_{2n})}{1+d(y_{2n+1}, y_{2n+1}).d(y_{2n+2}, y_{2n+1}).d(y_{2n+1}, y_{2n})}\right]$$

$$d(y_{2n+1}, y_{2n+2}) \le \alpha\ d(y_{2n}, y_{2n+1}) + \beta \max\{d(y_{2n}, y_{2n+1}), d(y_{2n}, y_{2n+1}), d(y_{2n+1}, y_{2n+1})\} + \gamma [d(y_{2n+1}, y_{2n+2}) + d(y_{2n+2}, y_{2n}) + d(y_{2n+1}, y_{2n+1})] + \eta\ d(y_{2n}, y_{2n+1}) + \xi\ d(y_{2n+1}, y_{2n})$$

$d(y_{2n+1}, y_{2n+2}) \leq (\alpha + \beta + \gamma + \eta + \xi)\, d(y_{2n}, y_{2n+1})$

That is $|\, d(y_{2n+1}, y_{2n+2}) \,| \leq (\alpha + \beta + \gamma + \eta + \xi)\, |\, d(y_{2n}, y_{2n+1}) \,|$ ………….. (3.1)

Similarly,

$$d(y_{2n+3}, y_{2n+4}) = d(Sx_{2n+2}, Tx_{2n+3}) \leq \alpha \left[\frac{d(AMx_{2n+2}, Sx_{2n+2}) + d(BDx_{2n+3}, Tx_{2n+3}).d(AMx_{2n+2}, Sx_{2n+2})}{1+d(Sx_{2n+2}, Tx_{2n+3})}\right]$$

$$+ \beta \max\left\{\frac{d(AMx_{2n+2}, BDx_{2n+3})}{1+d(AMx_{2n+2}, BDx_{2n+3})}, \frac{d(AMx_{2n+2}, Sx_{2n+2})}{1+d(AMx_{2n+2}, Sx_{2n+2})}, \frac{d(BDx_{2n+3}, Sx_{2n+2})}{1+d(BDx_{2n+3}, Sx_{2n+2})}\right\} + \gamma \left[\frac{d(BDx_{2n+3}, Tx_{2n+3})+d(Tx_{2n+3}, AMx_{2n+2})+d(Sx_{2n+2}, BDx_{2n+3})}{1+d(BDx_{2n+3}, Tx_{2n+3})+d(Tx_{2n+3}, AMx_{2n+2})+d(Sx_{2n+2}, BDx_{2n+3})}\right] + \eta \left[\frac{d(Tx_{2n+3}, BDx_{2n+3}).d(Sx_{2n+2}, AMx_{2n+2})}{d(Tx_{2n+3}, AMx_{2n+2})+d(Sx_{2n+2}, BDx_{2n+3})+d(BDx_{2n+3}, AMx_{2n+2})}\right] + \xi \left[\frac{d(Sx_{2n+2}, BDx_{2n+3})+d(BDx_{2n+3}, AMx_{2n+2})}{1+d(Sx_{2n+2}, BDx_{2n+3}).d(Tx_{2n+3}, BDx_{2n+3}).d(Sx_{2n+2}, AMx_{2n+2})}\right]$$

$$d(y_{2n+3}, y_{2n+4}) \leq \alpha \left[\frac{d(y_{2n+2}, y_{2n+3}) + d(y_{2n+3}, y_{2n+4}).d(y_{2n+2}, y_{2n+3})}{1+d(y_{2n+3}, y_{2n+4})}\right] + \beta \max\left\{\frac{d(y_{2n+2}, y_{2n+3})}{1+d(y_{2n+2}, y_{2n+3})}, \frac{d(y_{2n+2}, y_{2n+3})}{1+d(y_{2n+2}, y_{2n+3})}, \frac{d(y_{2n+3}, y_{2n+3})}{1+d(y_{2n+3}, y_{2n+3})}\right\} + \gamma \left[\frac{d(y_{2n+3}, y_{2n+4})+d(y_{2n+4}, y_{2n+2})+d(y_{2n+3}, y_{2n+3})}{1+d(y_{2n+3}, y_{2n+4})+d(y_{2n+4}, y_{2n+2})+d(y_{2n+3}, y_{2n+3})}\right] + \eta \left[\frac{d(y_{2n+4}, y_{2n+3}).d(y_{2n+3}, y_{2n+2})}{d(y_{2n+4}, y_{2n+2})+d(y_{2n+3}, y_{2n+3})+d(y_{2n+3}, y_{2n+2})}\right] + \xi \left[\frac{d(y_{2n+3}, y_{2n+3})+d(y_{2n+3}, y_{2n+2})}{1+d(y_{2n+3}, y_{2n+3}).d(y_{2n+4}, y_{2n+3}).d(y_{2n+3}, y_{2n+2})}\right]$$

$d(y_{2n+3}, y_{2n+4}) \leq \alpha\ d(y_{2n+2}, y_{2n+3}) + \beta \max\{d(y_{2n+2}, y_{2n+3}), d(y_{2n+2}, y_{2n+3}), d(y_{2n+3}, y_{2n+3})\} + \gamma\, [d(y_{2n+3}, y_{2n+4}) + d(y_{2n+4}, y_{2n+2}) + d(y_{2n+3}, y_{2n+3})] + \eta\, d(y_{2n+2}, y_{2n+3}) + \xi\, d(y_{2n+3}, y_{2n+2})$

$d(y_{2n+3}, y_{2n+4}) \leq (\alpha + \beta + \gamma + \eta + \xi)\, d(y_{2n+2}, y_{2n+3})$

That is $|\, d(y_{2n+3}, y_{2n+4}) \,| \leq (\alpha + \beta + \gamma + \eta + \xi)\, |\, d(y_{2n+2}, y_{2n+3}) \,|$ ………….. (3.2)

Therefore from (3.1) and (3.2) $d(y_n, y_{n+1}) \leq (\alpha + \beta + \gamma + \eta + \xi)\, d(y_{n-1}, y_n)$. If $\delta = \alpha + \beta + \gamma + \eta + \xi < 1$. Then it is concluded that $d(y_n, y_{n+1}) \leq \delta\, d(y_{n-1}, y_n)$

$d(y_n, y_{n+1}) \leq \delta^2\, d(y_{n-2}, y_{n-1}) \leq \delta^3\, d(y_{n-3}, y_{n-2}) \leq \ldots\ldots \leq \delta^n\, d(y_0, y_1)$

Now for all m>n, we have $d(y_m, y_n) \leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \ldots\ldots + d(y_{m-1}, y_m)$

$\leq \delta^n\, d(y_0, y_1) + \delta^{n+1}\, d(y_0, y_1) + \ldots\ldots + \delta^{m-1}\, d(y_0, y_1)$

$|\, d(y_m, y_n)\, | \le \frac{\delta^n}{1-\delta} |\, d(y_0, y_1)\, |$

Hence $|\, d(y_m, y_n)\, | \le \frac{\delta^n}{1-\delta} |\, d(y_0, y_1)\, | \to 0$ as $m,n\to\infty$. That is $\lim_{n\to\infty} |\, d(y_m, y_n)\, | = 0$.

