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MHD Boundary Layer Flow of Jeffrey Fluid over a Stretching/Shrinking SH EET through Porous Medium

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Abstract

The Magnetohydrodynamics boundary layer flow of Jeffrey fluid over a permeable stretching/shrinking sheet in the presence of porous medium is investigated. Using similarity transformations, the governing partial differential equations are reduced into nonlinear ordinary differential equations and solved it analytically. The effects of magnetic parameter, permeability parameter and Jeffrey parameter on velocity profiles while the local skin friction coefficients are presented through graphs. Our analysis reveals that the boundary layer thickness is studied in both stretching/shrinking sheet cases.

Keywords: MHD, Jeffrey parameter, permeability parameter, boundary layer, stretching/shrinking sheet.

INTRODUCTION:

It is well known that the industrial fluids commonly differ from the viscous fluid in view of their diverse rheological characteristics. Such type of fluids falls into the category of non-Newtonian liquids which are quite interesting in different applications. The examples of such applications include plastics manufacturing, wire and blade coating, dying of paper and textile, polymer industries, food processing, geophysics and chemical and petroleum processes. Materials such as drilling muds, apple sauce, foams, soaps, sugar solution pastes, clay coating, ketchup, lubricant, certain oils, colloidal and suspension solutions are the non-Newtonian fluids movement of biological fluids and many others. The simple Navier–Stokes equations are not suitable to characterize the flow behavior of non-Newtonian liquids. There does not exist a single relation which can predict the characteristics of all the non-Newtonian materials. Thus the different types of non-Newtonian models are suggested in the literature. The present fluid model is known as the Jeffrey fluid. Jeffrey fluid is one of the rate type materials. It shows the linear viscoelastic effect of fluid which has many applications in polymer industries. There are many examples of Jeffrey fluid including dilute polymer solution.

The flow over a stretching plate was first investigated by Crane [1] who gave an exact similarity solution in a closed analytical form for steady boundary layer flow of an incompressible viscous fluid. Wang [2] conferred the concept of the flow around the shrinking sheet in his study of unsteady film. The existence and uniqueness of the solution of steady viscous flow over a shrinking sheet was proved by Miklavčič and Wang [3]. Pavlov [4] first studied the MHD flow over a stretching surface in an electrically conducting fluid. [5] Mohamed Abd El-Aziz studied the Dual solutions in hydromagnetic stagnation point flow and heat transfer towards a stretching/shrinking sheet with non-uniform heat source/sink and variable surface heat flux. Azeem Shahzad et al. [6] reported on Unsteady axisymmetric flow and heat transfer over time-dependent radially stretching sheet. Turkyilmazoglu [7] analyzed the Flow of a micro polar fluid due to a porous stretching sheet and heat transfer. S. Baag et al. [8] discussed Entropy generation analysis for viscoelastic MHD flow over a stretching sheet embedded in a porous medium. Jawad Ahmed et. al. [9] investigated on MHD axisymmetric flow of power-law fluid over an unsteady stretching sheet with convective boundary conditions. N.F. Fauzi et al. [10] reported on Stagnation point flow and heat transfer over a nonlinear shrinking sheet with slip effects. A.K. Abdul Hakeem et al. [11] studied Magnetic field effect on second order slip flow of Nano fluid over a stretching/shrinking sheet with thermal radiation effect. Ruchika Dhanai et al. [12] examined on Multiple solutions of MHD boundary layer flow and heat transfer behavior of nanofluids induced by a power-law stretching/shrinking permeable sheet with viscous dissipation. Bhattacharya examined the Different aspects of the flow due to shrinking sheet were discussed in [13]– [15].

The most common and simplest model of non-Newtonian fluids is Jeffrey fluid which has time derivative instead of convected derivative. I. S. Awaludin et al. [16] studied the Stability analysis of stagnation-point flow over a stretching/shrinking sheet. Maria Imtiaz et al. [17] reported the MHD Convective Flow of Jeffrey Fluid Due to a Curved Stretching Surface with Homogeneous-Heterogeneous Reactions. N. Sandeep and C. Sulochana are [18] examined the Momentum and heat transfer behaviour of Jeffrey, Maxwell and Oldroyd-B nanofluids past a stretching surface with non-uniform heat source/sink. Kartini Ahmad et al. [19] studied Mixed convection Jeffrey fluid flow over an exponentially stretching sheet with magnetohydrodynamic effect. Tasawar Hayat et al. [20] discussed the MHD stagnation point flow of Jeffrey fluid by a radially stretching surface with viscous dissipation and Joule heating. Kalidas Das et al. [21] reported the Radiative flow of MHD Jeffrey fluid past a stretching sheet with surface slip and melting heat transfer. Hayat T et al. [22] examined Thermal and Concentration Stratifications Effects in Radiative Flow of Jeffrey Fluid over a Stretching Sheet.

FORMULATION OF THE PROBLEM

Consider the steady two dimensional MHD flow of an electrically conducting incompressible Jeffrey fluid flow over a stretching/shrinking sheet situated at y=0 the equations for the MHD Jeffrey fluid flow and the temperature are written in the usual notation as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\nu}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_o^2}{\rho}u-\frac{\nu}{k}u \tag{2}$$

Where u and v are the velocity components in x and y directions respectively, x is the distance along the sheet; $\nu=\left(\frac{\mu}{\rho}\right)$ is the kinematic fluid viscosity ρ is the fluid density, μ is the coefficient of fluid viscosity, σ is the constant electrical conductivity of the fluid,

λ_1 is the dimensionless Jeffrey parameter.

