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Solution of Partial Integro-Differential Equations by using Aboodh and Double Aboodh Transform Methods

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Abstract

Partial integro- differential equations (PIDE) occur in several fields of sciences and mathematics. The main purpose of this paper to study how to solve partial integro- differential equation (PIDE) by using various methods like Aboodh and Double Aboodh Transform.To solve PIDE by using Aboodh Transform (AT), first convert proposed PIDE to an ordinary differential equation (ODE) then solving this ODE by applying inverse AT we get an exact solution of the problem.To solve PIDE by using Double Aboodh Transform (DAT), first convert proposed PIDE to an algebraic equation, solving this algebraic and applying double inverse Aboodh Transform we obtain an exact solution of the problem.

These methods are useful tools for the solution of the differential and integral equation and linear system of differential and integral equation.

Keywords: Partial integro- differential equations (PIDE), ordinary differential equation (ODE), Aboodh Transform (AT), Double Aboodh Transform (DAT).

1. INTRODUCTION

In the last few years theory and application of partial integro- differential equations (PIDE) play an important role in the various fields of many problems of mathematical fields , engineering physics, biology, and social sciences [3-11].This explains a growing interest in the mathematics community to integro-differential equations and in particular to partial integro- differential equations. Therefore, it is very important to know various methods to solve such partial differential equations [1, 2].

One tool for solving linearPIDE's is Aboodh Transform method. It is one of the useful tools for solution of the differential, integral equation and linear system of differential and integral equation [1].

Second tool for is Double Aboodh Transform which is the higher version of Aboodh Transform to solve linear PIDE's [2].

In this paper, we solve single example of PIDE by using two different methods like Aboodh and Double Aboodh Transform.

2. PRELIMINARIES

2.1 Aboodh Transform:

Definition:

Let a function $f(t)$ defined for $t > 0$ then Aboodh transform of $f(t)$ is the function K defined as follows:

$$A[f(t)] = K(v) = \frac{1}{v}\int_0^\infty f(t)e^{-vt}dt \ , \quad t > 0$$

Theorem 1:

Aboodh transform of some partial derivatives are in the form:

$A[u(x,t)] = K(x,v)$

$$(i) A\left[\frac{\partial u(x,t)}{\partial t}\right] = vK(x,v) - \frac{u(x,0)}{v}.$$

$$(ii) A\left[\frac{\partial^2 u(x,t)}{\partial t^2}\right] = v^2K(x,v) - \frac{1}{v}\frac{\partial u(x,0)}{\partial t} - u(x,0).$$

$$(iii)A\left[\frac{\partial u(x,t)}{\partial x}\right] = K'(x,v) = \frac{d[K(x,v)]}{dx}.$$

$$(iv)A\left[\frac{\partial^2 u(x,t)}{\partial x^2}\right] = K''(x,v) = \frac{d^2[K(x,v)]}{dx^2}.$$

Theorem 2: (Convolution):

Let $f(t)$ and $g(t)$having Aboodh transform $\mathrm{M}(v)$ and $\mathrm{N}(v)$, then Aboodh transform of the convolution of f and g ,

$f(t) * g(t) = \int_0^\infty f(\tau)g(t-\tau)d\tau$, is given by:

$$A[f(t) * g(t)] = v\mathrm{M}(v)\mathrm{N}(v).$$

Solving PIDEs using Aboodh Transform Method:

Consider general linear PIDE,

$$\sum_{i=0}^{m} a_i \frac{\partial^i u}{\partial t^i} + \sum_{i=0}^{n} \mathrm{b}_i \frac{\partial^i u}{\partial x^i} + cu + \sum_{i=0}^{r} d_i \int_0^t k_i(t-s)\frac{\partial^i u(x,s)}{\partial x^i} ds + f(x,t) = 0. \qquad \rightarrow (1)$$

(With prescribed condition).

Where $f(x,t)$ and $k_i(t,s)$ are known functions. $a_i's$, $b_i's$, $d_i's$ and c are constants or the functions of x.