Hence $\{y_n\}$ is Cauchy sequence. Since X is completed, so $\{y_n\}$ is converges to some point z, that is $\lim_{n\to\infty} y_n = \lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} BDx_{2n+1} = \lim_{n\to\infty} Tx_{2n+1} = \lim_{n\to\infty} AMx_{2n+2} = z$. There exist some $u \in X$ such that $y_n \to u$ as $n\to\infty$.

Su = AMu = BDu = Tu = z. Since the pair (AM,S) and (BD,T) are weakly compatible.Then they commute at their coincidence point. Hence Sz = S(AMu) = AM(Su) = AMz and BDz = BD(Tu) = T(BDu) = Tz.

Now, we shall show that Tz = Sz. From (2) putting x = z and y = x_{2n+1} we have

$$d(Sz,Tx_{2n+1}) \le \alpha \left[\frac{d(AMz, Sz)+ d(BDx_{2n+1}, Tx_{2n+1}).d(AMz, Sz)}{1+d(Sz, Tx_{2n+1})}\right]+ \beta \max\left\{\frac{d(AMz, BDx_{2n+1})}{1+d(AMz, BDx_{2n+1})}, \frac{d(AMz, Sz)}{1+d(AMz, Sz)}, \frac{d(BDx_{2n+1}, Sz)}{1+d(BDx_{2n+1}, Sz)}\right\} + \gamma \left[\frac{d(BDx_{2n+1}, Tx_{2n+1})+d(Tx_{2n+1}, AMz)+d(Sz, BDx_{2n+1})}{1+d(BDx_{2n+1}, Tx_{2n+1})+d(Tx_{2n+1}, AMz)+d(Sz, BDx_{2n+1})}\right] + \eta \left[\frac{d(Tx_{2n+1}, BDx_{2n+1}).d(Sz, AMz)}{d(Tx_{2n+1}, AMz)+d(Sz, BDx_{2n+1})+d(BDx_{2n+1}, AMz)}\right] + \xi \left[\frac{d(Sz, BDx_{2n+1})+d(BDx_{2n+1}, AMz)}{1+d(Sz, BDx_{2n+1}).d(Tx_{2n+1}, BDx_{2n+1}).d(Sz, AMz)}\right]$$

$$d(Sz,y_{2n+2}) \le \alpha \left[\frac{d(Sz, Sz)+ d(y_{2n+1}, y_{2n+2}).d(Sz, Sz)}{1+d(Sz, y_{2n+2})}\right]+ \beta \max\left\{\frac{d(Sz, y_{2n+2})}{1+d(Sz, y_{2n+2})}, \frac{d(Sz, Sz)}{1+d(Sz, Sz)}, \frac{d(y_{2n+1}, Sz)}{1+d(y_{2n+1}, Sz)}\right\} + \gamma \left[\frac{d(y_{2n+1}, y_{2n+2})+d(y_{2n+2}, Sz)+d(Sz, y_{2n+1})}{1+d(y_{2n+1}, y_{2n+2})+d(y_{2n+2}, Sz)+d(Sz, y_{2n+1})}\right] + \eta \left[\frac{d(y_{2n+2}, y_{2n+1}).d(Sz, Sz)}{d(y_{2n+2}, Sz)+d(Sz, y_{2n+1})+d(y_{2n+1}, Sz)}\right] + \xi \left[\frac{d(Sz, y_{2n+1})+d(y_{2n+1}, Sz)}{1+d(Sz, y_{2n+1}).d(y_{2n+2}, y_{2n+1}).d(Sz, Sz)}\right]$$

Letting $n\to\infty$, we get

$$d(Sz, z) \le \alpha \left[\frac{d(Sz, Sz)+ d(z, z).d(Sz, Sz)}{1+d(Sz, z)}\right]+ \beta \max\left\{\frac{d(Sz, z)}{1+d(Sz, z)}, \frac{d(Sz, Sz)}{1+d(Sz, Sz)}, \frac{d(z, Sz)}{1+d(z, Sz)}\right\} + \gamma \left[\frac{d(z, z)+d(z, Sz)+d(Sz, z)}{1+d(z, z)+d(z, Sz)+d(Sz, z)}\right] + \eta \left[\frac{d(z, z).d(Sz, Sz)}{d(z, Sz)+d(Sz, z)+d(z, Sz)}\right] + \xi \left[\frac{d(Sz, z)+d(z, Sz)}{1+d(Sz, z).d(z, z).d(Sz, Sz))}\right]$$

$d(Sz, z) \le \beta\, d(Sz, z) + 2\gamma\, d(Sz, z) + 2\xi\, d(Sz, z)$.Then $d(Sz, z) \le (\beta + 2\gamma + 2\xi)d(Sz, z)$, that is $|d(Sz, z)| \le (\beta + 2\gamma + 2\xi)|d(Sz, z)|$, which is contradiction $\beta + 2\gamma + 2\xi < 1$.Therefore Sz = z, which implies AMz = z. Now we prove that Tz = z from (2) putting x = y = z, we get

$d(Sz,Tz) \leq \alpha\, [\frac{d(AMz,\ Sz)+ d(BDz,\ Tz).d(AMz,\ Sz)}{1+d(Sz,\ Tz)}]+ \beta \max\{\frac{d(AMz,\ BDz)}{1+d(AMz,\ BDz)}, \frac{d(AMz,\ Sz)}{1+d(AMz,\ Sz)}, \frac{d(BDz,\ Sz)}{1+d(BDz,\ Sz)}\} + \gamma\, [\frac{d(BDz,\ Tz)+d(Tz,\ AMz)+d(Sz,\ BDz)}{1+d(BDz,\ Tz)+d(Tz,\ AMz)+d(Sz,\ BDz)}] + \eta\, [\frac{d(Tz,\ BDz).d(Sz,\ AMz)}{d(Tz,\ AMz)+d(Sz,\ BDz)+d(BDz,\ AMz)}] + \xi\, [\frac{d(Sz,\ BDz)+d(BDz,\ AMz)}{1+d(Sz,\ BDz).d(Tz,\ BDz).d(Sz,\ AMz)}]$