Boundary conditions are

$$u=U_w\ ,\ v=-v_w \text{ at } y=0;\ u\to 0 \text{ as } y\to\infty \tag{3}$$

Where U_w is stretching/shrinking velocity of the sheet with $U_w=cx$ for stretching sheet case and $U_w=-cx$ for shrinking sheet case with c(>0) being the stretching/shrinking constant. Here v_w is the wall mass transfer velocity with $v_w > 0$ for mass suction and $v_w < 0$ for mass injection.

The following transformation are introduced

$$\psi=\sqrt{c\nu}\,xf(\eta) \text{ and } \eta=y\sqrt{c/\nu} \tag{4}$$

Where ψ is the stream function defined in the usual notation as

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{5}$$

and η is the similarity variable.

For relations of (4), the mass conservation of equation (1) satisfied automatically, and the momentum equation (2) takes the following forms

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = \frac{\nu}{1+\lambda_1}\frac{\partial^3 \psi}{\partial y^3} - \frac{\sigma B_0^2}{\rho}\frac{\partial \psi}{\partial y} - \frac{\nu}{k}\frac{\partial \psi}{\partial y} \tag{6}$$

Using (4) and (5) equation (6) finally takes the following forms

$$\left(\frac{1}{1+\lambda_1}\right) f''' + f f'' - f'^2 - (M + \lambda_2) f' = 0 \tag{7}$$

Where primes denote differentiation with respect to η and $M = \sigma B_0^2 / \rho c$ is the magnetic parameter, $\lambda_2 = \dfrac{\nu}{ck}$ is porous parameter, f is a dimensionless steam function and f' is the dimensionless velocity.

The boundary conditions are described as follows.

For stretching sheet

$$f(\eta) = S, \; f'(\eta) = 1 \text{ at } \eta = 0 \; ; \; f'(\eta) \to 0 \text{ as } \eta \to \infty \tag{8}$$

For shrinking sheet

$$f(\eta) = S, \; f'(\eta) = -1 \text{ at } \eta = 0 \; ; \; f'(\eta) \to 0 \text{ as } \eta \to \infty \tag{9}$$

where $S = v_w/(c\nu)^{1/2}$ is wall mass transfer parameter with S>0 (i.e.$v_w > 0$) for wall mass suction and S<0 (i.e.$v_w < 0$) for wall mass injection

SOLUTION OF THE PROBEM

a) Stretching sheet case

Assume that the analytic solution of Eq. (7) along with (8) can be written as (similar to Vajravelu and Rollins [23], Pop and Na [24], and Fang et al. [25]):

$$f(\eta) = a + b e^{-\lambda \eta} \tag{10}$$

where a, b and λ are constants with $\lambda > 0$. Substituting the relation (10) into equation (7) and (8), we obtain

For MHD flow of Newtonian fluid, the exact solution is obtained by

$$b = \frac{1}{\lambda} \quad , \quad a = S + \frac{1}{\lambda} \quad \text{and}$$

$$\lambda = \frac{S + \sqrt{S^2 + 4\left(\frac{1}{1+\lambda_1}\right)(1 + M + \lambda_2)}}{2\left(\frac{1}{1+\lambda_1}\right)} \tag{11}$$

Now the exact analytical solution reduces to

$$f(\eta) = S + \frac{2\left(\frac{1}{1+\lambda_1}\right)}{S + \sqrt{S^2 + 4\left(\frac{1}{1+\lambda_1}\right)(1 + M + \lambda_2)}} - \frac{2\left(\frac{1}{1+\lambda_1}\right)}{S + \sqrt{S^2 + 4\left(\frac{1}{1+\lambda_1}\right)(1 + M + \lambda_2)}} \exp\left(-\frac{S + \sqrt{S^2 + 4\left(\frac{1}{1+\lambda_1}\right)(1 + M + \lambda_2)}}{2\left(\frac{1}{1+\lambda_1}\right)} \eta \right) \tag{12}$$

$$f'(\eta) = \exp\left(-\frac{S + \sqrt{S^2 + 4\left(\frac{1}{1+\lambda_1}\right)(1 + M + \lambda_2)}}{2\left(\frac{1}{1+\lambda_1}\right)} \eta \right)$$

$$f''(0) = -\lambda = -\frac{S + \sqrt{S^2 + 4\left(\frac{1}{1+\lambda_1}\right)(1 + M + \lambda_2)}}{2\left(\frac{1}{1+\lambda_1}\right)}$$

Shrinking sheet case

The shrinking sheet flow is more interesting than the stretching sheet flow.