Taking Aboodh transform on both sides of PIDE (1) with respect to t we get,

$$\sum_{i=0}^{m} a_i\, A\left\{\frac{\partial^i u}{\partial t^i}\right\} + \sum_{i=0}^{n} \mathrm{b}_i\, A\left\{\frac{\partial^i u}{\partial x^i}\right\} + cA\{u\} + \sum_{i=0}^{r} d_i A\left\{\int_0^t k_i(t-s) * \frac{\partial^i u(x,s)}{\partial x^i} ds\right\} + A\{f(x,t)\} = 0 .$$

Using theorem 1 and theorem 2 for Aboodh transform, we get:

$$\sum_{i=0}^{m}\left[a_i v^i \bar{u}(x,v) - \sum_{k=0}^{i-1} \frac{1}{v^{2-i+k}} u^{(i-k)}(x,o)\right] + \sum_{i=0}^{n} \mathrm{b}_i \frac{d^i \bar{u}(x,v)}{dx^i} + c\bar{u}(x,v) + \sum_{i=0}^{r} d_i v \bar{k}_i(v) \frac{d^i \bar{u}(x,v)}{dx^i} + \bar{f}(x,v) = 0 . \qquad \rightarrow (2)$$

Where:

$\bar{u}(x,v) = A[u(x,v)]$, $\bar{f}(x,v) = A[f(x,t)]$, $\bar{k}_i(v) = A[k_i(t)]$.

Equation (2) is an ordinary differential equation in $\bar{u}(x,v)$.

Solving this ODE and taking inverse Aboodh transform of $\bar{u}(x,v)$, we get solution $u(x,t)$ of (1) .

Illustrative example:

Example:

Consider the PIDE,

$$u_{tt} = u_x + 2\int_0^t (t-s).\mathrm{u}(x,s)ds - 2e^x \ . \rightarrow (3)$$

With initial conditions

$$u(x,0) = e^x \ , u_t(x,0) = 0 \ . \quad \rightarrow (4)$$

And boundary condition

$$u(0,t) = cost \ . \quad \rightarrow (5)$$

Solution:

Taking Aboodh transform with respect to t on both sides of (3):

$$v^2\bar{u}\,(x,v) - u(x,0) - \frac{1}{v}\,u_t(x,0) = \bar{u}_x + v\left(\frac{2}{v^3}.\bar{u}\right) - \frac{2e^x}{v^2}$$

$$\bar{u}_x + \frac{2}{v^2}\bar{u} - v^2\bar{u} - \frac{2}{v^2}e^x + e^x = 0 \ .$$

$$\frac{d\bar{u}}{dx} + \left(\frac{2}{v^2} - v^2\right)\bar{u} = e^x\left(\frac{2}{v^2} - 1\right) \ . \quad \rightarrow (6)$$

Therefore the solution of (6) is,

$$\bar{u} = \frac{e^x}{v^2+1} + C\,e^{-\left(\frac{2}{v^2}-v^2\right)x} . \quad \rightarrow (7)$$

From boundary condition (5)

$$\bar{u}\,(0,v) = A\{u(0,t)\} = A\{cost\} = \frac{1}{v^2+1} . \quad \rightarrow (8)$$

Using (7) and (8) to get: C = 0.

Then equation (7) becomes,

$$\bar{u}(x,v) = \frac{v^2}{v^2+1}e^x. \quad \to (9)$$

Applying inverse Aboodh transform on both sides of (9):

$$u(x,t) = A^{-1}\{u(x,v)\} = e^x A^{-1}\left\{\frac{1}{v^2+1}\right\} = e^x \, cost \ .$$

$$\therefore \ u(x,t) = e^x \, cost \ .$$

2.2 Double Aboodh Transform:

Definition:

Let $f(x,t)$, where $t, x \in R^+$ be a function, which can be expressed as a convergent infinite series then, its double Aboodh transform given by:

$$A_2[f(x,t),u,v] = K(u,v) = \frac{1}{uv}\int_0^\infty \int_0^\infty f(x,t)e^{-(ux+vt)}dx\,dt \quad , \qquad x,t > 0$$

Where, u &v are complex values.

