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⋕ frg Connectedness in Fine- Topological Spaces

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Abstract

Rajak K. have introduced fine-topological space which is a special case of generalized topological space. In this paper, we have introduced ⋕ frg -open sets, ⋕ frg -open sets and define some new continuous functions. Also, we have introduced ⋕ frg -connectedness and their properties are studied.

Keywords: ⋕ frg -open set, ⋕ frg -closed sets, ⋕ frg -compact spaces, ⋕ frg -complete accumulation point, ⋕ frg -connectedness, fine-open sets, fine-closed sets.

Mathematics Subject Classification: 54XX, 54CXX, 54A05.

1. INTRODUCTION

Many researchers have investigated the basic properties of connectedness. The productivity and fruitfulness of these notions of connectedness motivated mathematicians to generalize these notions. In the course of these attempts many stronger and weaker forms of and connectedness have been introduced and investigated.

Regular open sets and strong regular open sets have been introduced and investigated by Stone [18] and Tong [19] respectively. Levine [7], Biswas [3], Cameron [4], Sundaram and Sheik john[19], Bhattacharyya and Lahiri [2], Nagaveni [9], Pushpalatha [16], Gnanambal [5], Gnanambal and Balachandran [6], Palaniappan and Rao[10], Maki, Devi and Balachandran [8], Benchalli and Wali [1] and S. Syed Ali Fathima and M. Mariasingam [20] introduced and investigated semi open sets, generalized closed sets, regular semi open sets, weakly closed sets, semi generalized closed sets, weakly generalized closed sets, strongly generalized closed sets, generalized pre-regular closed sets, regular generalized closed sets, generalized a-generalized closed sets, Rw-closed sets and #regular generalized closed sets respectively.

Rajak K. [17], have investigated a special case of generalized topological space called fine topological space. In this space, Powar P. L. and Rajak K. [11 – 15] have defined a new class of open sets namely fine-open sets which contains all α−open sets, β−open sets, semi-open sets, pre-open sets, regular open sets etc. By using these fine-open sets they have defined fine-irresolute mappings which includes pre-continuous functions, semi-continuous function, α−continuous function, β−continuous functions, α−irresolute functions, β−irresolute functions, etc (cf. [11]-[15]).

The aim of this paper, we have introduced fgsp-open sets, ♯ frg -open sets and define some new continuous functions. Also, we have introduced ♯ frg -connectedness and their properties are studied.

2. PRELIMINARIES

Throughout this paper (X, τ), (Y, σ) are topological spaces with no separation axioms assumed unless otherwise stated. Let A ⊆ X. The closure of A and the interior of A will be denoted by Cl(A) and Int(A) respectively.

Definition: 2.1 A subset A of a space X is called

(1) a regular open set[18]if A = intcl (A) and a regular closed set if A = clint (A).

(2) regular semi open[4]if there is a regular open U such U ⊆A ⊆ cl(U).

(3) regular generalized closed set (briefly, rg-closed)[10] if cl (A) ⊆U whenever A⊆U and U is regular open in X.

(4) rw-closed [1] if cl(A) ⊆ U whenever A⊆U and U is regular semi open.

Definition 2.2 Let A be a subset of X. Then

(i) rg-interior [1] of A is the union of all rg-open sets contained in A.

(ii) rg-closure [1] of A is the intersection of all rg-closed sets containing A.

The rg-interior [respectively rg-closure] of A is denoted by rgInt(A) [respectively rgCl(A)].

Definition: 2.3 A subset A of a space X is called #regular generalized closed (briefly #rg-closed) [20] set if cl(A) ⊆U whenever A ⊆U and U is rw-open. We denote the set of all #rg– closed sets in X by #RGC(X).

Definition 2.4 The #rg-closure [20] of a set A is the intersection of all #rg -closed sets containing A and is denoted by #rg Cl(A).

Definition 2.5 The #rg -interior [20] of a set A is the union of all #rg -open sets contained in A and is denoted by #rg Int(A).

