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# On pseudo-projective curvature tensor with respect to semi-symmetric non-metric connection in a Kenmotsu manifold

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**Abstract**

The object of the present paper is to study some new results on pseudo-projective curvature tensor in a Kenmotsu manifold with respect to semi-symmetric non-metric connection.

**AMS subject classification:** 53C05, 53C25, 53D15.
**Keywords:** Kenmotsu manifold, semi-symmetric non-metric connection, $\eta$-Einstein manifold, pseudo-projective curvature, pseudo-projectively flat manifold, locally pseudo-projectively-$\phi$-symmetric manifold.

## 1. Introduction

In 1924, the idea of semi-symmetric linear connection on a differentiable manifold was introduced by Friedmann and Schouten [10]. In 1930, Bartolotti [5] gave a geometrical meaning of such a connection. In 1932, H. A. Hayden [12] introduced semi-symmetric metric connection in a Riemannian manifolds. A systematic study of the semi-symmetric metric connection in a Riemannian manifold was started by Yano [23]. The semi-symmetric non-metric connection in a Riemannian manifold was studied in the papers [1, 8].

In [20], Tanno classified almost contact metric manifold $M$ whose automorphism group attains the maximum dimension. For such a manifold, the sectional curvature of plane section containing $\xi$ is a constant, say $c$. (1) If $c > 0$, $M$ is a homogeneous Sasakian manifold of constant $\phi$-sectional curvature. (2) If $c = 0$, $M$ is global Riemannian product of a line or circle with a Kähler manifold of constant holomorphic sectional curvature. (3) If $c < 0$, $M$ is warped product space $\mathbb{R} \times_f \mathbb{C}^n$. In 1972, Kenmotsu [14] characterized the differential geometric properties of manifolds of class (3); the structure so obtained is now known as Kenmotsu structure. A Kenmotsu structure is not Sasakian (see [14]). Kenmotsu manifolds have also been studied in several papers [9, 13, 18] and the reference therein. Kenmotsu manifolds with semi symmetric metric and non metric connections have been studied by various authors (see [2, 3, 4, 16, 17, 19, 21, 22]).

In this paper we obtain some new results on pseudo-projective curvature tensor in a Kenmotsu manifold with respect to semi-symmetric non-metric connection. The paper is organized as follows : In section 2, we give a brief introduction of a Kenmotsu manifold and define semi-symmetric non-metric connection. In section 3, we find the curvature tensor, Ricci tensor and scalar curvature in a Kenmotsu manifold with respect to semi-symmetric non-metric connection. In section 4, it is shown that the pseudo-projectively flat Kenmotsu manifold with respect to $\bar{\nabla}$ is an $\eta$- Einstein manifold. In section 5, we consider a Kenmotsu manifold with respect to semi-symmetric non-metric connection satisfying the condition $\bar{\tilde{P}}(\xi, X) \cdot \bar{S} = 0$. In section 6, a locally pseudo-projectively-$\phi$-symmetric Kenmotsu manifold with respect to semi-symmetric non-metric connection have been studied.

## 2. Kenmotsu manifold

An $n$-dimensional smooth manifold $(M, g)(n = 2m + 1 > 1)$ is said to be an almost contact metric manifold [14], if it admits a (1,1) tensor field $\phi$, a structure vector field $\xi$, an 1-form $\eta$ and the Riemannian metric $g$ which satisfy

$$\phi^2(X) = -X + \eta(X)\xi, \tag{2.1}$$

$$g(X, \xi) = \eta(X), \;\; \eta(\phi X) = 0, \;\; \phi(\xi) = 0, \;\; \eta(\xi) = 1, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.3}$$

for arbitary vector fields $X, Y$ on $M$.
An almost contact metric manifold $M(\phi, \xi, \eta, g)$ is said to be Kenmotsu manifold if [14]