Theorem 1:

Double Aboodh transform of first and second order partial derivatives are in the form:

$$(i) A_2\left\{\frac{\partial f}{\partial x}\right\} = uK(u,v) - \frac{1}{u} K(0,v).$$

$$(ii) A_2\left\{\frac{\partial^2 f}{\partial x^2}\right\} = u^2K(u,v) - K(0,v) - \frac{1}{u}\frac{\partial[K(0,v)]}{\partial x} .$$

$$(iii) A_2\left\{\frac{\partial f}{\partial t}\right\} = vK(u,v) - \frac{1}{v} K(u,0) .$$

$$(iv) A_2\left\{\frac{\partial^2 f}{\partial t^2}\right\} = v^2K(u,v) - K(u,0) - \frac{1}{v}\frac{\partial[K(u,0)]}{\partial t} .$$

$$(v) A_2\left\{\frac{\partial^2 f}{\partial x \partial t}\right\} = \frac{1}{uv}f(0,0) - \frac{u}{v}K(u,0) + uvK(u,v) - \frac{v}{u} K(0,v) .$$

Proof:

$$(i)\ A_2\left\{\frac{\partial f}{\partial x}\right\} = \frac{1}{uv}\int_0^{\infty}\int_0^{\infty} e^{-(ux+vt)}\frac{\partial f(x,t)}{\partial x}\,dx\,dt\,.$$

$$= \frac{1}{v}\int_0^{\infty} e^{-vt}\left[\frac{1}{u}\int_0^{\infty} e^{-ux}\frac{\partial f(x,t)}{\partial x}\,dx\right]dt \to (*)$$

$$\mathrm{I} = \int_0^{\infty} e^{-ux}\frac{\partial f(x,t)}{\partial x}\,dx$$

Integration by part:

$$u = e^{-ux}\quad,\ du = -ue^{-ux}dx\quad, dv = \frac{\partial f(x,t)}{\partial x}\,dx\,, v = f(x,t)\,.$$

$$\mathrm{I} = e^{-ux}f(x,t)|_0^{\infty} + u\int_0^{\infty} e^{-ux}f(x,t)\,dx$$

$$= -f(0,t) + u\int_0^{\infty} e^{-ux}f(x,t)\,dx. \quad \to (**)$$

Substitution (**) in (*):

$$= \frac{1}{v}\int_0^{\infty} e^{-vt}\left[\frac{1}{u}\left(-f(0,t) + u\int_0^{\infty} e^{-ux}f(x,t)\,dx\right)\right]dt$$

$$= -\frac{1}{uv}\int_0^{\infty} e^{-vt}f(0,t)dt + \frac{1}{v}\int_0^{\infty} e^{-vt}e^{-ux}f(x,t)\,dx\,dt\,.$$

$$= -\frac{1}{u}K(0,v) + uK(u,v)\,.$$

$$A_2\left\{\frac{\partial f}{\partial x}\right\} = uK(u,v) - \frac{1}{u}\,K(0,v).$$

$$(ii)\ A_2\left\{\frac{\partial^2 f}{\partial x^2}\right\} = \frac{1}{uv}\int_0^{\infty}\int_0^{\infty} e^{-(ux+vt)}\frac{\partial^2 f(x,t)}{\partial x^2}\,dx\,dt\,.$$

$$= \frac{1}{v}\int_0^{\infty} e^{-vt}\left[\frac{1}{u}\int_0^{\infty} e^{-ux}\frac{\partial^2 f(x,t)}{\partial x^2}\,dx\right]dt \to (*)$$

$$\mathrm{I} = \frac{1}{u}\int_0^{\infty} e^{-ux}\frac{\partial^2 f(x,t)}{\partial x^2}\,dx$$