Remark 2.6 Every #rg -closed set is rg -closed.

Definition 2.7 A function f: X → Y is said to be #rg -continuous [20] if $f^{-1}(V)$ is #rg -closed in X for every closed set V of Y (c.f. [2]).

Definition 2.8 A function f : X → Y is said to be #rg -irresolute [20] if $f^{-1}(V)$ is #rg -closed in X for every #rg -closed set V of Y (c.f. [2]).

Definition 2.9 A topological space X is said to be #rg -connected if X cannot be expressed as a disjoint union of two non-empty #rg -open sets..

Definition 2.10 A topological space X is said to be #Trg -space if every #rg -closed subset of X is closed subset of X (c.f. [2]).

Definition 2.11 Let (X, τ) be a topological space we define

$\tau(A_\alpha) = \tau_\alpha$ (say) = $\{G_\alpha(\neq X) : G_\alpha \cap A_\alpha \neq \phi$, for $A_\alpha \in \tau$ and $A_\alpha \neq \phi, X$, for some $\alpha \in J$, where J is the index set.$\}$ Now, we define $\tau_f = \{\phi, X, \cup_{\alpha \in J} \{\tau_\alpha\}\}$

The above collection τ_f of subsets of X is called the fine collection of subsets of X and (X, τ, τ_f) is said to be the fine space X generated by the topology τ on X (cf. [15]).

Definition 2.12 A subset U of a fine space X is said to be a fine-open set of X, if U belongs to the collection τ_f and the complement of every fine-open sets of X is called the fine-closed sets of X and we denote the collection by F_f (cf. [15]).

Definition 2.13 Let A be a subset of a fine space X, we say that a point x ∈ X is a fine limit point of A if every fine-open set of X containing x must contains at least one point of A other than x (cf. [15]).

Definition 2.14 Let A be the subset of a fine space X, the fine interior of A is defined as the union of all fine-open sets contained in the set A i.e. the largest fine-open set contained in the set A and is denoted by fInt(cf. [15])

Definition 2.15 Let A be the subset of a fine space X, the fine closure of A is defined as the intersection of all fine-closed sets containing the set A i.e. the smallest fine-closed set containing the set A and is denoted by fcl(cf. [15]).

Definition 2.16 A function f : $(X, \tau, \tau_f) \to (Y, \tau', \tau_{f'})$ is called fine-irresolute (or f-irresolute) if $f^{-1}(V)$ is fine-open in X for every fine-open set V of Y (cf. [15]).

3. #frg -CONNECTEDNESS

In this section we have defined #frg -connectedness in fine-topological space.

Definition 3.1 A subset A of a space X is called fine regular generalized closed set (briefly, rg-closed)[13] if fcl (A) ⊆U whenever A⊆U and U is regular open in X. The family of all frg-open sets (respectively frg-closed sets) of (X, τ) is denoted by frgO(X, τ) [respectively frgCL(X, τ)].

Example 3.2 Let X = {a, b, c} with the topology τ = {∅, X,{a}, {a, c}}, τ_f = {∅, X, {a}, {c}, {a, b}, {a, c}, {b, c}}, F = {∅, X, {a}, {b}, {c}, {a, b}, {b, c}}. It may be easily check that the only frg-open sets are P(X).

Definition 3.3 Let A be a subset of X. Then

(i) frg-interior of A is the union of all frg-open sets contained in A.

(ii) frg-closure of A is the intersection of all frg-closed sets containing A.

The frg-interior [respectively frg-closure] of A is denoted by $f\mathrm{rg}(A)$ [respectively$f\mathrm{rg}cl(A)$].

Example 3.4 Let X = {a, b, c} with the topology τ = {∅, X,{a}, {a, c}}, τ_f = {∅, X, {a}, {c}, {a, b}, {a, c}, {b, c}}, F = {∅, X, {a}, {b}, {c}, {a, b}, {b, c}}. Let $S=\{a,b\}$ the frg-interior of S is {a, b} and the fb-closure of S is $\{a,b\}$.