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.4}$$

$$(\nabla_X \phi)(Y) = -g(X, \phi Y) - \eta(Y)\phi X, \tag{2.5}$$

where $\nabla$ is the Riemannian connection of $g$.
Also the following relations hold in a Kenmotsu manifold [14]

$$(\nabla_X \eta)(Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.6}$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.7}$$

$$R(\xi, X)Y = -R(X, \xi)Y = \eta(Y)X - g(X, Y)\xi, \tag{2.8}$$

$$\eta(R(X, Y)Z) = g(X, Z)\eta(Y) - g(Y, Z)\eta(X), \tag{2.9}$$

$$S(X, \xi) = -2n\eta(X). \tag{2.10}$$

**Definition 2.1.** A Kenmotsu manifold $M$ is said to be a $\eta$-Einstein if its Ricci tensor $S$ of type (0,2) is of the form

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y), \tag{2.11}$$

where $a$ and $b$ are smooth functions on $M$. If $b = 0$, then an $\eta$-Einstein manifold becomes to Einstein manifold.

**Definition 2.2.** The pseudo-projective curvature tensor $\tilde{P}$ in a Kenmotsu manifold M is defined by [7]

$$\begin{aligned}\tilde{P}(X, Y)Z = \alpha R(X, Y)Z &+ \beta[S(Y, Z)X - S(X, Z)Y] \\ &- \frac{r}{(2n+1)}\left[\frac{\alpha}{2n} + \beta\right][g(Y, Z)X - g(X, Z)Y],\end{aligned} \tag{2.12}$$

where $\alpha$ and $\beta$ are constants such that $\alpha, \beta \neq 0$, $R$ is the curvature tensor, $S$ is the Ricci tensor and $r$ is the scalar curvature.

If $\alpha = 1$ and $\beta = -\frac{1}{2n}$, then (2.12) takes the form

$$\tilde{P}(X, Y)Z = R(X, Y)Z - \frac{1}{2n}[S(Y, Z)X - S(X, Z)Y] = P(X, Y)Z, \tag{2.13}$$

where $P$ is the projective curvature tensor [15]. Thus the projective curvature tensor $P$ is the particular case of pseudo-projective curvature tensor $\tilde{P}$. The manifold is said to be pseudo-projectively flat if $\tilde{P}$ vanishes identically on $M$.

Let $\nabla$ be a linear connection in an $n$-dimensional differentiable manifold $M$. The torsion tensor $T$ and the curvature tensor $R$ of $\nabla$ are given respectively by

$$\begin{aligned}T(X, Y) &= \nabla_X Y - \nabla_Y X - [X, Y], \\ R(X, Y)Z &= \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z.\end{aligned}$$

The connection $\nabla$ is said to be symmetric if its torsion tensor $T$ vanishes, otherwise it is non-symmetric. The connection $\nabla$ is said to be metric connection if there is a Riemannian metric $g$ in $M$ such that $\nabla g = 0$, otherwise it is non-metric. It is well known that a linear connection is symmetric and metric if it is the Levi-Civita connection.

A linear connection $\nabla$ is said to be semi-symmetric connection [11] if its torsion tensor $T$ is of the form

$$T(X, Y) = \eta(Y)X - \eta(X)Y, \tag{2.14}$$

where $\eta$ is a 1-form.

A relation between semi-symmetric non-metric connection $\bar{\nabla}$ and the Levi-Civita connection $\nabla$ have been obtained by Barua and Mukhopadhyay [6] and is given by

$$\bar{\nabla}_X Y = \nabla_X Y - \eta(X)Y + g(X, Y)\xi. \tag{2.15}$$

Using (2.15), the torsion tensor $T$ of $M$ with respect to the connection $\bar{\nabla}$ is given by

$$T(X, Y) = \bar{\nabla}_X Y - \bar{\nabla}_Y X - [X, Y] = \eta(Y)X - \eta(X)Y. \tag{2.16}$$

Hence, a relation satisfying (2.16) is called a semi-symmetric connection. Further using (2.15), we have

$$\begin{aligned}(\bar{\nabla}_X g)(Y, Z) &= \bar{\nabla}_X g(Y, Z) - g(\bar{\nabla}_X Y, Z) - g(Y, \bar{\nabla}_X Z)\\ &= 2\eta(X)g(Y, Z) - \eta(Y)g(X, Z) - \eta(Z)g(X, Y)\\ &\neq 0.\end{aligned} \tag{2.17}$$

The connection $\bar{\nabla}$ defined by (2.15) satisfying (2.16) and (2.17) is a type of semi-symmetric non-metric connection.