Integration by part:

$$u = e^{-ux} \quad , \; du = -ue^{-ux}dx \quad , dv = \frac{\partial^2 f(x,t)}{\partial x^2}\, dx \, , v = \frac{\partial f(x,t)}{\partial x} \, .$$

$$\mathrm{I} = \frac{1}{u}\left[e^{-ux}\frac{\partial f(x,t)}{\partial x}\Big|_0^{\infty} + u\int_0^{\infty} e^{-ux}\frac{\partial f(x,t)}{\partial x}\, dx \right]$$

$$= -\frac{1}{u}\frac{\partial f(0,t)}{\partial x} + \int_0^{\infty} e^{-ux}\frac{\partial f(x,t)}{\partial x}\, dx. \qquad \to (**)$$

Substitution (**) in (*):

$$= \frac{1}{v}\int_0^{\infty} e^{-vt}\left[-\frac{1}{u}\frac{\partial f(0,t)}{\partial x} + \int_0^{\infty} e^{-ux}\frac{\partial f(x,t)}{\partial x}\, dx \right] dt$$

$$= -\frac{1}{u}\frac{\partial [K(0,v)]}{\partial x} + \frac{1}{v}\int_0^{\infty} e^{-vt}e^{-ux}\frac{\partial f(x,t)}{\partial x}\, dxdt$$

$$= -\frac{1}{u}\frac{\partial [K(0,v)]}{\partial x} + u\left[uK(u,v) \; - \frac{1}{u}K(0,v) \right]$$

$$A_2\left\{ \frac{\partial^2 f}{\partial x^2} \right\} = u^2 K(u,v) \; - K(0,v) - \frac{1}{u}\frac{\partial [K(0,v)]}{\partial x} \, .$$

$$(iii)\; A_2\left\{ \frac{\partial f}{\partial t} \right\} = \frac{1}{uv}\int_0^{\infty}\int_0^{\infty} e^{-(ux+vt)}\frac{\partial f(x,t)}{\partial t}\, dt\, dx \, .$$

$$= \frac{1}{u}\int_0^{\infty} e^{-ux}\left[\frac{1}{v}\int_0^{\infty} e^{-vt}\frac{\partial f(x,t)}{\partial t}\, dt \right] dx \; \to (*)$$

$$\mathrm{I} = \frac{1}{v}\int_0^{\infty} e^{-vt}\frac{\partial f(x,t)}{\partial t}\, dt$$

Integration by part:

$$u = e^{-vt} \quad , \; du = -ve^{-vt}dt \quad , dv = \frac{\partial f(x,t)}{\partial t}\, dt \, , v = f(x,t) \, .$$

$$\mathrm{I} = \frac{1}{v}\left[e^{-vt} f(x,t)|_0^{\infty} + v\int_0^{\infty} e^{-vt} f(x,t)\, dt \right]$$

$$= \frac{1}{v}\left[-f(x,0) + v\int_0^\infty e^{-vt} f(x,t)\,dt. \right]$$

$$= -\frac{1}{v} f(x,0) + \int_0^\infty e^{-vt} f(x,t)\,dt. \quad \rightarrow (**)$$

Substitution (**) in (*):

$$= \frac{1}{u}\int_0^\infty e^{-ux}\left[-\frac{1}{v} f(x,0) + \int_0^\infty e^{-vt} f(x,t)\,dt \right] dx$$

$$= -\frac{1}{v}\int_0^\infty \frac{1}{u} e^{-ux} f(x,0) dx + \frac{1}{u}\int_0^\infty e^{-ux} e^{-vt} f(x,t)\,dt\ \ dx$$

$$= -\frac{1}{v} K(u,0) + vK(u,v)\,.$$

$$A_2\left\{\frac{\partial f}{\partial t}\right\} = vK(u,v)\ -\frac{1}{v}\ K(u,0).$$

$$(iv)\ A_2\left\{\frac{\partial^2 f}{\partial t^2}\right\} = \frac{1}{uv}\int_0^\infty\int_0^\infty e^{-(ux+vt)}\frac{\partial^2 f(x,t)}{\partial t^2}\ dt\,dx\,.$$

$$= \frac{1}{u}\int_0^\infty e^{-ux}\left[\frac{1}{v}\int_0^\infty e^{-vt}\frac{\partial^2 f(x,t)}{\partial t^2}\ dt\right] dx$$

$$\mathrm{I} = \frac{1}{v}\int_0^\infty e^{-vt}\frac{\partial^2 f(x,t)}{\partial t^2}\ dt$$