$d(z,Tz) \leq \alpha\, [\frac{d(z,\ z)+ d(Tz,\ Tz).d(z,\ z)}{1+d(z,\ Tz)}]+ \beta \max\{\frac{d(z,\ Tz)}{1+d(z,\ Tz)}, \frac{d(z,\ z)}{1+d(z,\ z)}, \frac{d(Tz,\ z)}{1+d(Tz,\ z)}\} + \gamma\, [\frac{d(Tz,\ Tz)+d(Tz,\ z)+d(z,\ Tz)}{1+d(Tz,\ Tz)+d(Tz,\ z)+d(z,\ Tz)}] + \eta\, [\frac{d(Tz,\ Tz).d(z,\ z)}{d(Tz,\ z)+d(z,\ Tz)+d(Tz,\ z)}] + \xi\, [\frac{d(z,\ Tz)+d(Tz,\ z)}{1+d(z,\ Tz).d(Tz,\ Tz).d(z,\ z)}]$

$d(z,Tz) \leq \beta\, d(z, Tz) + 2\gamma\, d(z, Tz) + 2\xi\, d(z, Tz)$. Then $d(z,Tz) \leq (\beta + 2\gamma + 2\xi)\, d(Tz, z)$. That is

$|d(z,Tz)| \leq (\beta + 2\gamma + 2\xi)|d(Tz,z)|$, which is contradiction $\beta + 2\gamma + 2\xi < 1$.Therefore $Tz = z$, since $BDz = Tz$, which implies $BDz = z$. Now we prove that $Mz = z$ from (2) putting $x = Mz$ and $y = z$, we get

$d(S(Mz), Tz) \leq \alpha\, [\frac{d(AM(Mz),\ S(Mz))+ d(BDz,\ Tz).d(AM(Mz),\ S(Mz))}{1+d(S(Mz),\ Tz)}]+ \beta \max\{\frac{d(AM(Mz),\ BDz)}{1+d(AM(Mz),\ BDz)}, \frac{d(AM(Mz),\ S(Mz))}{1+d(AM(Mz),\ S(Mz))}, \frac{d(BDz,\ S(Mz))}{1+d(BDz,\ S(Mz))}\} + \gamma\, [\frac{d(BDz,\ Tz)+d(Tz,\ AM(Mz))+d(S(Mz),\ BDz)}{1+d(BDz,\ Tz)+d(Tz,\ AM(Mz))+d(S(Mz),\ BDz)}] + \eta\, [\frac{d(Tz,\ BDz).d(S(Mz),\ AM(Mz))}{d(Tz,\ AM(Mz))+d(S(Mz),\ BDz)+d(BDz,\ AM(Mz))}] + \xi\, [\frac{d(S(Mz),\ BDz)+d(BDz,\ AM(Mz))}{1+d(S(Mz),\ BDz).d(Tz,\ BDz).d(S(Mz),\ AM(Mz))}]$

$d(Mz, z) \leq \alpha\, [\frac{d(Mz,\ Mz)+ d(z,\ z).d(Mz,\ Mz))}{1+d(Mz,\ z)}]+ \beta \max\{\frac{d(Mz,\ z)}{1+d(Mz,\ z)}, \frac{d(Mz,\ Mz)}{1+d(Mz,\ Mz)}, \frac{d(z,\ Mz)}{1+d(z,\ Mz)}\} + \gamma\, [\frac{d(z,\ z)+d(z,\ Mz)+d(Mz,\ z)}{1+d(z,\ z)+d(z,\ Mz)+d(Mz,\ z)}] + \eta\, [\frac{d(z,\ z).d(Mz,\ Mz)}{d(z,\ Mz)+d(Mz,\ z)+d(z,\ Mz)}] + \xi\, [\frac{d(Mz,\ z)+d(z,\ Mz)}{1+d(Mz,\ z).d(z,\ z).d(Mz,\ Mz)}]$

$d(Mz, z) \leq \beta\, d(Mz, z) + 2\gamma\, d(Mz, z) + 2\xi\, d(Mz, z)$. Then $d(z,Tz) \leq (\beta + 2\gamma + 2\xi)\, d(Mz, z)$. That is $|d(Mz,z)| \leq (\beta + 2\gamma + 2\xi)|d(Mz,z)|$, which is contradiction $\beta + 2\gamma + 2\xi < 1$.Therefore $Mz = z$, since $AMz = z$, which implies $Az = z$. Now we prove that $Dz = z$ from (2) putting $x = z$ and $y = Dz$, we get

$d(Sz, T(Dz)) \leq \alpha\, [\frac{d(AMz,\ Sz)+ d(BD(Dz),\ T(Dz)).d(AMz,\ Sz)}{1+d(Sz,\ T(Dz))}]+ \beta$
$\max\{\frac{d(AMz,\ BD(Dz))}{1+d(AMz,\ BD(Dz))}, \frac{d(AMz,\ Sz)}{1+d(AMz,\ Sz)}, \frac{d(BD(Dz),\ Sz)}{1+d(BD(Dz),\ Sz)}\} + \gamma$
$[\frac{d(BD(Dz),\ T(Dz))+d(T(Dz),\ AMz)+d(Sz,\ BD(Dz))}{1+d(BD(Dz),\ T(Dz))+d(T(Dz),\ AMz)+d(Sz,\ BD(Dz))}] + \eta$
$[\frac{d(T(Dz),\ BD(Dz)).d(Sz,\ AM(Dz))}{d(T(Dz),\ AMz)+d(Sz,\ BD(Dz))+d(BD(Dz),\ AMz)}] + \xi$
$[\frac{d(Sz,\ BD(Dz))+d(BD(Dz),\ AMz)}{1+d(Sz,\ BD(Dz)).d(T(Dz),\ BD(Dz)).d(Sz,\ AMz)}]$

$d(z, Dz) \leq \alpha\, [\frac{d(z,\ z)+ d(Dz,\ Dz).d(z,\ z)}{1+d(z,\ Dz)}]+ \beta \max\{\frac{d(z,\ Dz)}{1+d(z,\ Dz)}, \frac{d(z,\ z)}{1+d(z,\ z)}, \frac{d(Dz,\ z)}{1+d(Dz,\ z)}\} + \gamma$
$[\frac{d(Dz,\ Dz)+d(Dz,\ z)+d(z,\ Dz)}{1+d(Dz,\ Dz)+d(Dz,\ z)+d(z,\ Dz)}] + \eta\, [\frac{d(Dz,\ Dz).d(z,\ Dz)}{d(Dz,\ z)+d(z,\ Dz)+d(Dz,\ z)}] + \xi$
$[\frac{d(z,\ Dz)+d(Dz,\ z)}{1+d(z,\ Dz).d(Dz,\ Dz).d(z,\ z)}]$

$d(z, Dz) \leq (\beta + 2\gamma + 2\xi)\, d(z, Dz)$. That is $|d(z, Dz)| \leq (\beta + 2\gamma + 2\xi)|d(z, Dz)|$, which is contradiction $\beta + 2\gamma + 2\xi < 1$.Therefore $Dz = z$, since $BDz = z$, which implies $Bz = z$.

