EOQ Model with Time Induced Demand, Trade Credits and Price Discount on Shortages: A Periodic Review

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Abstract

Inventory control is the most important application of operations research. In traditional EOQ models demand rate is considered constant. But in actual practice or any type of business transaction it is in dynamic stage. In this study a periodic review inventory model with time induced demand and is non-increasing function of time under trade credits is developed. We formulate and analyze the method of determining the optimal order quantity and total profit. Inventory manager offers price discount when there is no stock in hand to customer who is interested to backorder their demand. Shortages are allowed and fully backlogged. The model maximizes the total profit. Numerical examples are provided to illustrate the model. Sensitivity analysis has also been provided with the help of several key parameters. The second and third order approximations are used to find closed from solution.

Keywords: Inventory model; shortage; price discount; trade credits; time-dependent demand

1. INTRODUCTION

In the tradition EOQ models it was considered that buyer must pay for purchase products suddenly receiving them. But in practice a vendor frequently offers their retailers a trade credits for settling the amount owed to them. Generally, there is no interest charged, if the outstanding amount is paid within the trade credit. The trade
credit is beneficial for seller as well buyer. During the past few decades several inventory modelers have studied their inventory models with permissible delay in payments. Goyal [1] explored an EOQ model under trade credits. Teng [2] provided an appropriate pricing and lot sizing model for a retailer when the supplier gives a trade credits. Aggarwal & Jaggi [3] generalized Goyal’s model for deteriorating products. Hwang and Shinn [4] established the optimal pricing and lot sizing for the retailer under the condition of trade credits. Khanra et al [5] considered an economic order quantity model for a deteriorating item with time induced demand under permissible delay in payments. Teng et al. [6] established an EOQ under trade credit financing with increasing demand. Chung [7] explored an “alternative approach to determine the economic order quantity under permissible delay in payments”. Related research papers can be found in Chung [8], Chung & Liao [9], Teng and Chang [10], Huang and Hsu [11], Liao et al [12], Ouyang et al. [13], Teng and Chung [14], Soni [15], Tripathi and Kumar [16] and there references.

In Classical inventory model demand rate is considered constant. However, in reality the demand is in dynamic state. Silver & Meal [17] considered an EOQ model for variable demand. Donaldon [18] was to first to provide a fully analytical solution to the problem of inventory replenishment with a linearly time dependent demand. Tripathi and Pandey [19] considered “an inventory model for deteriorating items with Weibull distribution time-dependent demand rate under permissible delay in payments”. Min et al. [20] developed a lot-sizing model for deteriorating items with a current stock-dependent demand and delay in payment. Tripathi & Tomar [21] established the possible effects of a temporary price discount offered by a supplier on a retailer’s replenishment policy for deteriorating items with linear time-dependent demand rate. Research work in this direction came from Dave & Patel [22], Chung & Ting [23], Gowsami & Chaudhuri [24], Jalan et al. [25], Lin et al [26] and others.

Generally, buyers have to wait for some daily life useful products in case of unavailability. The reason is the outstanding quality of the item or specific characteristics. Ghiami et al [27] investigated a two echelon supply chain model for deteriorating inventory in which the retailer’s warehouse has a limited capacity. Pal & Chandra [28] studied a periodic review inventory model with stock-dependent demand under shortages and trade credits. Tripathi [29] developed an EOQ model for deteriorating item with linearly time dependent demand rate under inflation and time discounting over a finite planning horizon under shortages. Yang [30], Law and Wee [31], Dye [32], Jaggi et al [33], Ouyang & Chang [34], Wee et al [35], Luong & Karim [36] developed their EOQ models under shortages.
2. NOTATIONS AND ASSUMPTION

The following notations are adopted:

- \( A \): ordering cost/ order
- \( p \): purchase cost/ unit/unit time
- \( C_1 \): back order cost/ unit/ unit time
- \( C_2 \): cost of a lost sale
- \( \beta_0 \): marginal profit/ unit
- \( \beta \): price discount on unit backorder offered
- \( I_e \): interest earned/ unit time
- \( I_p \): interest payable/ unit time beyond the trade credit \((I_p > I_e)\)
- \( s \): selling price/ unit
- \( \alpha_0 \): upper bound on backorder ratio, \( 0 \leq \alpha_0 \leq 1 \)
- \( \alpha \): fraction of the demand during stock-out time which is accepted to be backlogged
- \( t_1 \): time taken from stock in hand \( 0 \leq t_1 \leq T \)
- \( T \): length of a replenishment cycle
- \( I_m \): maximum stock height in a replenishment cycle
- \( I_b \): maximum shortage (backorder)
- \( Q(t) \): inventory level at time \( t \)
- \( \lambda \): deterioration rate

The assumptions are as follows:

(i) Lead time is negligible

(ii) Shortages are allowed and fraction \( \alpha \) of unmet demands in the stock out is backlogged.

(iii) Demand rate \( D(t) \) at time \( t \) is \( D(t) = \begin{cases} a - bt, & \text{for } 0 \leq t \leq t_1, a > 0, a > b > 0 \\ a, & \text{for } t_1 \leq t \leq T \end{cases} \).

(iv) Single item is considered.

(v) Back order fraction \( \alpha \) is proportional to the price discount \( \beta \) offered by vendor. Thus

\[ \alpha = \frac{\alpha_0}{\beta_0} \beta, \text{ where } 0 \leq \beta \leq \beta_0. \]
3. MATHEMATICAL MODEL AND OPTIMAL SOLUTION

Let us consider that the starting of the first reorder time, the stock level is zero before ordering, the order quantity during the period \((0, t_1)\) is \(I_m\). The planning horizon is divided into reorder time intervals, each of length \(T\). Orders are placed at time points \(T, 2T, 3T, \ldots\), the order quantity being just sufficient to bring the stock height to a certain level \(I_m\).

Decrease of inventory level \(Q(t)\) occurs due to both demand and deterioration in interval \((0, t_1)\). The shortages occurs during \((t_1 , T)\) in which a fraction \(\alpha\) is backlogged. The change of inventory level with respect to time is considered as:

\[
\frac{dQ(t)}{dt} + \lambda Q(t) = -(a - bt) , \quad 0 \leq t_1 \leq T
\]  

and

\[
\frac{dQ(t)}{dt} = -a\alpha , \quad t_1 \leq t \leq T
\]

under the condition \(Q(t_1) = 0\)

The solution of (1) and (2) with the help of (3) are

\[
Q(t) = \frac{1}{\lambda} \left\{ \left( a + \frac{b}{\lambda} \right) \left( e^{\lambda(t_1 - t)} - 1 \right) - b \left( t_1 e^{\lambda(t_1 - t)} - t \right) \right\}
\]  

and

\[
Q(t) = a\alpha(t_1 - t)
\]

respectively.

Thus \(I_m = Q(0) = \frac{1}{\lambda} \left\{ \left( a + \frac{b}{\lambda} \right) \left( e^{\lambda(t_1)} - 1 \right) - b \left( t_1 e^{\lambda(t_1)} \right) \right\}\)

And \(I_b = -Q(T) = a\alpha(T - t_1)\)

The sales revenue \(SR\) during \([0, T]\) is

\[
s \left\{ \int_0^t (a - bt) dt + \int_{t_1}^T a\alpha dt \right\} = s \left\{ t_1 \left( a - \frac{bt_1}{2} \right) + a\alpha(T - t_1) \right\}
\]  

The holding cost \(HC\) during \([0, t_1]\)

\[
h \int_0^{t_1} Q(t) dt = h \left[ \left( a + \frac{b}{\lambda} \right) \left( \frac{e^{\lambda t} - 1}{\lambda} - t_1 \right) - b \left( \frac{t_1 (e^{\lambda t} - 1)}{\lambda} - \frac{t_1^2}{2} \right) \right]
\]  

Total number of backorders \(BO\) during \([t_1, T]\) is
EOQ Model with Time Induced Demand, Trade Credits and Price Discount...

\[ C_1 \int_{\frac{T}{2}}^{T} Q(t)dt = \frac{a\alpha C_1(T-t_1)^2}{2} \]  \hspace{1cm} (10)