Integration by part:

$$u = e^{-vt}\ \ ,\ \ du = -ve^{-vt}dt\ \ ,dv = \frac{\partial^2 f(x,t)}{\partial t^2}\ dt\,,v = \frac{\partial f(x,t)}{\partial t}\,.$$

$$\mathrm{I} = \frac{1}{v}\left[e^{-vt}\frac{\partial f(x,t)}{\partial t}\Big|_0^\infty + v\int_0^\infty e^{-vt}\frac{\partial f(x,t)}{\partial t}\ dt \right]$$

$$= -\frac{1}{v}\frac{\partial f(x,0)}{\partial t} + \int_0^\infty e^{-vt}\frac{\partial f(x,t)}{\partial t}\ dt\,. \quad \rightarrow (**)$$

Substitution (**) in (*):

$$=\frac{1}{u}\int_0^\infty e^{-ux}\left[-\frac{1}{v}\frac{\partial f(x,0)}{\partial t}+\int_0^\infty e^{-vt}\frac{\partial f(x,t)}{\partial t}\,dt\right]dx$$

$$=-\frac{1}{v}\frac{\partial[K(u,0)]}{\partial t}+v\left[vK(u,v)\ -\frac{1}{v}K(u,0)\right]$$

$$=-\frac{1}{v}\frac{\partial[K(u,0)]}{\partial t}+v^2\,K(u,v)-K(u,0)$$

$$A_2\left\{\frac{\partial^2 f}{\partial t^2}\right\}=v^2K(u,v)\ -K(u,0)-\frac{1}{v}\frac{\partial[K(u,0)]}{\partial t}\,.$$

$$(v)\ A_2\left\{\frac{\partial^2 f}{\partial x\partial t}\right\}=\frac{1}{uv}\int_0^\infty\int_0^\infty e^{-(ux+vt)}\frac{\partial^2 f(x,t)}{\partial x\partial t}\,dt\,dx\,.$$

$$=\frac{1}{v}\int_0^\infty e^{-vt}\left[\frac{1}{u}\frac{\partial}{\partial t}\int_0^\infty e^{-ux}\frac{\partial f(x,t)}{\partial x}\,dx\right]dt$$

Integration by part:

$$u=\frac{1}{u}e^{-vt}\ \ ,\ du=-\frac{v}{u}e^{-vt}dt\ \ ,dv=\frac{\partial}{\partial t}\int_0^\infty e^{-ux}\frac{\partial f(x,t)}{\partial x}\,dx\,,$$

$$v=\int_0^\infty e^{-ux}\frac{\partial f(x,t)}{\partial x}\,dx\ .$$

$$=\frac{1}{v}\left[\frac{1}{u}e^{-vt}\int_0^\infty e^{-ux}\frac{\partial f(x,t)}{\partial x}\,dx\,\Big|_0^\infty+\frac{v}{u}\int_0^\infty e^{-vt}\int_0^\infty e^{-ux}\frac{\partial f(x,t)}{\partial x}\,dx\ dt\right]$$

$$=\frac{1}{uv}e^{-vt}\int_0^\infty e^{-ux}\frac{\partial f(x,t)}{\partial x}\,dx\,\Big|_0^\infty+v\left[\frac{1}{uv}\int_0^\infty\int_0^\infty e^{-(ux+vt)}\frac{\partial f(x,t)}{\partial x}\,dx\,dt\right]$$

$$=\frac{1}{v}e^{-vt}\frac{1}{u}\int_0^\infty e^{-ux}\frac{\partial f(x,t)}{\partial x}\,dx\,\Big|_0^\infty+v\left[uK(u,v)\ -\frac{1}{u}\,K(0,v)\right].\quad\rightarrow(*)$$

$$\mathrm{I}=\frac{1}{u}\int_0^\infty e^{-ux}\frac{\partial f(x,t)}{\partial x}\,dx$$