Therefore z is a unique common fixed point of A, B, D, M, S and T.

Uniqueness: Let u be an another common fixed point of A, B, D, M, S and T. Then, we have

$d(Sz,Tu) \leq \alpha\, [\frac{d(AMz,\ Sz)+ d(BDu,\ Tu).d(AMz,\ Sz)}{1+d(Sz,\ Tu)}]+ \beta$
$\max\{\frac{d(AMz,\ BDu)}{1+d(AMz,\ BDu)}, \frac{d(AMz,\ Sz)}{1+d(AMz,\ Sz)}, \frac{d(BDu,\ Sz)}{1+d(BDu,\ Sz)}\} + \gamma$
$[\frac{d(BDu,\ Tu)+d(Tu,\ AMz)+d(Sz,\ BDu)}{1+d(BDu,\ Tu)+d(Tu,\ AMz)+d(Sz,\ BDu)}] + \eta\, [\frac{d(Tu,\ BDu).d(Sz,\ AMz)}{d(Tu,\ AMz)+d(Sz,\ BDu)+d(BDu,\ AMz)}] + \xi$
$[\frac{d(Sz,\ BDu)+d(BDu,\ AMz)}{1+d(Sz,\ BDu).d(Tu,\ BDu).d(Sz,\ AMz)}]$

$d(z, u) \leq \alpha\, [\frac{d(z,\ z)+ d(u,\ u).d(z,\ z)}{1+d(z,\ u)}]+ \beta \max\{\frac{d(z,\ u)}{1+d(z,\ u)}, \frac{d(z,\ z)}{1+d(z,\ z)}, \frac{d(u,\ z)}{1+d(u,\ z)}\} + \gamma$
$[\frac{d(u,\ u)+d(u,\ z)+d(z,\ u)}{1+d(u,\ u)+d(u,\ z)+d(z,\ u)}] + \eta\, [\frac{d(u,\ u).d(z,\ z)}{d(u,\ z)+d(z,\ u)+d(u,\ z)}] + \xi\, [\frac{d(z,\ u)+d(u,\ z)}{1+d(z,\ u).d(u,\ u).d(z,\ z)}]$

$d(z, u) \leq (\beta + 2\gamma + 2\xi)\, d(z, u)$. That is $|d(z,u)| \leq (\beta + 2\gamma + 2\xi)|d(z,u)|$, which is contradiction $\beta + 2\gamma + 2\xi < 1$.Therefore $z = u$, therefore z is a unique common fixed point of A, B, D, M, S and T.

Case II: we consider the case : $d(Tx_{2n+1}, AMx_{2n}) + d(Sx_{2n}, BDx_{2n+1}) + d(BDx_{2n+1}, AMx_{2n}) = 0$ (for any n) implies that $d(Sx_{2n}, Tx_{2n+1})$. So that $y_{2n} = Sx_{2n} = y_{2n+1} = BDx_{2n+1} = Tx_{2n+1} = AMx_{2n+2} = y_{2n+2}$.Thus we have $y_{2n+1} = Sx_{2n} = AMx_{2n} = y_{2n}$, there exists n_1 and m_1, such that $n_1 = Sm_1 = AMm_1 = m_1$.Similarly $y_{2n+2} = Tx_{2n+1} = BDx_{2n+1} = y_{2n+1}$, there exists n_2 and m_2 such that $n_2= Tm_2 = BD_{m2} = m_2$. As $d(Tm_2, AMm_1) + d(Sm_1, BDm_2) + d(BDm_2, AMm_1) = 0$ implies that , $d(Sm_1, Tm_2) = 0$, so that $n_1 = Sm_1 = AMm_1 = Tm_2 = BDm_2 = n_2$. Which is turn yields that $n_1 = Sn_1 = AMm_1 = AMn_1$ similarly, one can also have $n_2 = Tn_2 = BDn_2$.As $n_1 = n_2$, implies $n_1 = Sn_1 = Tn_1 = BDn_1$, therefore $n_1 = Sn_1 = An_1 = Mn_1 = Tn_1 = Bn_1 = Dn_1$. Hence, $n_1 = n_2$, is common fixed point.

Uniqueness : Let v_1 is an another common fixed point of A, B, D, M, S and T.Then we have $v_1 = Sv_1 = Av_1 = Mv_1 = Tv_1 = Bv_1 = Dv_1$.Therefore $d(Tv_1, AMv_1) + d(Sn_1, BDv_1) + d(BDv_1, AMn_1) = 0$, so that $d(n_1,v_1) = 0$.Hence this implies that $n_1 = v_1$. Hence n_1 is a unique common fixed point of A, B, D, M, S and T.

Corollary: Let (X, d) be a complex valued metric space and D, M, S and T be four self mappings in X satisfying the condition:

1. $S(X) \subset D(X)$ and $T(X) \subset M(X)$.