Definition 3.5 A subset A of a space X is called # fine regular generalized closed (briefly #frg-closed) set if fcl(A) ⊆U whenever A ⊆U and U is frw-open. The family of all #frg-open [respectively #frg -closed] sets of (X, τ) is denoted by #frg O(X, τ) respectively #frg CL(X, τ)].

Remark 3.6 Every #rg -open set is #frg -open and every #rg -open and fb-open set is fine-open.

Example 3.7 Let X = {a, b, c} with the topology τ = { ∅, X, {a, b}}, τf = {∅, X, {a}, {a, b}, {a, c}, {b}, {b, c}}, Ff = {∅, X, {b, c}, {c}, {b}, {a, c}, {a}}. It can be easily check that the only #frg-open sets are { X, ϕ, {a} , $\{b\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$} .

Remark 3.8 Every #frg -open set is frg -open.

Definition 3.9 The #frg -closure of a set A is the intersection of all #frg −closed sets containing A and is denoted by #frgC(A).

Definition 3.10 The #frg -interior of a set A is the union of all #frg -open sets contained in A and is denoted by #frg (A) .

Example 3.11 Let X = {a, b, c} with the topology τ = { ∅, X, {a}}, τf = {∅, X, {a}, {a, b}, {a, c}}, Ff = {∅, X, {b, c}, {c}, {b}}. Let S={ b} the #frg-interior of S is ∅ and the #frg-closure of S is {b}.

Definition 3.12 A function f: X → Y is said to be #frg -continuous if $f^{-1}(V)$ is #frg -closed in X for every fine-closed set V of Y.

Example 3.13 Let X = {a, b, c} with the topology τ= { ϕ,X , {a} , {a,c}} ,τ_f={ ϕ,X, $\{a\}$, $\{a,b\}$, $\{a,c\}$,{ $b\}$, $\{b,c\}$ }, F_f={ϕ X,$\{a\}$,$\{b\}$,$\{c\}$,$\{a,c\}$,$\{b,c\}$ } and let Y = {1, 2, 3} with the topology τ'= {ϕ, Y, {1} , {1,2} , τf'={ϕ, Y, {1}, {1, 2}, {1, 3}, {2}, {2, 3} }. We define a mapping f:X→Y by f(a) = 1, f(b) = 2, f(c) = 3. It may be easily check that the only #frg-open sets of Y are {ϕ, Y, {1} , {1,2} , {1,3} ,{ 2} ,{ 2,3}} whose pre-images are {ϕ, X, $\{a\}$, $\{a,b\}$, $\{a,c\}$, $\{b\}$, $\{b,c\}$} which are fine-closed in X. Hence, f is #frg-continuous.

Definition 3.14 A function f: X → Y is said to be #frg -irresolute if $f^{-1}(V)$ is #frg -closed in X for every #frg -closed set V of Y.

Example 3.15 Let X = {a, b,c} with the topology τ= {ϕ, X,$\{a,b\}$} ,τ_f={ϕ, X, $\{a\}$,

$\{a,b\}$, $\{a,c\}$, $\{b\}$, $\{b,c\}$ } and let Y = {1, 2, 3} with the topology τ'= {ϕ,Y, {1} , {1,2}} ,$\tau_{f'}$= {ϕ,Y, {1} , {1,2} , {1,3} , {2} ,{ 2,3}} . We define a mapping f:X→Y by f(a) = 1, f(b) = 2, f(c) = 3. It may be easily checked that the only #frg-open sets of Y are {ϕ,Y, {1} , {1,2} , {1,3} ,{ 2} , {2,3} whose pre-images are {ϕ, X, $\{a\}$, $\{a,b\}$, $\{a,c\}$, $\{b\}$,{ b,c}} which are #frg-open in X. Hence, f is #frg -irresolute.