Total number of lost sales \( LS \) during \([t_1, T]\) is

\[ C_2 a(1-\alpha)(T-t_1) \]  \hspace{1cm} (11)

Two cases may arise regarding the permissible delay in payments \((m \leq t_1 \text{ and } m > t_1)\)

**Case I \((m \leq t_1)\)**

Since credit period is shorter than time for stock in hand, thus vendor can use the sales revenue to earn interest at rate \( I_e \) in \([0, t_1]\). The interest earned \( IE_1 \) by the buyer is

\[ = sL \int_{0}^{t_1} Q(t)dt = sL \left[ \left( a + \frac{b}{\lambda} \right) \left( e^{\frac{b}{\lambda} - 1} - \frac{t_1}{\lambda} \right) - b \left( -\frac{t_1}{\lambda} - \frac{t_1^2}{2} \right) \right] \]  \hspace{1cm} (12)

and the interest payable \( IP_1 \) by the vendor beyond the fixed credit period is

\[ = pL \int_{t_1}^{T} Q(t)dt = pL \left[ \left( a + \frac{b}{\lambda} \right) \left( e^{\frac{b}{\lambda} - 1} - (t_1 - m) \right) - b \left( -\frac{t_1}{\lambda} - \frac{(t_1 - m)^2}{2} \right) \right] \]  \hspace{1cm} (13)

Therefore, the total profit \( TP_1(t_1,T) \) per unit cycle time is

\[ TP_1(t_1,T) = \frac{1}{T} \left\{ SR - (A + HC + BO + LS + IP_1 - IE_1) \right\} \]  \hspace{1cm} (14)

**Case II \((m > t_1)\)**

Since credit period is longer than \( t_1 \), seller pays no interest, but earns interest \( IE_2 \) with rate \( I_e \). Thus

\[ IE_2 = sL \left[ \left( a + \frac{b}{\lambda} \right) \left( e^{\frac{b}{\lambda} - 1} - t_1 \right) - b \left( -\frac{t_1}{\lambda} - \frac{t_1^2}{2} \right) + \frac{\lambda aa(m-t_1)^2}{2} \right] \]  \hspace{1cm} (15)

Hence, the total profit \( TP_2(t_1,T) \) per unit cycle time is

\[ TP_2(t_1,T) = \frac{1}{T} \left\{ SR - (A + HC + BO + LS - IE_2) \right\} \]  \hspace{1cm} (16)

With the help of above discussion we get the following properties:

**Property 1:** The optimal cycle time is an increasing function of time for positive inventory:

**Proof:** It is obvious from \((A_{15}), (A_{25})\) and \((A_{35})\).
Property 2: The optimal cycle time is convex function of $t_1$.

Proof: It is obvious from $(A_{16})$, $(A_{23})$ and $(A_{34})$.

Property 3: If $\alpha = 1$, the total profit is greater than that of $0 < \alpha < 1$.

Proof: In case of $\alpha = 1$, $LS = 0$. The total profit for both cases become

$$TP_1^I(t_1, T) = \frac{1}{T}\left\{SR - (A + HC + BO + IP_1 - IE)\right\}$$

(17)

$$TP_1^I(t_1, T) = \frac{1}{T}\left\{SR - (A + HC + BO - IE)\right\}$$

(18)

From (14) and (17), we get

$$TP_1^I(t_1, T) - TP_1^I(t_1, T) = \frac{LS}{T} > 0 \Rightarrow TP_1^I(t_1, T) > TP_1^I(t_1, T)$$

(19)

From (16) and (18), we get

$$TP_2^I(t_1, T) - TP_2^I(t_1, T) = \frac{LS}{T} > 0 \Rightarrow TP_2^I(t_1, T) > TP_2^I(t_1, T)$$

(20)

4. NUMERICAL EXAMPLES

Case I: Consider the parameter values $s = 50$, $a = 15$, $b = 1.5$, $\alpha = 0.7$, $\theta = 0.05$, $C_1 = 50$, $C_2 = 20$, $h = 40$, $I_e = 0.03$, $I_r = 0.05$, $A = 100$, $p = 50$, $m = 0.1$ in appropriate unite. We get, $t_1^* = 0.541542$ year, $T^* = 0.62013$ year, $Q = 8.8353$ units, $TP^* = $390.111.