## 3. Curvature tensor on Kenmotsu manifold with respect to semi-symmetric non-metric connection

The curvature tensor $\bar{R}$ of semi-symmetric non-metric connection $\bar{\nabla}$ in $M$ is defined by

$$\bar{R}(X, Y)Z = \bar{\nabla}_X \bar{\nabla}_Y Z - \bar{\nabla}_Y \bar{\nabla}_X Z - \bar{\nabla}_{[X,Y]} Z, \tag{3.1}$$

Using equations (2.6) and (2.15) in (3.1), we get

$$\begin{aligned}\bar{R}(X, Y)Z = R(X, Y)Z + g(Y, Z)X - g(X, Z)Y + \eta(X)g(Y, Z)\xi\\ - \eta(Y)g(X, Z)\xi,\end{aligned} \tag{3.2}$$

where

$$R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z$$

is the Riemannian curvature tensor of the connection $\nabla$. Taking the inner product of (3.2) with $U$, it follows that

$$\begin{aligned}\tilde{\bar{R}}(X, Y, Z, U) &= \tilde{R}(X, Y, Z, U) + g(Y, Z)g(X, U) - g(X, Z)g(Y, U)\\ &+ \eta(X)\eta(U)g(Y, Z) - \eta(Y)\eta(U)g(X, Z),\end{aligned} \tag{3.3}$$

where $U \in \chi(M)$, $\tilde{\bar{R}}(X, Y, Z, U) = g(\bar{R}(X, Y)Z, U$ and
$\tilde{R}(X, Y, Z, U) = g(R(X, Y)Z, U)$. Let $\{e_1, \ldots, e_{2n+1}\}$ be a local orthonormal basis of the tangent space at a point of the manifold $M$. Then by putting $X = U = e_i$ in (3.3) and taking summation over $i$, $1 \leq i \leq 2n + 1$ and also using (2.2), we get

$$\bar{S}(Y, Z) = S(Y, Z) + (2n + 1)g(Y, Z) - \eta(Y)\eta(Z), \tag{3.4}$$

where $\bar{S}$ and $S$ denote the Ricci tensors of $M$ with respect to $\bar{\nabla}$ and $\nabla$ respectively.
Again, let $\{e_1, \ldots, e_{2n+1}\}$ be a local orthonormal basis of vector fields in $M$. Then by

putting $Y = Z = e_i$ in (3.4) and taking summation over $i$, $1 \leq i \leq 2n+1$ and also using (2.2), it follows that

$$\bar{r} = r + (2n+1)(2n+1) - 1, \tag{3.5}$$

where $\bar{r}$ and $r$ are the scalar curvature of connections $\bar{\nabla}$ and $\nabla$, respectively on $M$.
In a (2n+1)-dimensional Kenmotsu manifold with respect to semi-symmetric non-metric connection, the following relation hold [3]:

$$\bar{R}(\xi, Y)Z = g(Y, Z)\xi - \eta(Y)\eta(Z)\xi, \tag{3.6}$$

$$\eta(\bar{R}(X, Y)Z) = \eta(X)g(Y, Z) - \eta(Y)g(X, Z), \tag{3.7}$$

$$\bar{S}(X, \xi) = 0. \tag{3.8}$$

## 4. Pseudo-projectively flat Kenmotsu manifold with respect to semi-symmetric non-metric connection

Analogous to the definition (2.2),the pseudo-projective curvature tensor $\bar{\tilde{P}}$ in a Kenmotsu manifold with respect to semi-symmetric non-metric connection is given by

$$\begin{aligned}\bar{\tilde{P}}(X, Y)Z &= \alpha\bar{R}(X, Y)Z + \beta[\bar{S}(Y, Z)X - \bar{S}(X, Z)Y] \\ &- \frac{\bar{r}}{2n+1}\left(\frac{\alpha}{2n} + \beta\right)[g(Y, Z)X - g(X, Z)Y].\end{aligned} \tag{4.1}$$