Definition 3.16 A topological space X is said to be #frg -connected if X cannot be expressed as a disjoint union of two non-empty #frg -open sets. A subset of X is #frg -connected if it is #frg -connected as a subspace.

Example 3.17 Let X = {a, b, c} with the topology τ = {X, ϕ, {a}}, τ_f={ϕ, X, $\{a\}$, $\{a,b\}$, $\{a,c\}$ } and #frg- open sets are { X, ϕ , {a}, {a,b}, {a,c}}. Then it is #frg connected space.

Definition 3.18 A topological space X is said to be #fTrg -space if every #frg -closed subset of X is fine-closed subset of X.

Example 3.19 Let X = {a, b, c} and let τ = {X, ϕ, {a}, {a, c}}, τ_f= { ϕ,X, $\{a\}$, $\{a,b\}$, $\{a,c\}$,{ b} , $\{b,c\}$ } and #rg- open sets are { X, ϕ, {a}, {a, c}} . Then it is #rg-connected.

Theorem 3.20 If f : X → Y is a #frg -continuous and X is #frg -connected, then Y is fine-connected.

Proof: Suppose that Y is not fine-connected. Let Y = A∪B where A and B are disjoint non-empty fine-open set in Y. Since f is #frg -continuous and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty #frg -open sets in X. This contradicts the fact that X is #frg -connected. Hence, Y is fine -connectedness in topological spaces.

Theorem 3.21 If f : X → Y is a #frg -irresolute surjection and X is #frg -connected, then Y is #frg -connected.

Proof: Suppose that Y is not #frg -connected. Let Y = A ∪ B where A and B are disjoint non-empty #frg -open set in Y. Since f is #frg -irresolute and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty #frg -open sets in X. This contradicts the fact that X is #frg -connected. Hence Y is #frg-connected.

Theorem 3.22 In a fine-topological space (X, τ, τ_f) with at least two points, if frgO(X, τ, τf) = frgCL(X, τ, τf) then X is not #frg -connected.

Proof: By hypothesis we have frgO(X, τ, τf)) = frgCL(X, τ, τf)) and every #frg-closed set is frg -closed, there exists some non-empty proper subset of X which is both #frg -open and #frg -closed in X. So by last Theorem 3.21 we have X is not #frg -connected.

Theorem 3.23 Suppose that X is a #fTrg -space then X is fine-connected if and only if it is #frg -connected.

Proof: Suppose that X is fine-connected. Then X cannot be expressed as disjoint union of two non-empty proper subsets of X. Suppose X is not a #frg -connected space. Let A and B be any two #frg -open subsets of X such that X = A∪B, where A ∩ B = ∅ and A ⊂ X, B ⊂ X. Since X is #fTrg -space and A,B are #frg -open, A,B are open subsets of X, which contradicts that X is fine-connected. Therefore X is #frg -connected. Conversely, every fine-open set is #frg -open. Therefore every #frg -connected space is fine-connected.

Theorem 3.24 If the #frg -open sets C and D form a separation of X and if Y is #frg -connected subspace of X, then Y lies entirely within C or D.

Proof: Since C and D are both #frg -open in X the sets C ∩ Y and D ∩ Y are #frg -open in Y these two sets are disjoint and their union is Y. If they were both non-empty, they would constitute a separation of Y. Therefore, one of them is empty. Hence Y must lie entirely in C or in D.

Theorem 3.25 Let A be a #frg -connected subspace of X. If A ⊂ B ⊂ #frg ClA then B is also #frg -connected.

Proof: Let A be #frg -connected and let A ⊂ B ⊂ #frg Cl(A). Suppose that B = C∪D is a separation of B by fgb-open sets. Then by Theorem 3.24 above A must lie entirely in C or in D. Suppose that A ⊂ C, then #frg Cl(A) ⊆ #frgCl(C). Since #frg Cl(C) and D are disjoint, B cannot intersect D. This contradicts the fact that D is non-empty subset of B. So D = ∅ which implies B is #frg -connected.

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