Putting the equation (12) into equation (9) and (11), we obtain

$$b = \frac{1}{\lambda} \ , a = S - \frac{1}{\lambda} \quad \text{and}$$

$$\lambda = \frac{S \pm \sqrt{S^2 - 4\left(\frac{1}{1+\lambda_1}\right)(1 - M - \lambda_2)}}{2\left(\frac{1}{1+\lambda_1}\right)} \tag{13}$$

Now the exact analytical solution reduces to

$$f(\eta) = S + \frac{2\left(\frac{1}{1+\lambda_1}\right)}{S \pm \sqrt{S^2 - 4\left(\frac{1}{1+\lambda_1}\right)(1 - M - \lambda_2)}} - \frac{2\left(\frac{1}{1+\lambda_1}\right)}{S \pm \sqrt{S^2 - 4\left(\frac{1}{1+\lambda_1}\right)(1 - M - \lambda_2)}} \exp\left(- \frac{S \pm \sqrt{S^2 - 4\left(\frac{1}{1+\lambda_1}\right)(1 - M - \lambda_2)}}{2\left(\frac{1}{1+\lambda_1}\right)} \eta \right) \tag{14}$$

$$f'(\eta) = \exp\left(- \frac{S \pm \sqrt{S^2 - 4\left(\frac{1}{1+\lambda_1}\right)(1 - M - \lambda_2)}}{2\left(\frac{1}{1+\lambda_1}\right)} \eta \right)$$

$$f''(0) = \lambda = \frac{S \pm \sqrt{S^2 - 4\left(\frac{1}{1+\lambda_1}\right)(1 - M - \lambda_2)}}{2\left(\frac{1}{1+\lambda_1}\right)}$$

For MHD flow of Jeffrey fluid the existence and uniqueness of the similarity solution strongly depends on magnetic parameter M. Three cases arise to be considered which are $M + \lambda_2 < 1,\ M + \lambda_2 = 1, M + \lambda_2 > 1$

Case 1: $M + \lambda_2 < 1$

The steady flow is possible when the following condition is satisfied.

$$S^2 \geq 4\left(\frac{1}{1+\lambda_1}\right)(1 - (M + \lambda_2)) \tag{15}$$

Also, the similarity solution is unique if

$S^2 = 4\left(\frac{1}{1+\lambda_1}\right)\left(1-(M+\lambda_2)\right)$ and it is of dual nature if

$S^2 < 4\left(\frac{1}{1+\lambda_1}\right)\left(1-(M+\lambda_2)\right)$ no similarity found.

Case 2: $M+\lambda_2 = 1$

The steady flow of Jeffrey fluid occurs for mass suction only. i.e. for S>0 and in this case the solution is always unique.

Case 3: $M+\lambda_2 > 1$

There is no restriction on the steady flow of Jeffrey fluid over a shrinking sheet, i.e. the similarity solution exists for any values of the mass transfer parameter, Jeffrey parameter, porous parameter and the solution is unique

Local Skin Friction:

The physical quantities of interest are the local skin friction coefficient C_f . Which is defined as

$$C_f = \frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\frac{\rho U_w^2}{2}} \tag{16}$$

which in the present case, can be expressed in the following form

$$C_f = \frac{2}{\sqrt{\mathrm{Re}}}\frac{f''(0)}{1+\lambda_1} \tag{17}$$

$$\text{Where} \quad \mathrm{Re} = \frac{U_w x}{\nu} \tag{18}$$

Numerical values of the function $\frac{f''(0)}{1+\lambda_1}$ which represent the wall shear stress at the surface respectively for various values of the parameter

RESULTS AND DISCUSSION

The objective of the present paper is to study the MHD boundary layer flow of Jeffrey fluid over a stretching/shrinking sheet through a porous medium. The analytical solution is performed for different values of dimensionless parameter involved in the equations such as the Jeffrey parameter λ_1, Magnetic parameter M, suction/injection parameter S. To illustrate the computed results, some figures are plotted and physical explanation are given.

Stretching sheet case

The variation in velocity $f'(\eta)$ for different values Jeffrey parameter λ_1 is shown in figure 1. We observed that the velocity is decreases with increasing Jeffrey parameter λ_1 and consequently the thickness of the boundary layer decreases.

The variation in velocity $f'(\eta)$ for different values of Jeffrey parameter λ_1 with mass injection S. Shown in figure 2. We observe that the velocity is decreases with increasing Jeffrey parameter λ_1 and consequently the thickness of the boundary layer decreases.

The variation in velocity $f'(\eta)$ for different value of magnetic parameter M with mass suction/injection are shown in figure 3 and figure 4. We observe that the velocity is decreases with increasing M. Reduction is caused by the Lorentz force, a mechanical force arising due to the interaction of magnetic and electric fields for the motion of an electrically conduction fluid. The Lorentz force increases when M increases and consequently boundary layer thickness in decreases.

The local skin friction coefficient $f''(0)$ against S. for different values of Jeffrey parameter λ_1 as shown figure 7. We observe that the skin friction coefficient $f''(0)$ is decreases with increasing Jeffrey parameter λ_1.

The variation in local skin friction coefficient $f''(0)$ against s for different values of magnetic parameter M is shown in figure 8. We observe that the local skin friction coefficient $f''(0)$ is decreases with increasing M.

The influence of porous parameter λ_2 with suction/injection on velocity are shown in fig 5 and 6. We observe that the velocity decreases with increasing porous parameter λ_2.