2. For each $x, y \in X$, such that $x \neq y$, $d(Ty, Mx) + d(Sx, Dy) + d(Dy, Mx) \neq 0$, where $\alpha, \beta, \gamma, \eta$ and ξ are non negative real number with $\alpha + \beta + 2\gamma + \eta + \xi < 1$, or $d(Sx, Ty) = 0$ if $d(Ty, Mx) + d(Sx, Dy) + d(Dy, Mx) = 0$, such that

$$d(Sx, Ty) \precsim \alpha \left[\frac{d(Mx, Sx)+ d(Dy, Ty).d(Mx, Sx)}{1+d(Sx, Ty)}\right]+ \beta \max\left\{\frac{d(Mx, Dy)}{1+d(Mx, Dy)}, \frac{d(Mx, Sx)}{1+d(Mx, Sx)}, \frac{d(Dy, Sx)}{1+d(Dy, Sx)}\right\} + \gamma \left[\frac{d(Dy, Ty)+d(Ty, Mx)+d(Sx, Dy)}{1+d(Dy, Ty)+d(Ty, Mx)+d(Sx, Dy)}\right] + \eta \left[\frac{d(Ty, Dy).d(Sx, Mx)}{d(Ty, Mx)+d(Sx, Dy)+d(Dy, Mx)}\right] + \xi \left[\frac{d(Sx, Dy)+d(Dy, Mx)}{1+d(Sx, Dy).d(Ty, Dy).d(Sx, Mx)}\right].$$

3. The pair (M, S) and (D, T) are weakly compatible.

Then D, M, S and T have a unique common fixed point.

Corollary : If (M,S) and (D, T) are four commuting self mappings defined on a complete complex valued metric space (X, d) satisfying the condition:

$$d(Sx, Ty) \precsim \alpha \left[\frac{d(M^m x, S^m x)+ d(D^n y, T^n y).d(M^m x, S^m x)}{1+d(S^m x, T^n y)}\right]+ \beta \max\left\{\frac{d(M^m x, D^n y)}{1+d(M^m x, D^n y)}, \frac{d(M^m x, S^m x)}{d(M^m x, S^m x)}, \frac{d(D^n y, S^m x)}{d(D^n y, S^m x)}\right\} + \gamma \left[\frac{d(D^n y, T^n y)+d(T^n y, M^m x)+d(S^m x, D^n y)}{1+d(D^n y, T^n y)+d(T^n y, M^m x)+d(S^m x, D^n y)}\right] + \eta \left[\frac{d(T^n y, D^n y).d(S^m x, M^m x)}{d(T^n y, M^m x)+d(S^m x, D^n y)+d(D^n y, M^m x)}\right] + \xi \left[\frac{d(S^m x, D^n y)+d(D^n y, M^m x)}{1+d(S^m x, D^n y).d(T^n y, D^n y).d(S^m x, M^m x)}\right].$$

For each $x, y \in X$, such that $x \neq y$, where , where $\alpha, \beta, \gamma, \eta$ and ξ are non negative real number with $\alpha + \beta +2\gamma + \eta + \xi < 1$, or if $d(S^m x, T^n y) = 0$ if $d(T^n y, M^m x) +$

$d(S^m x, D^n y) + d(D^n y, M^m x) = 0$,Then D, M, S and T have a unique common fixed point.

Theorem 3.2. Let A, B, D, M, S and T be self mappings of a complete complex valued metric space (X, d) satisfying conditions:

1. $S(X) \subset BD(X)$ and $T(X) \subset AM(X)$.

2. For each $x, y \in X$, where $\alpha, \beta, \gamma, \eta$ and ξ are non negative real number with $\alpha + \beta + 2\gamma + \eta + \xi < 1$, such that

$$d(Sx, Ty) \le \alpha \left[\frac{d(AMx, Sx) + d(BDy, Ty).d(AMx, Sx)}{1+d(Sx, Ty)}\right] + \beta \max\left\{\frac{d(AMx, BDy)}{1+d(AMx, BDy)}, \frac{d(AMx, Sx)}{1+d(AMx, Sx)}, \frac{d(BDy, Sx)}{1+d(BDy, Sx)}\right\} + \gamma \left[\frac{d(BDy, Ty)+d(Ty, AMx)+d(Sx, BDy)}{1+d(BDy, Ty)+d(Ty, AMx)+d(Sx, BDy)}\right] + \eta \left[\frac{d(Ty, BDy).d(Sx, AMx)}{1 + d(Ty, AMx)+d(Sx, BDy)+d(BDy, AMx)}\right] + \xi \left[\frac{d(Sx, BDy)+d(BDy, AMx)}{1+d(Sx, BDy).d(Ty, BDy).d(Sx, AMx)}\right]$$

3. (AM, S) are compatible, and AM or S is continuous and (BD, T) are weakly compatible.

4. (BD, T) are compatible, and BD or T is continuous and (AM, S) are weakly compatible. Then A, B, D, M, S and T have a unique common fixed point.

Proof : By above theorem $\{y_n\}$ is a Cauchy sequence. Since X is completed, so$\{y_n\}$ is converges to some point z. thus subsequence $\{Sx_{2n}\}$, $\{BDx_{2n+1}\}$, $\{Tx_{2n+1}\}$ and $\{AMx_{2n+2}\}$ also converges to z, that is

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} BDx_{2n+1} = \lim_{n\to\infty} AMx_{2n+2} = \lim_{n\to\infty} Tx_{2n+1} \quad (3.3)$$

Assume that S is continuous. Since (AM, S) are compatible, we have

$$\lim_{n\to\infty} AM(Sx_{2n+2}) = \lim_{n\to\infty} S(AMx_{2n+2}) = Sz \quad (3.4)$$

Now putting $x = x_{2n+2}$, $y = x_{2n+1}$ then we have

$$d(AM(Sx_{2n}),Tx_{2n+1}) \le \alpha \left[\frac{d(AMx_{2n+2}, Sx_{2n+2}) + d(BDx_{2n+1}, Tx_{2n+1}).d(AMx_{2n+2}, Sx_{2n+2})}{1+d(Sx_{2n}, Tx_{2n+1})}\right] + \beta \max\left\{\frac{d(AMx_{2n+2}, BDx_{2n+1})}{1+d(AMx_{2n+2}, BDx_{2n+1})}, \frac{d(AMx_{2n+2}, Sx_{2n+2})}{1+d(AMx_{2n+2}, Sx_{2n+2})}, \frac{d(BDx_{2n+1}, Sx_{2n+2})}{1+d(BDx_{2n+1}, Sx_{2n+2})}\right\} + \gamma \left[\frac{d(BDx_{2n+1}, Tx_{2n+1})+d(Tx_{2n+1}, AMx_{2n+2})+d(Sx_{2n+2}, BDx_{2n+1})}{1+d(BDx_{2n+1}, Tx_{2n+1})+d(Tx_{2n+1}, AMx_{2n+2})+d(Sx_{2n+2}, BDx_{2n+1})}\right] + \eta \left[\frac{d(Tx_{2n+1}, BDx_{2n+1}).d(Sx_{2n+2}, AMx_{2n+2})}{1+d(Tx_{2n+1}, AMx_{2n+2})+d(Sx_{2n+2}, BDx_{2n+1})+d(BDx_{2n+1}, AMx_{2n+2})}\right] + \xi \left[\frac{d(Sx_{2n+2}, BDx_{2n+1})+d(BDx_{2n+1}, AMx_{2n+2})}{1+d(Sx_{2n+2}, BDx_{2n+1}).d(Tx_{2n+1}, BDx_{2n+1}).d(Sx_{2n+2}, AMx_{2n+2})}\right]$$