Let us assume that the manifold $M$ with respect to semi-symmetric non-metric connection is pseudo-projectively flat, that is, $\bar{\tilde{P}} = 0$. Then from (4.1), it follows that

$$\begin{aligned}\bar{R}(X, Y)Z &= -\frac{\beta}{\alpha}[\bar{S}(Y, Z)X - \bar{S}(X, Z)Y] \\ &+ \frac{\bar{r}}{\alpha(2n+1)}\left(\frac{\alpha}{2n} + \beta\right)[g(Y, Z)X - g(X, Z)Y].\end{aligned} \tag{4.2}$$

which on using (3.2),(3.4) and (3.5) takes the form

$$\begin{aligned}&R(X, Y)Z \\ = &\left[\frac{r + (2n+1)^2 - 1}{(2n+1)\alpha}\left(\frac{\alpha}{2n} + \beta\right) - \frac{\beta}{\alpha}(2n+1) - 1\right]\{g(Y, Z)X - g(X, Z)Y\} \\ - &\frac{\beta}{\alpha}[S(Y, Z)X - S(X, Z)Y + \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X] \\ + &\eta(Y)g(X, Z)\xi - \eta(X)g(Y, Z)\xi.\end{aligned} \tag{4.3}$$

Taking inner product of (4.3) with $\xi$ and using (2.2), we have

$$\begin{aligned}&\eta(R(X, Y)Z) \\ = &\left[\frac{r + (2n+1)^2 - 1}{(2n+1)\alpha}\left(\frac{\alpha}{2n} + \beta\right) - \frac{\beta}{\alpha}(2n+1) - 1\right]\{g(Y, Z)\eta(X) - g(X, Z)\eta(Y)\} \\ - &\frac{\beta}{\alpha}[S(Y, Z)\eta(X) - S(X, Z)\eta(Y)] + \eta(Y)g(X, Z) - \eta(X)g(Y, Z).\end{aligned} \tag{4.4}$$

By virtue of (2.9), we obtain

$$S(Y,Z)\eta(X) = S(X,Z)\eta(Y) + \left[\frac{r+(2n+1)^2-1}{(2n+1)\beta}\left(\frac{\alpha}{2n}+\beta\right) - (2n+1) - \frac{\alpha}{\beta}\right]\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}. \tag{4.5}$$

Now putting $X = \xi$ in the above equation, we get

$$\begin{aligned} S(Y,Z) &= \left[\frac{r+(2n+1)^2-1}{(2n+1)\beta}\left(\frac{\alpha}{2n}+\beta\right) - (2n+1) - \frac{\alpha}{\beta}\right] g(Y,Z) \\ &+ \left[\frac{\alpha}{\beta} + 1 - \frac{r+(2n+1)^2-1}{(2n+1)\beta}\left(\frac{\alpha}{2n}+\beta\right)\right]\eta(Y)\eta(Z). \end{aligned} \tag{4.6}$$

Thus, the manifold $M$ under the consideration is an $\eta$-Einstein manifold. Hence, we can state the following theorem.

**Theorem 4.1.** A $(2n+1)$-dimensional pseudo-projectively flat Kenmostu manifold with respect to semi-symmetric non-metric connection $\bar{\nabla}$ is an $\eta$-Einstein manifold.

If $\alpha = 1$ and $\beta = -\dfrac{1}{2n}$, then (4.6) takes the form

$$S(Y,Z) = -g(Y,Z) + (1-2n)\eta(Y)\eta(Z). \tag{4.7}$$

Thus, the manifold $M$ under the consideration is an $\eta$-Einstein manifold. Hence, we can state the following proposition.