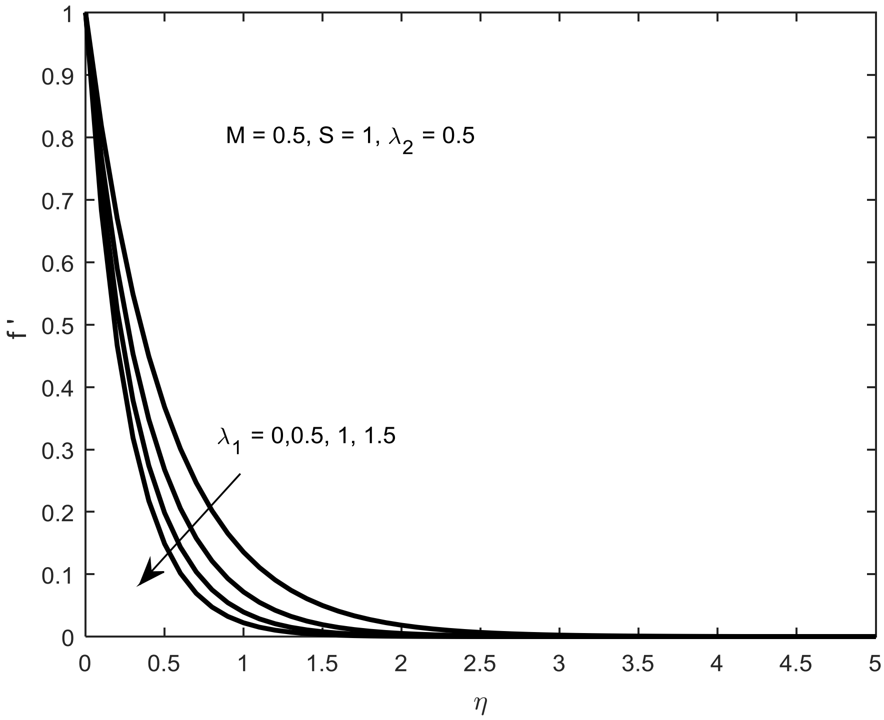

Fig 1. The velocity profiles for different values of Jeffrey parameter λ_1 with suction

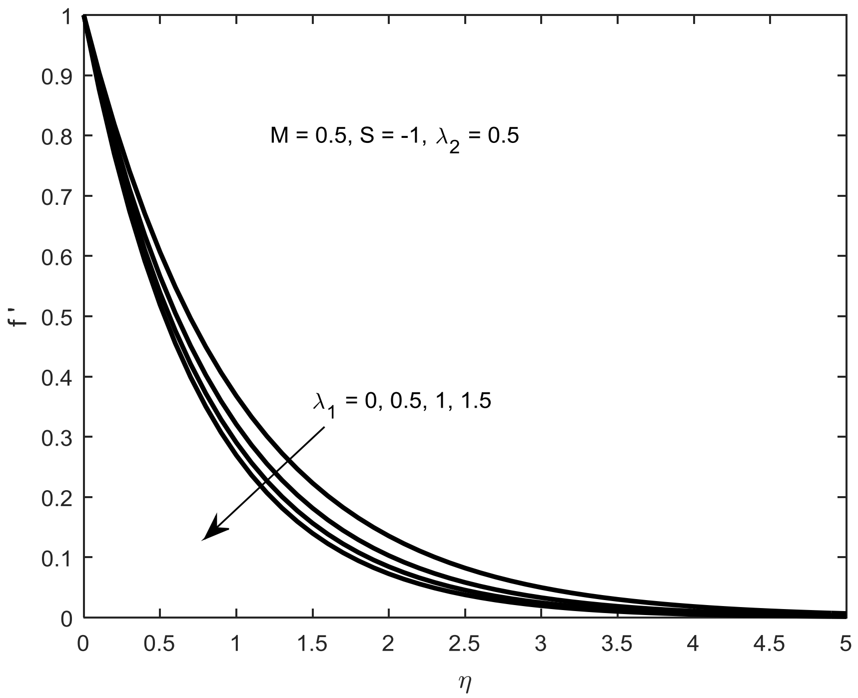

Fig 2. The velocity profiles for different values of Jeffrey parameter λ_1 with injection

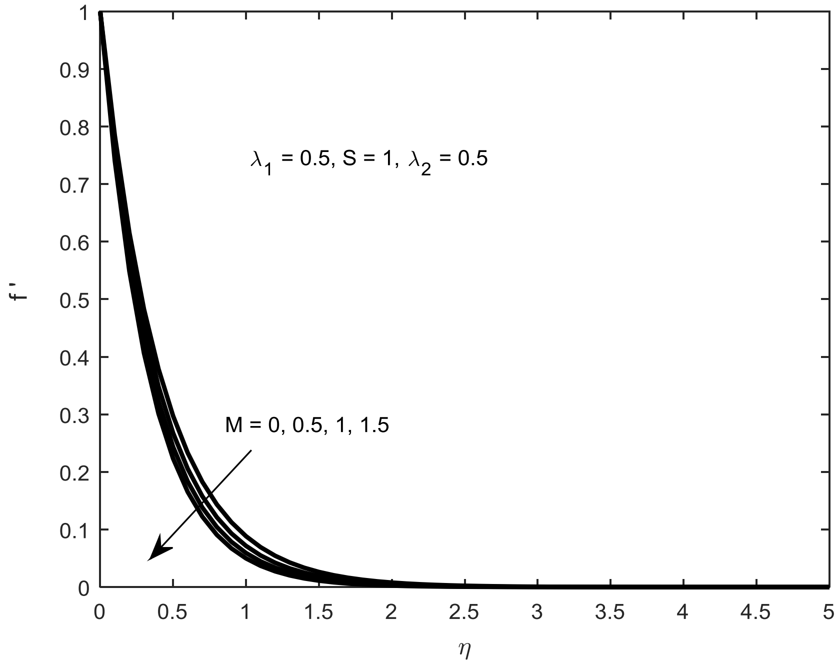

Fig 3. The velocity profiles for different values of Magnetic parameter M with suction