Letting $n\to\infty$, in the above inequality and using (3.3) and (3.4), we get

$$d(Sz, z) \le \alpha\left[\frac{d(z, z)+ d(z, z).d(z, z)}{1+d(z, z)}\right]+ \beta \max\left\{\frac{d(z, z)}{1+d(z, z)}, \frac{d(z, z)}{1+d(z, z)}, \frac{d(z, z)}{1+d(z, z)}\right\} + \gamma \left[\frac{d(z, z)+d(z, z)+d(z, z)}{1+d(z, z)+d(z, z)+d(z, z)}\right] + \eta\left[\frac{d(z, z).d(z, z)}{1+ d(z, z)+d(z, z)+d(z, z)}\right] + \xi\left[\frac{d(z, z)+d(z, z)}{1+d(z, z).d(z, z).d(z, z)}\right]$$

$d(Sz, z) \precsim 0$ that is $|d(Sz, z)| \le 0$, hence $Sz = z$. Now putting $x = z$ and $y = x_{2n+1}$ in (2) we have

$$d(Sz,Tx_{2n+1}) \le \alpha\left[\frac{d(AMz, Sz)+ d(BDx_{2n+1}, Tx_{2n+1}).d(AMz, Sz)}{1+d(Sz, Tx_{2n+1})}\right]+ \beta \max\left\{\frac{d(AMz, BDx_{2n+1})}{1+d(AMz, BDx_{2n+1})}, \frac{d(AMz, Sz)}{1+d(AMz, Sz)}, \frac{d(BDx_{2n+1}, Sz)}{1+d(BDx_{2n+1}, Sz)}\right\} + \gamma \left[\frac{d(BDx_{2n+1}, Tx_{2n+1})+d(Tx_{2n+1}, AMz)+d(Sz, BDx_{2n+1})}{1+d(BDx_{2n+1}, Tx_{2n+1})+d(Tx_{2n+1}, AMz)+d(Sz, BDx_{2n+1})}\right] + \eta \left[\frac{d(Tx_{2n+1}, BDx_{2n+1}).d(Sz, AMz)}{1+d(Tx_{2n+1}, AMz)+d(Sz, BDx_{2n+1})+d(BDx_{2n+1}, AMz)}\right] + \xi \left[\frac{d(Sz, BDx_{2n+1})+d(BDx_{2n+1}, AMz)}{1+d(Sz, BDx_{2n+1}).d(Tx_{2n+1}, BDx_{2n+1}).d(Sz, AMz)}\right]$$

Letting $n\to\infty$, we get

$$d(z, z) \le \alpha\left[\frac{d(AMz, z)+ d(z, z).d(AMz, z)}{1+d(z, z)}\right]+ \beta \max\left\{\frac{d(AMz, z)}{1+d(AMz, z)}, \frac{d(AMz, z)}{1+d(AMz, z)}, \frac{d(z, z)}{1+d(z, z)}\right\} + \gamma \left[\frac{d(z, z)+d(z, AMz)+d(z, z)}{1+d(z, z)+d(z, AMz)+d(z, z)}\right] + \eta\left[\frac{d(z, z).d(z, AMz)}{1+d(z, AMz)+d(z, z)+d(z, AMz)}\right] + \xi \left[\frac{d(z, z)+d(z, AMz)}{1+d(z, z).d(z, z).d(z, AMz))}\right]$$

$d(z, z) \le \alpha\, d(AMz, z) + \beta\, d(AMz, z) + \gamma\, d(AMz, z)$

$d(z, z) \le (\alpha + \beta + \gamma)\, d(AMz, z)$, that is $0 \le (\alpha + \beta + \gamma)\, d(AMz, z)$.Then $|d(AMz, z)| \ge 0$, hence $AMz = z$.

Since $S(X) \subset BD(X)$, there exist a point $w \in X$ such that $Sz = BDw$. Suppose that $BDw \neq Tw$. Now to prove $BDw = Tw$ and given that $Sz = z = BDw$. From (2) putting $x = z$ and $y = w$, obtain

$$d(Sz, Tw) \le \alpha\left[\frac{d(AMz, Sz)+ d(BDz, Tw).d(AMz, Sz)}{1+d(Sz, Tw)}\right]+ \beta \max\left\{\frac{d(AMz, BDw)}{1+d(AMz, BDw)}, \frac{d(AMz, Sz)}{1+d(AMz, Sz)}, \frac{d(BDw, Sw)}{1+d(BDw, Sw)}\right\} + \gamma \left[\frac{d(BDw, Tw)+d(Tw, AMz)+d(Sz, BDw)}{1+d(BDw, Tw)+d(Tw, AMz)+d(Sz, BDw)}\right] + \eta\left[\frac{d(Tw, BDw).d(Sz, AMz)}{1+d(Tw, AMz)+d(Sz, BDw)+d(BDw, AMz)}\right] + \xi \left[\frac{d(Sz, BDw)+d(BDw, AMz)}{1+d(Sz, BDw).d(Tw, BDw).d(Sz, AMz)}\right]$$

$$d(z,Tw) \leq \alpha\, [\frac{d(z,\ z)+ d(z,\ Tw).d(z,\ z)}{1+d(z,\ Tw)}]+ \beta \max\{\frac{d(z,\ z)}{1+d(z,\ z)}, \frac{d(z,\ z)}{1+d(z,\ z)}, \frac{d(z,\ z)}{1+d(z,\ z)}\} + \gamma [\frac{d(z,\ Tw)+d(Tw,\ z)+d(z,\ z)}{1+d(z,\ Tw)+d(Tw,\ z)+d(z,\ z)}] + \eta\, [\frac{d(Tw,\ z).d(z,\ z)}{1+d(Tw,\ z)+d(z,\ z)+d(z,\ z)}] + \xi [\frac{d(z,\ z)+d(z,\ z)}{1+d(z,\ z).d(Tw,\ z).d(z,\ z)}]$$