**Proposition 4.2.** A $(2n+1)$-dimensional projectively flat Kenmostu manifold with respect to semi-symmetric non-metric connection $\bar{\nabla}$ is an $\eta$-Einstein manifold.

Next by using (3.2, (3.4) and (3.5) in equation (4.1), we get

$$\begin{aligned} &\bar{\tilde{P}}(X,Y)Z \\ =\ & \alpha[R(X,Y)Z + g(Y,Z)X - g(X,Z)Y + \eta(X)g(Y,Z)\xi - \eta(Y)g(X,Z)\xi] \\ +\ & \beta[S(Y,Z)X - S(X,Z)Y + (2n+1)\{g(Y,Z)X - g(X,Z)Y\} - \eta(Y)\eta(Z)X \\ +\ & \eta(X)\eta(Z)Y] - \frac{\{r+(2n+1)^2-1\}}{2n+1}\left(\frac{\alpha}{2n}+\beta\right)[g(Y,Z)X - g(X,Z)Y]. \end{aligned} \tag{4.8}$$

On simplification we obtain from (4.8) that

$$\begin{aligned} &\bar{\tilde{P}}(X,Y)Z \\ =\ & \tilde{P}(X,Y)Z + \left(\frac{\beta-\alpha}{2n+1}\right)[g(Y,Z)X - g(X,Z)Y] \\ +\ & \alpha\,[\eta(X)g(Y,Z)\xi - \eta(Y)g(X,Z)\xi] + \beta[\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X], \end{aligned} \tag{4.9}$$

where, $\tilde{P}(X, Y)Z$ is the pseudo-projective curvature tensor with respect to the Levi-Civita connection and is given by (2.12). Next, taking $Z = \xi$ in the (4.9) and using (2.2), we get

$$\bar{\tilde{P}}(X, Y)\xi = \tilde{P}(X, Y)\xi + \left(\frac{\beta - \alpha}{2n+1}\right)[\eta(Y)X - \eta(X)Y] + \beta[\eta(X)Y - \eta(Y)X] \quad (4.10)$$

Suppose that the vector fields $X$ and $Y$ are orthogonal to $\xi$, then (4.10) reduces to

$$\bar{\tilde{P}}(X, Y)\xi = \tilde{P}(X, Y)\xi. \quad (4.11)$$

Hence, we can state the following theorem:

**Theorem 4.3.** A $(2n+1)$-dimensional Kenmotsu manifold is $\xi$-pseudo-projectively flat with respect to semi-symmetric non-metric connection if and only if the manifold is also $\xi$-pseudo-projectively flat with respect to the Levi-Civita connection provided that the vector fields are horizontal vector fields.

## 5. Kenmotsu manifold satisfying $\bar{\tilde{P}}(\xi, X) \cdot \bar{S} = 0$ with respect to semi-symmetric non-metric connection

In this section, we consider Kenmotsu manifold with respect to a semi-symmetric non-metric connection $\bar{\nabla}$ satisfying $\bar{\tilde{P}}(\xi, X) \cdot \bar{S} = 0$. Then

$$(\bar{\tilde{P}}(\xi, X) \cdot \bar{S}(Y, Z)) = 0, \quad (5.1)$$

So,

$$\bar{S}(\bar{\tilde{P}}(\xi, X)Y, Z) + \bar{S}(Y, \bar{\tilde{P}}(\xi, X)Z) = 0. \quad (5.2)$$

By virtue of (2.2), (3.4), (3.6) and 3.8, we obtain

$$\begin{aligned} \bar{\tilde{P}}(\xi, X)Y &= \alpha\big[g(X, Y)\xi - \eta(X)\eta(Y)\xi\big] \\ &+ \beta\big[S(X, Y)\xi + (2n+1)g(X, Y)\xi - \eta(X)\eta(Y)\xi\big] \\ &- \frac{\{r + (2n+1)^2 - 1\}}{2n+1}\left(\frac{\alpha}{2n} + \beta\right)[g(X, Y)\xi - \eta(Y)X]. \end{aligned} \quad (5.3)$$