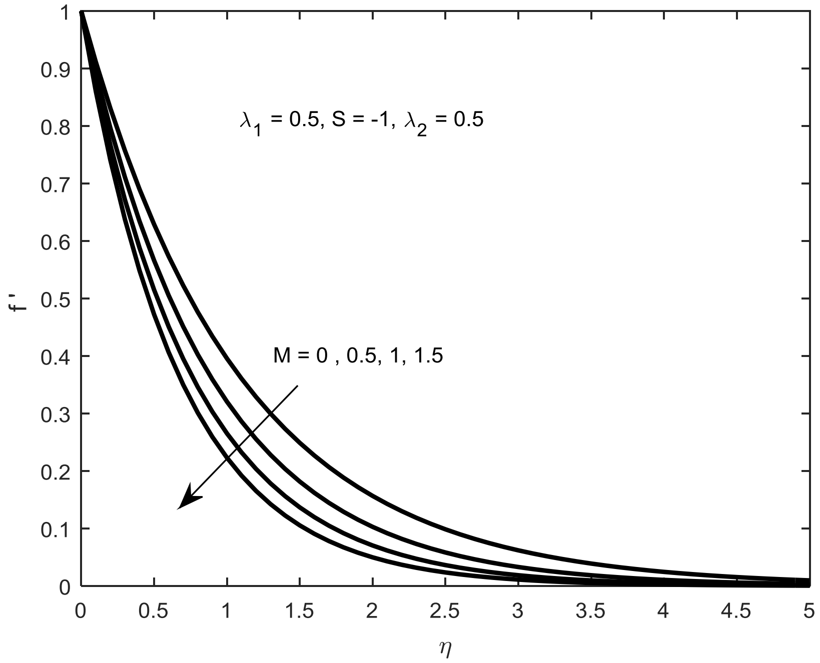

Fig 4 The velocity profiles for different values of Magnetic parameter M with injection

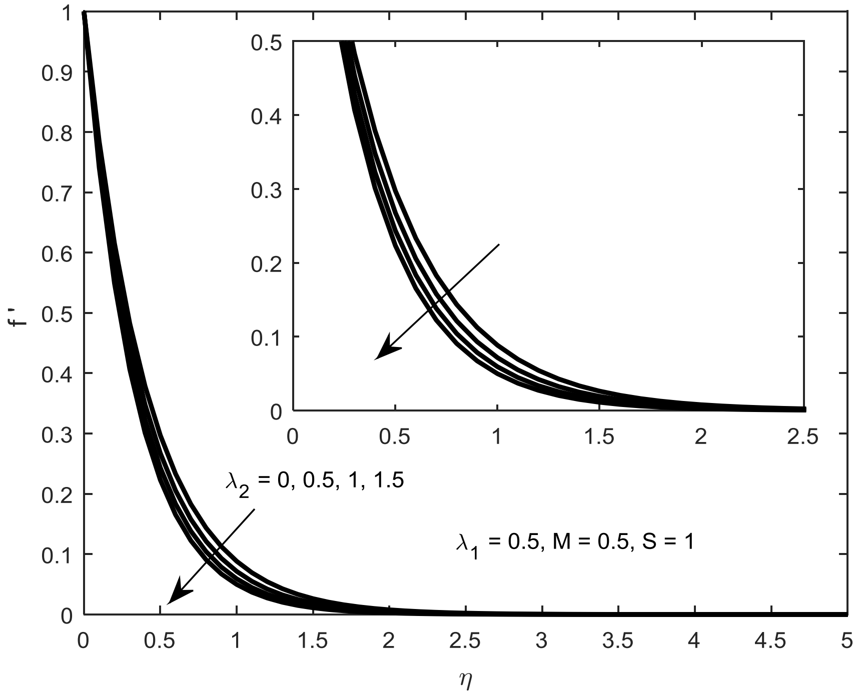

Fig 5 The velocity profiles for different values of porous parameter λ_2 with suction