Integration by part:

$$u = e^{-ux} \quad , \quad du = -ue^{-ux}\,dx \quad , dv = \frac{\partial f(x,t)}{\partial x}\,dx\,, v = f(x,t)\,.$$

$$= \frac{1}{u}\left[e^{-ux}\,f(x,t)|_0^\infty + u\int_0^\infty e^{-ux} f(x,t)\,dx\right]$$

$$= \frac{1}{u}[-f(0,t)] + u\left[\frac{1}{u}\int_0^\infty e^{-ux} f(x,t)\,dx\right]$$

$$= -\frac{1}{u} f(0,t) + uK(u,t). \qquad \rightarrow (**)$$

Substitution (**) in (*):

$$= \frac{1}{v} e^{-vt}\left[-\frac{1}{u} f(0,t) + uK(u,t)\right]\Big|_0^\infty + v\left[uK(u,v) - \frac{1}{u}\,K(0,v)\right]$$

$$= -\frac{1}{v}\left[-\frac{1}{u} f(0,0) + uK(u,0)\right] + uvK(u,v) - \frac{v}{u}\,K(0,v)$$

$$= \frac{1}{uv}\,f(0,0) - \frac{u}{v}\,K(u,0) + uvK(u,v) - \frac{v}{u}\,K(0,v)\,.$$

$$A_2\left\{\frac{\partial^2 f}{\partial x \partial t}\right\} = \frac{1}{uv} f(0,0) - \frac{u}{v} K(u,0) + uvK(u,v) - \frac{v}{u}\,K(0,v)\,.$$

Theorem 2: (Convolution)

Let $f(x,t)$ and $g(x,t)$ be the functions having Double Aboodh transform $\mathrm{M}(u,v)$ and $\mathrm{N}(u,v)$, then the Double Aboodh transform of convolution of $f(x,t)$ and $g(x,t) is$,

$$A_2[(f * g)(x,t)\,;(u,v)] = uv\mathrm{M}(u,v)\mathrm{N}(u,v).$$

Solving PIDE's using Double Aboodh transform Method:

Consider the general linear partial integro-differential equation,

$$\sum_{i=0}^{m} a_i \frac{\partial^i u}{\partial t^i} + \sum_{i=0}^{n} b_i \frac{\partial^i u}{\partial x^i} + cu + \sum_{i=0}^{r} d_i \int_0^t k_i(t-s)\frac{\partial^i u(x,s)}{\partial x^i}\,ds + f(x,t) = 0\,. \qquad \rightarrow (10)$$

(With prescribed condition).

Where $f(x,t)$ and $k_i(t,s)$ are known functions. a_i , b_i , d_i and c are constants or the functions of x.

Taking double Aboodh transform on both sides of PIDE (10) with respect to t we get,

$$\sum_{i=0}^{m} a_i A_2\left\{\frac{\partial^i u}{\partial t^i}\right\} + \sum_{i=0}^{n} b_i A_2\left\{\frac{\partial^i u}{\partial x^i}\right\} + cA_2\{u\} + \sum_{i=0}^{r} d_i A_2\left\{\int_0^t k_i(t-s) * \frac{\partial^i u(x,s)}{\partial x^i} ds\right\} + A_2\{f(x,t)\} = 0 .$$