Now, by substituting (5.3) in (5.2), we have

$$\begin{aligned} &\alpha\big[g(X, Y)\bar{S}(\xi, Z) - \eta(X)\eta(Y)\bar{S}(\xi, Z)\big] \\ &+ \beta\big[S(X, Y)\bar{S}(\xi, Z) + (2n+1)g(X, Y)\bar{S}(\xi, Z) - \eta(X)\eta(Y)\bar{S}(\xi, Z)\big] \\ &- \frac{\{r + (2n+1)^2 - 1\}}{2n+1}\left(\frac{\alpha}{2n} + \beta\right)[g(X, Y)\bar{S}(\xi, Z) - \eta(Y)\bar{S}(X, Z)] \\ &+ \alpha\big[g(X, Z)\bar{S}(Y, \xi) - \eta(X)\eta(Z)\bar{S}(Y, \xi)\big] \\ &+ \beta\big[S(X, Z)\bar{S}(Y, \xi) + (2n+1)g(X, Z)\bar{S}(Y, \xi) - \eta(X)\eta(Z)\bar{S}(Y, \xi)\big] \\ &- \frac{\{r + (2n+1)^2 - 1\}}{2n+1}\left(\frac{\alpha}{2n} + \beta\right)[g(X, Z)\bar{S}(Y, \xi) - \eta(Z)\bar{S}(Y, X)] = 0. \end{aligned} \quad (5.4)$$

Now using (3.8) in (5.4), we obtain

$$\eta(Y)\bar{S}(X, Z) + \eta(Z)\bar{S}(Y, X) = 0. \tag{5.5}$$

Putting $Z = \xi$ in the above equation and using (2.2), (3.4) and (3.8), we obtain

$$S(Y, X) = -(2n + 1)g(Y, X) + \eta(Y)\eta(X). \tag{5.6}$$

That is, $M$ is an $\eta$-Einstein manifold. Hence, we can state the following theorem.

**Theorem 5.1.** A $(2n + 1)$-dimensional Kenmostu manifold $M$ with respect to the semi-symmetric non-metric connection $\bar{\nabla}$ satisfying $\bar{\tilde{P}}(\xi, X) \cdot \bar{S} = 0$, is an $\eta$-Einstein manifold.

## 6. Locally pseudo-projectively-$\phi$-symmetric Kenmotsu manifold with respect to semi-symmetric non-metric connection

In this section, we consider locally pseudo-projectively-$\phi$-symmetric Kenmotsu manifold with respect to the semi-symmetric non-metric connection.

A $(2n+1)$-dimensional Kenmotsu manifold $M$ is said to be locally pseudo-projectively-$\phi$-symmetric with respect to the semi-symmetric non-metric connection if

$$\phi^2((\bar{\nabla}_W\bar{\tilde{P}})(X, Y)Z) = 0, \tag{6.1}$$

for all vector fields $X$, $Y$, $Z$ orthogonal to $\xi$, where $\bar{\tilde{P}}$ is the pseudo-projective curvature tensor of the semi-symmetric non-metric connection $\bar{\nabla}$. From relation (2.15) we have

$$(\bar{\nabla}_W\bar{\tilde{P}})(X, Y)Z = (\nabla_W\bar{\tilde{P}})(X, Y)Z - \eta(\bar{\tilde{P}}(X, Y)Z)W + g(W, \bar{\tilde{P}}(X, Y)Z)\xi. \tag{6.2}$$