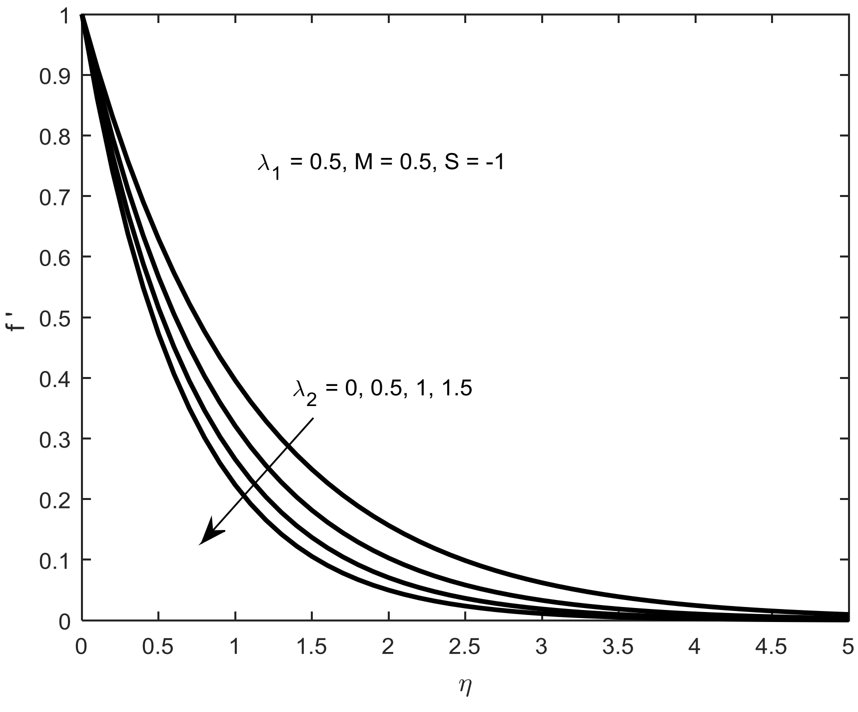

Fig 6 The velocity profiles for different values of porous parameter λ_2 with injection

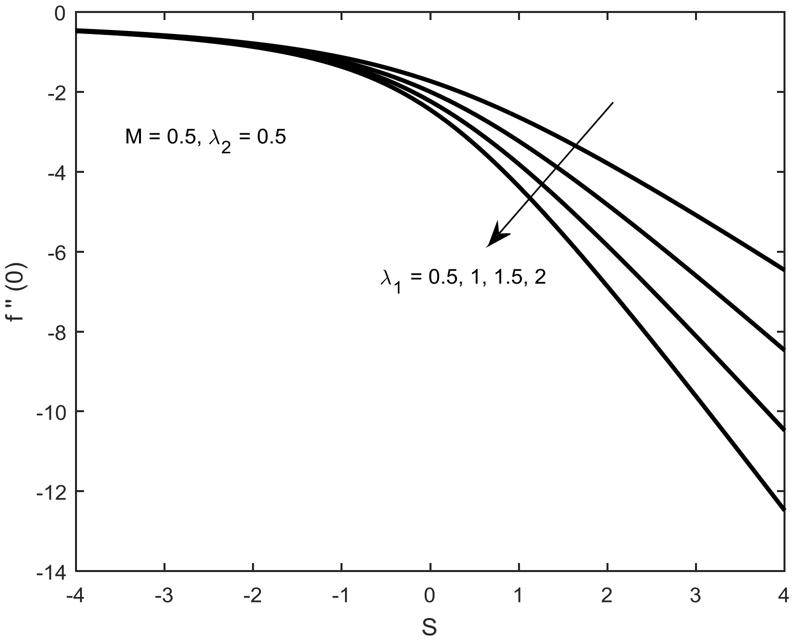

Fig 7. Skin friction coefficient $f''(0)$ against S for different values of Jeffrey parameter λ_1.

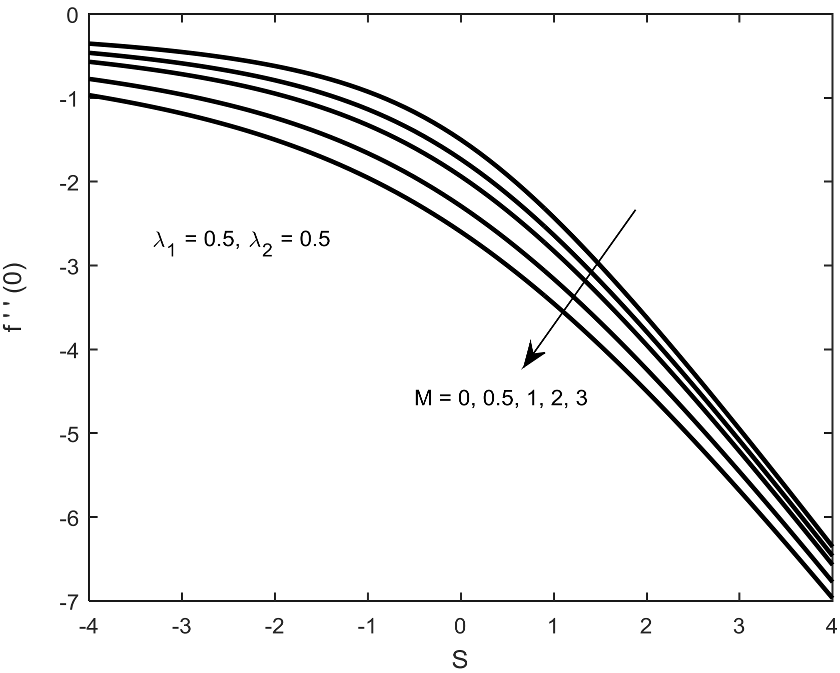

Fig 8. Skin friction coefficient $f''(0)$ against S for different values of Magnetic parameter M

Shrinking sheet case

The variation in velocity $f'(\eta)$ for different values Jeffrey parameter λ_1 is shown in figure 9. We observed that the velocity is increases with increasing Jeffrey parameter λ_1 and consequently the thickness of the boundary layer decreases.