Using theorem 1 and theorem 2 for double Aboodh transform we get,

$$\sum_{i=0}^{m} a_i \left\{v^i \bar{u}(x,v) - \sum_{k=0}^{i-1} \frac{1}{v^{2-i+k}} A_x\left\{\frac{\partial^k \bar{u}(x,o)}{\partial t^k}\right\}\right\} + \sum_{i=0}^{n} b_i \left\{v^i \bar{u}(x,v) - \sum_{j=0}^{i-1} \frac{1}{u^{2-i+j}} A_t\left\{\frac{\partial^j \bar{u}(0,t)}{\partial x^j}\right\}\right\} + c\bar{u}(x,v) + \sum_{i=0}^{r} d_i v \bar{k}_i(v)\left\{u^i \bar{u}(x,v) - \sum_{j=0}^{i-1} \frac{1}{u^{2-i+j}} A_t\left\{\frac{\partial^j \bar{u}(0,t)}{\partial x^j}\right\}\right\} + \bar{f}(x,v) = 0 . \quad \rightarrow (11)$$

Where:

$$\bar{u}(x,v) = A_2[u(x,v)] , \bar{f}(x,v) = A_2[f(x,v)] , \bar{k}_i(v) = A_2[k_i(t)]$$

Equation (11) is an algebraic equation in $\bar{u}(x,v)$.

Solving algebraic equation and taking inverse double Aboodh transform of $\bar{u}(x,v)$, we get an exact solution $u(x,t)$.

Illustrative example:

Example:

Consider the PIDE,

$$u_{tt} = u_x + 2\int_0^t (t-s).\mathrm{u}(x,s)ds - 2e^x \ . \rightarrow (12)$$

With initial conditions

$$u(x,0) = e^x \ , u_t(x,0) = 0 \ . \ \rightarrow (13)$$

And boundary condition

$$u(0\,,t) = cost\ . \quad \rightarrow (14)$$

Solution:

Taking double Aboodh transform of equation (12):

$$v^2 K(u,v) - K(u,0) - \frac{1}{v} K_t(u,0) = uK(u,v) - \frac{1}{u} K(0,v) + \frac{2}{v^2} K(u,v) - \frac{2}{u^2 v^2 (1-\frac{1}{u})}. \quad \rightarrow (15)$$

Moreover, single Aboodh transform of initial conditions (13) & boundary condition (14) are given by:

$$K(u,0) = \frac{1}{u^2(1-\frac{1}{u})} \quad , \quad K_t(u,0) = 0 \ , \ K(0,v) = \frac{1}{v^2+1}$$

Then equation (15) becomes

$$v^2 K(u,v) - \frac{1}{u^2\left(1-\frac{1}{u}\right)} = uK(u,v) - \frac{1}{u(v^2+1)} + \frac{2}{v^2} K(u,v) - \frac{2}{u^2 v^2\left(1-\frac{1}{u}\right)}.$$

$$\left(u + \frac{2}{v^2} - v^2\right) K(u,v) = \frac{2}{u^2 v^2\left(1-\frac{1}{u}\right)} + \frac{1}{u(v^2+1)} - \frac{1}{u^2\left(1-\frac{1}{u}\right)}.$$

$$\left(\frac{uv^2+2-v^4}{v^2}\right) K(u,v) = \frac{2(v^2+1)+uv^2\left(1-\frac{1}{u}\right)-v^2(v^2+1)}{u^2 v^2 (1-\frac{1}{u})(v^2+1)}$$

$$K(u,v) = \frac{1}{u^2(1-\frac{1}{u})} \cdot \frac{1}{v^2+1}. \quad \rightarrow (16)$$

Applying inverse double Aboodh transform of equation (16), we get an exact solution:

$$u(x,t) = e^x\, cost\ .$$

3. CONCLUSION

PIDE's are used in modeling various phenomena in science , engineering and social sciences .The methods of Aboodh and Double Aboodh Transforms are successfully used to solve a general linear PIDE's. In Aboodh Transform general linear PIDE's are solve by using convolution kernel. In double Aboodh Transform by using an algebraic equation, we solve general linear PIDE's.

Finally, we get exact solutions of such PIDE after a few steps of calculations.

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