$d(z,Tw) \leq 2\,\gamma\, d(z,Tw)$ that is $|d(z,\ Tw)| \leq\ 2\,\gamma|d(z,\ Tw)|$,which is contradiction to $2\,\gamma < 1$.Therefore Tw = z. Hence BDw = z = Tw.Thus BDw = Tw.Since BD and T are weakly compatible then BDz = BD(Tw) = T(BDw) = Tw. Thus z is a coincidence point of BD and T. Now to prove Tz = z from (2) putting x = z and y = z.

$$d(Sz,Tz) \leq \alpha\, [\frac{d(AMz,\ Sz)+ d(BDz,\ Tz).d(AMz,\ Sz)}{1+d(Sz,\ Tz)}]+ \beta \max\{\frac{d(AMz,\ BDz)}{1+d(AMz,\ BDz)}, \frac{d(AMz,\ Sz)}{1+d(AMz,\ Sz)}, \frac{d(BDz,\ Sz)}{1+d(BDz,\ Sz)}\} + \gamma\, [\frac{d(BDz,\ Tz)+d(Tz,\ AMz)+d(Sz,\ BDz)}{1+d(BDz,\ Tz)+d(Tz,\ AMz)+d(Sz,\ BDz)}] + \eta\, [\frac{d(Tz,\ BDz).d(Sz,\ AMz)}{1+d(Tz,\ AMz)+d(Sz,\ BDz)+d(BDz,\ AMz)}] + \xi\, [\frac{d(Sz,\ BDz)+d(BDz,\ AMz)}{1+d(Sz,\ BDz).d(Tz,\ BDz).d(Sz,\ AMz)}]$$

$$d(z,Tz) \leq \alpha\, [\frac{d(z,\ z)+ d(Tz,\ Tz).d(z,\ z)}{1+d(z,\ Tz)}]+ \beta \max\{\frac{d(z,\ Tz)}{1+d(z,\ Tz)}, \frac{d(z,\ z)}{1+d(z,\ z)}, \frac{d(Tz,\ z)}{1+d(Tz,\ z)}\} + \gamma [\frac{d(Tz,\ Tz)+d(Tz,\ z)+d(z,\ Tz)}{1+d(Tz,\ Tz)+d(Tz,\ z)+d(z,\ Tz)}] + \eta\, [\frac{d(Tz,\ Tz).d(z,\ z)}{1+d(Tz,\ z)+d(z,\ Tz)+d(Tz,\ z)}] + \xi [\frac{d(z,\ Tz)+d(Tz,\ z)}{1+d(z,\ Tz).d(Tz,\ Tz).d(z,\ z)}].$$

$d(z,Tz) \leq\ \beta\, d(z, Tz) + 2\,\gamma\, d(z, Tz) + 2\xi\, d(z, Tz)$. Then $d(z,Tz) \leq\ (\beta + 2\,\gamma + 2\xi)\, d(Tz, z)$. That is

$|d(z, Tz)\ | \leq\ (\beta\ + 2\,\gamma\ + 2\,\xi\,)|d(Tz, z)|$, which is contradiction $\beta\ + 2\,\gamma\ + 2\,\xi <1$.Therefore Tz = z, since BDz = Tz, which implies BDz = z. Now we prove that Mz = z from (2) putting x = Mz and y = z, we get

$$d(S(Mz), Tz) \leq \alpha\, [\frac{d(AM(Mz),\ S(Mz))+ d(BDz,\ Tz).d(AM(Mz),\ S(Mz))}{1+d(S(Mz),\ Tz)}]+ \beta \max\{\frac{d(AM(Mz),\ BDz)}{1+d(AM(Mz),\ BDz)}, \frac{d(AM(Mz),\ S(Mz))}{1+d(AM(Mz),\ S(Mz))}, \frac{d(BDz,\ S(Mz))}{1+d(BDz,\ S(Mz))}\} + \gamma [\frac{d(BDz,\ Tz)+d(Tz,\ AM(Mz))+d(S(Mz),\ BDz)}{1+d(BDz,\ Tz)+d(Tz,\ AM(Mz))+d(S(Mz),\ BDz)}] + \eta [\frac{d(Tz,\ BDz).d(S(Mz),\ AM(Mz))}{1+d(Tz,\ AM(Mz))+d(S(Mz),\ BDz)+d(BDz,\ AM(Mz))}] + \xi [\frac{d(S(Mz),\ BDz)+d(BDz,\ AM(Mz))}{1+d(S(Mz),\ BDz).d(Tz,\ BDz).d(S(Mz),\ AM(Mz))}]$$

$$d(Mz, z) \leq \alpha\, [\frac{d(Mz,\ Mz)+ d(z,\ z).d(Mz,\ Mz))}{1+d(Mz,\ z)}]+ \beta \max\{\frac{d(Mz,\ z)}{1+d(Mz,\ z)}, \frac{d(Mz,\ Mz)}{1+d(Mz,\ Mz)}, \frac{d(z,\ Mz)}{1+d(z,\ Mz)}\} + \gamma\, [\frac{d(z,\ z)+d(z,\ Mz)+d(Mz,\ z)}{1+d(z,\ z)+d(z,\ Mz)+d(Mz,\ z)}] + \eta\, [\frac{d(z,\ z).d(Mz,\ Mz)}{1+d(z,\ Mz)+d(Mz,\ z)+d(z,\ Mz)}] + \xi [\frac{d(Mz,\ z)+d(z,\ Mz)}{1+d(Mz,\ z).d(z,\ z).d(Mz,\ Mz)}]$$

d(Mz, z) ≤ β d(Mz, z) + 2 γ d(Mz, z) + 2 ξ d(Mz, z). Then d(z,Tz) ≤ (β + 2 γ + 2ξ) d(Mz, z). That is $|d(Mz, z)| \le (\beta + 2\gamma + 2\xi)|d(Mz, z)|$, which is contradiction β + 2 γ + 2 ξ <1.Therefore Mz = z, since AMz = z, which implies Az = z. Now we prove that Dz = z from (2) putting x = z and y = Dz, we get