Now differentiate (4.9) covariantly with respect to $W$, we get

$$\begin{aligned}(\nabla_W\bar{\tilde{P}})(X, Y)Z &= (\nabla_W\tilde{P})(X, Y)Z \\ &+ \alpha[\{g(Y, Z)(\nabla_W\eta)(X) - g(X, Z)(\nabla_W\eta)(Y)\}\xi \\ &+ \{g(Y, Z)\eta(X) - g(X, Z)\eta(Y)\}\nabla_W\xi] \\ &+ \beta[\{(\nabla_W\eta)(X)Y - (\nabla_W\eta)(Y)X\}\eta(Z) \\ &+ \{\eta(X)Y - \eta(Y)X\}(\nabla_W\eta)(Z)].\end{aligned} \tag{6.3}$$

By taking into account of (6.3)in (6.2), we obtain

$$\begin{aligned}&(\bar{\nabla}_W\bar{\tilde{P}})(X, Y)Z \\ = \; &(\nabla_W\tilde{P})(X, Y)Z + \alpha[\{g(Y, Z)(\nabla_W\eta)(X) - g(X, Z)(\nabla_W\eta)(Y)\}\xi \\ + \; &\{g(Y, Z)\eta(X) - g(X, Z)\eta(Y)\}\nabla_W\xi] + \beta[\{(\nabla_W\eta)(X)Y - (\nabla_W\eta)(Y)X\}\eta(Z) \\ + \; &\{\eta(X)Y - \eta(Y)X\}(\nabla_W\eta)(Z)] - \eta(\bar{\tilde{P}}(X, Y)Z)W + g(W, \bar{\tilde{P}}(X, Y)Z)\xi.\end{aligned} \tag{6.4}$$

Applying $\phi^2$ on both sides of the equations (6.4) and using equations (2.1), we get

$$\begin{aligned}
& \phi^2((\bar{\nabla}_W \bar{\tilde{P}})(X,Y)Z) \\
= \;& \phi^2((\nabla_W \tilde{P})(X,Y)Z) + \alpha[\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}\phi^2(\nabla_W \xi)] \\
+ \;& \beta[\{(\nabla_W \eta)(X)\phi^2(Y) - (\nabla_W \eta)(Y)(\phi^2 X)\}\eta(Z) \\
+ \;& \{\eta(X)(\phi^2 Y) - \eta(Y)(\phi^2 X)\}(\nabla_W \eta)(Z)] - \eta(\bar{\tilde{P}}(X,Y)Z)(\phi^2 W). \qquad (6.5)
\end{aligned}$$

Using the relations (2.2), (2.4), (2.5), (3.7) and (4.1) in equations (6.5), we have

$$\begin{aligned}
& \phi^2((\bar{\nabla}_W \bar{\tilde{P}})(X,Y)Z) \\
= \;& \phi^2((\nabla_W \tilde{P})(X,Y)Z) + \alpha[\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}\{-W + \eta(W)\xi\}] \\
+ \;& \beta[\{(g(X,W) - \eta(X)\eta(W))(-Y + \eta(Y)\xi) \\
- \;& (g(Y,W) - \eta(Y)\eta(W))(-X + \eta(X)\xi)\}\eta(Z) \\
+ \;& \{\eta(X)(-Y + \eta(Y)\xi) - \eta(Y)(-X + \eta(X)\xi)\}(g(Z,W) - \eta(Z)\eta(W))] \\
- \;& \{\alpha[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)] + \beta[\bar{S}(Y,Z)\eta(X) - \bar{S}(X,Z)\eta(Y)] \\
- \;& \frac{\bar{r}}{2n+1}\left(\frac{\alpha}{2n} + \beta\right)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]\}(-W + \eta(W)\xi). \qquad (6.6)
\end{aligned}$$

If $X, Y, Z, W$ are orthogonal to $\xi$ in (6.6), then we obtain

$$\phi^2((\bar{\nabla}_W \bar{\tilde{P}})(X,Y)Z) = \phi^2((\nabla_W \tilde{P})(X,Y)Z).$$

Hence we can state the following:

**Theorem 6.1.** A (2n+1)-dimensional Kenmostu manifold is locally pseudo-projectively-$\phi$-symmetric with respect to semi-symmetric non-metric connection $\bar{\nabla}$ if and only if it is so with respect to Levi-Civita connection $\nabla$.

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