The variation in velocity $f'(\eta)$ for different value of magnetic parameter M is shown in figure 10. We observe that the velocity is increases with increasing M.
The effect of physical parameter on velocity field for the flow of magnetic parameter M and Jeffrey parameter λ_1 are shown in figure 11 and figure 12. Examine the dual velocity profiles. It reveals that due to the magnetic field the boundary layer thickness reduces for the first solution and increase for the second solution in figure 11 also the opposite behavior in velocity profiles exhibit in figure 12.

The local skin friction coefficient $f''(0)$ against S for different values of Jeffrey parameter λ_1 as shown figure 13. We observe that the skin friction coefficient $f''(0)$ is decreases with increasing Jeffrey parameter λ_1.

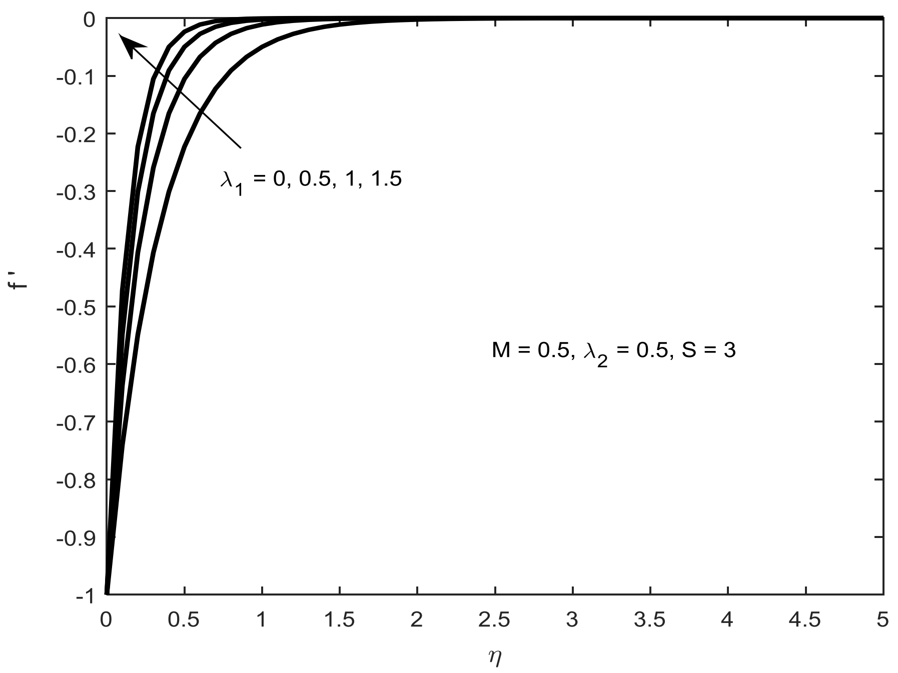

Fig 9. Velocity profile for different values of Jeffrey parameter λ_1

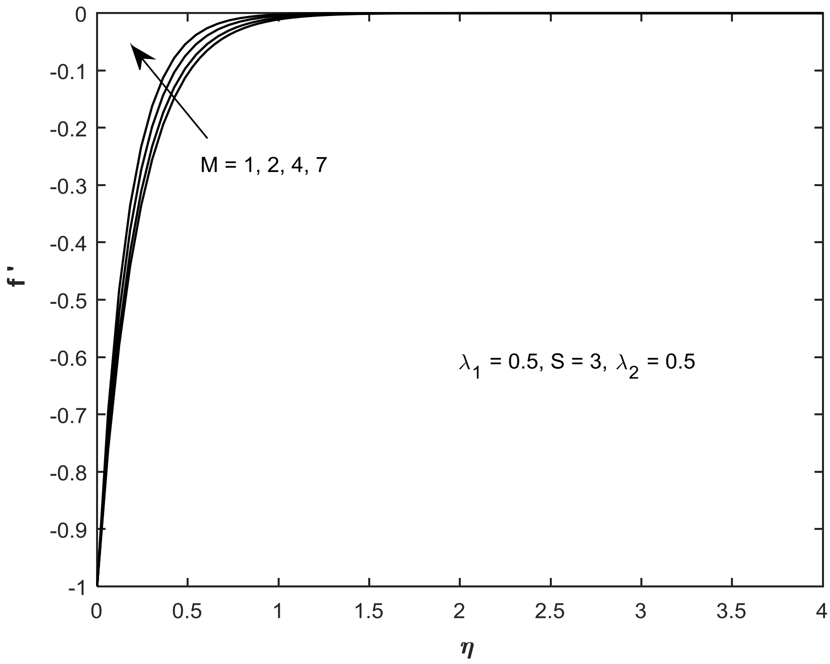

Fig 10. Velocity profiles for different values of Magnetic parameter *M*

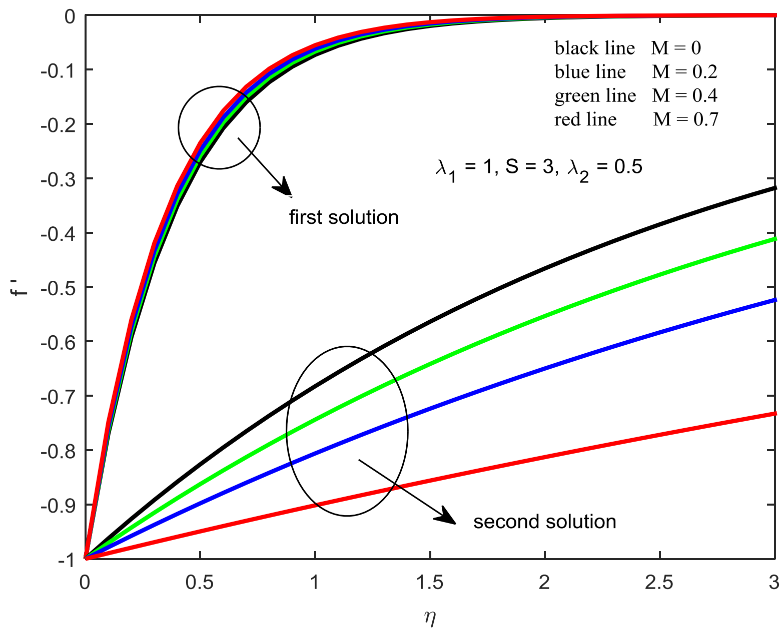

Fig 11. Dual Velocity profile for several values of M

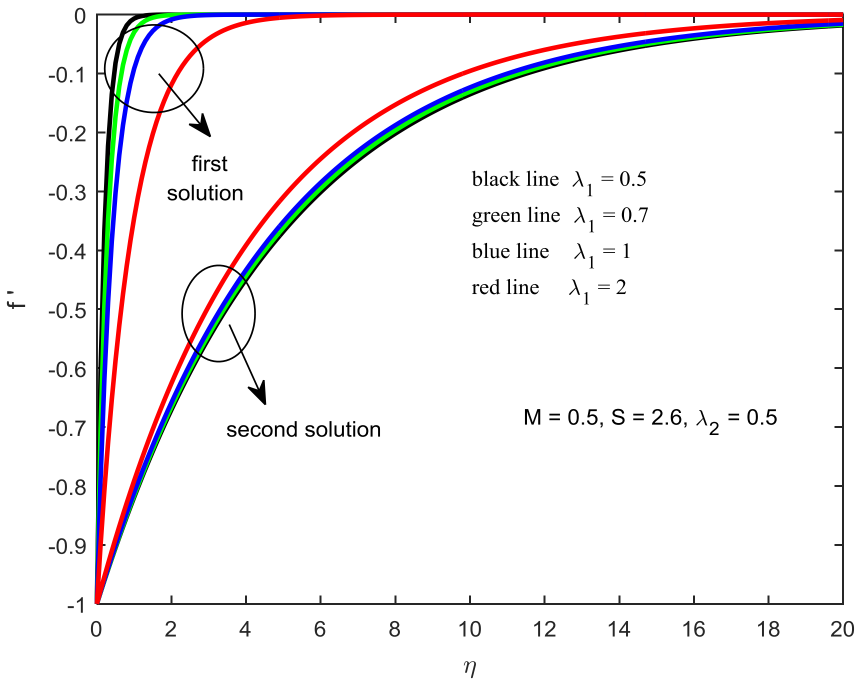

Fig 12. Dual Velocity profile for several values of λ_1

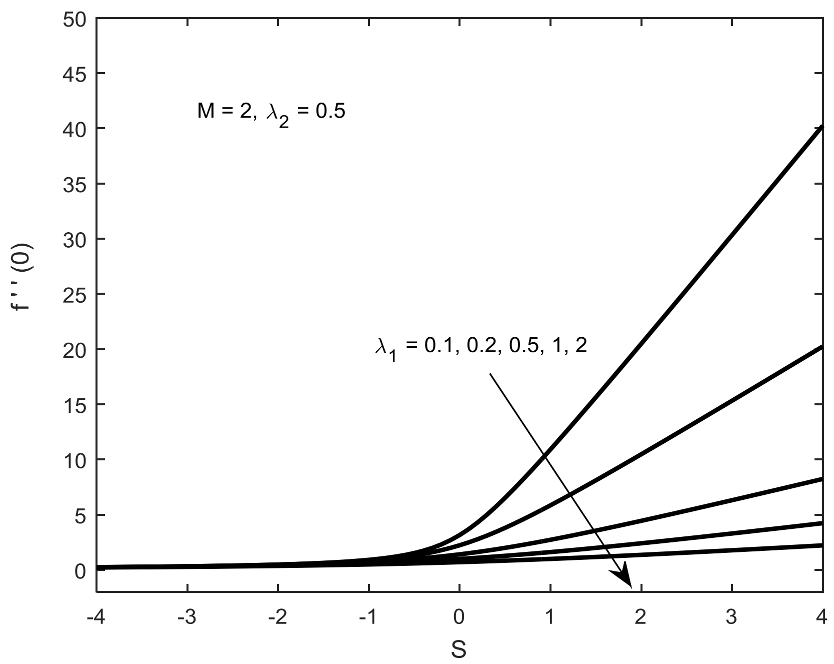

Fig 13. Skin friction coefficient $f''(0)$ against S for different values of Jeffrey parameter λ_1

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