$$d(Sz, T(Dz)) \le \alpha \left[\frac{d(AMz,\ Sz) + d(BD(Dz),\ T(Dz)).d(AMz,\ Sz)}{1+d(Sz,\ T(Dz))}\right] + \beta \max\left\{\frac{d(AMz,\ BD(Dz))}{1+d(AMz,\ BD(Dz))}, \frac{d(AMz,\ Sz)}{1+d(AMz,\ Sz)}, \frac{d(BD(Dz),\ Sz)}{1+d(BD(Dz),\ Sz)}\right\} + \gamma \left[\frac{d(BD(Dz),\ T(Dz))+d(T(Dz),\ AMz)+d(Sz,\ BD(Dz))}{1+d(BD(Dz),\ T(Dz))+d(T(Dz),\ AMz)+d(Sz,\ BD(Dz))}\right] + \eta \left[\frac{d(T(Dz),\ BD(Dz)).d(Sz,\ AM(Dz))}{1+d(T(Dz),\ AMz)+d(Sz,\ BD(Dz))+d(BD(Dz),\ AMz)}\right] + \xi \left[\frac{d(Sz,\ BD(Dz))+d(BD(Dz),\ AMz)}{1+d(Sz,\ BD(Dz)).d(T(Dz),\ BD(Dz)).d(Sz,\ AMz)}\right]$$

$$d(z, Dz) \le \alpha \left[\frac{d(z,\ z) + d(Dz,\ Dz).d(z,\ z)}{1+d(z,\ Dz)}\right] + \beta \max\left\{\frac{d(z,\ Dz)}{1+d(z,\ Dz)}, \frac{d(z,\ z)}{1+d(z,\ z)}, \frac{d(Dz,\ z)}{1+d(Dz,\ z)}\right\} + \gamma \left[\frac{d(Dz,\ Dz)+d(Dz,\ z)+d(z,\ Dz)}{1+d(Dz,\ Dz)+d(Dz,\ z)+d(z,\ Dz)}\right] + \eta \left[\frac{d(Dz,\ Dz).d(z,\ Dz)}{1+d(Dz,\ z)+d(z,\ Dz)+d(Dz,\ z)}\right] + \xi \left[\frac{d(z,\ Dz)+d(Dz,\ z)}{1+d(z,\ Dz).d(Dz,\ Dz).d(z,\ z)}\right]$$

d(z, Dz) ≤ (β + 2γ + 2ξ) d(z, Dz). That is $|d(z, Dz)| \le (\beta + 2\gamma + 2\xi)|d(z, D z)|$, which is contradiction β + 2 γ + 2 ξ <1.Therefore Dz = z, since BDz = z, which implies Bz =z.Therefore by combining all the above result, we conclude that z is a unique common fixed point of A, B, D, M, S and T.

Uniqueness: Let u be an another common fixed point of A, B, D, M, S and T. Then, we have

$$d(Sz,Tu) \le \alpha \left[\frac{d(AMz,\ Sz) + d(BDu,\ Tu).d(AMz,\ Sz)}{1+d(Sz,\ Tu)}\right] + \beta \max\left\{\frac{d(AMz,\ BDu)}{1+d(AMz,\ BDu)}, \frac{d(AMz,\ Sz)}{1+d(AMz,\ Sz)}, \frac{d(BDu,\ Sz)}{1+d(BDu,\ Sz)}\right\} + \gamma \left[\frac{d(BDu,\ Tu)+d(Tu,\ AMz)+d(Sz,\ BDu)}{1+d(BDu,\ Tu)+d(Tu,\ AMz)+d(Sz,\ BDu)}\right] + \eta \left[\frac{d(Tu,\ BDu).d(Sz,\ AMz)}{1+d(Tu,\ AMz)+d(Sz,\ BDu)+d(BDu,\ AMz)}\right] + \xi \left[\frac{d(Sz,\ BDu)+d(BDu,\ AMz)}{1+d(Sz,\ BDu).d(Tu,\ BDu).d(Sz,\ AMz)}\right]$$

$$d(z, u) \le \alpha \left[\frac{d(z,\ z) + d(u,\ u).d(z,\ z)}{1+d(z,\ u)}\right] + \beta \max\left\{\frac{d(z,\ u)}{1+d(z,\ u)}, \frac{d(z,\ z)}{1+d(z,\ z)}, \frac{d(u,\ z)}{1+d(u,\ z)}\right\} + \gamma \left[\frac{d(u,\ u)+d(u,\ z)+d(z,\ u)}{1+d(u,\ u)+d(u,\ z)+d(z,\ u)}\right] + \eta \left[\frac{d(u,\ u).d(z,\ z)}{1+d(u,\ z)+d(z,\ u)+d(u,\ z)}\right] + \xi \left[\frac{d(z,\ u)+d(u,\ z)}{1+d(z,\ u).d(u,\ u).d(z,\ z)}\right]$$

d(z, u) ≤ (β + 2γ + 2ξ) d(z, u). That is $|d(z, u)| \le (\beta + 2\gamma + 2\xi)|d(z, u)|$, which is contradiction β + 2 γ + 2 ξ <1.Therefore z = u, therefore z is a unique common fixed point of A, B, D, M, S and T.

Corollary: Let (X, d) be a complex valued metric space and D, M, S and T be four self mappings in X satisfying the condition:

1. $S(X) \subset D(X)$ and $T(X) \subset M(X)$.

2. For each $x, y \in X$, such that $x \neq y$, $d(Ty, Mx) + d(Sx, Dy) + d(Dy, Mx) \neq 0$, where $\alpha, \beta, \gamma, \eta$ and ξ are non negative real number with $\alpha + \beta + 2\gamma + \eta + \xi < 1$, or $d(Sx, Ty) = 0$ if $d(Ty, Mx) + d(Sx, Dy) + d(Dy, Mx) = 0$, such that

$$d(Sx, Ty) \precsim \alpha \left[\frac{d(Mx, Sx)+ d(Dy, Ty).d(Mx, Sx)}{1+d(Sx, Ty)}\right]+ \beta \max\left\{\frac{d(Mx, Dy)}{1+d(Mx, Dy)}, \frac{d(Mx, Sx)}{1+d(Mx, Sx)}, \frac{d(Dy, Sx)}{1+d(Dy, Sx)}\right\} + \gamma \left[\frac{d(Dy, Ty)+d(Ty, Mx)+d(Sx, Dy)}{1+d(Dy, Ty)+d(Ty, Mx)+d(Sx, Dy)}\right] + \eta \left[\frac{d(Ty, Dy).d(Sx, Mx)}{d(Ty, Mx)+d(Sx, Dy)+d(Dy, Mx)}\right] + \xi \left[\frac{d(Sx, Dy)+d(Dy, Mx)}{1+d(Sx, Dy).d(Ty, Dy).d(Sx, Mx)}\right].$$

3. The pair (M, S) and (D, T) are weakly compatible.

Then D, M, S and T have a unique common fixed point.

CONCLUSION

In this paper we proved fixed point theorem and common fixed point theorem in complex valued metric space through concept of compatibility and continuity.

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