Case II:

Consider the parameter values $s = 50$, $a = 15$, $b = 1.5$, $\alpha = 0.7$, $\lambda = 0.05$, $C_1 = 50$, $C_2 = 20$, $h = 400$, $I_e = 0.03$, $I_r = 0.05$, $A = 100$, $p = 50$, $m = 0.1$ in appropriate unite. We get $t_1^* = 0.0979101$ year, $T^* = 0.618461$ year, $Q = 6.93082$ units and $TP^* = $161.484.

The following figures (Case 1 & 2) 1 and 2 show that the total profit is concave with respect to cycle time:

Fig 1 (Case 1) can be drawn considering the following parameter values $s = 50$, $a = 15$, $b = 1.5$, $\alpha = 0.7$, $\lambda = 0.05$, $C_1 = 50$, $C_2 = 20$, $h = 50$, $I_e = 0.03$, $I_r = 0.05$, $A = 100$, $p = 50$, $m = 0.1$, $t_1 = 0.2$, in appropriate units.
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Fig 1: (Case 1): Graph between cycle time $T$ and Total profit $TP$

Fig 2 (Case 2) consider the following parameter values $s = 100$, $a = 10$, $b = 1.5$, $\alpha = 0.7$, $\lambda = 0.05$, $C_1 = 50$, $C_2 = 20$, $h = 50$, $I_c = 0.03$, $I_r = 0.05$, $A = 100$, $p = 50$, $m = 0.1$, $t_1 = 0.05$ in appropriate units.

Fig. 2. (Case 2) Graph between cycle time $T$ and Total profit $TP$
5. SENSITIVITY ANALYSIS

In real life the and business management the future planning is uncertain. Seller wants to more profit for the products kept in hand. The sensitivity analysis is beneficial for vendor and buyer both. We consider the variation of $t_1$, $T$, $Q$ and $TP$ with the variation of $s$, $h$, $A$, $C_1$, $C_2$, $m$ and $p$. The numerical date is taken from numerical examples 1 and 2 for case I and II respectively keeping remaining parameters same.

Table 1: Variation of $t_1$, $T$, $Q$ and $TP$ with several parameters

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<th>Case II</th>
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| $h$    | $t_1$   | $T$    | $Q$  | $TP$   | $a$    | $t_1$   | $T$    | $Q$  | $TP$   |
| 45     | 0.509895 | 0.619887 | 8.70332 | 373.825 | 16     | 0.0962043 | 0.597176 | 7.14690 | 183.229 |
| 50     | 0.481704 | 0.619718 | 8.58556 | 359.315 | 17     | 0.0946481 | 0.577743 | 7.35490 | 205.330 |
| 55     | 0.456434 | 0.619597 | 8.47979 | 346.307 | 18     | 0.0932200 | 0.559903 | 7.55554 | 227.759 |
| 60     | 0.433657 | 0.619508 | 8.38422 | 334.582 | 19     | 0.0919031 | 0.543445 | 7.74933 | 250.493 |
| 65     | 0.413023 | 0.619439 | 8.29740 | 323.960 | 20     | 0.0906832 | 0.528194 | 7.93675 | 273.508 |

| $A$    | $t_1$   | $T$    | $Q$  | $TP$   | $C_1$  | $t_1$   | $T$    | $Q$  | $TP$   |
| 105    | 0.553515 | 0.646660 | 9.16262 | 382.382 | 42     | 0.0946403 | 0.670111 | 7.45866 | 181.290 |
| 110    | 0.565035 | 0.672161 | 9.47718 | 374.957 | 44     | 0.0954953 | 0.65585 | 7.31272 | 175.918 |
| 115    | 0.576151 | 0.696745 | 9.78037 | 367.801 | 46     | 0.0963242 | 0.642555 | 7.17679 | 170.963 |
| 120    | 0.586904 | 0.720504 | 10.0733 | 360.999 | 48     | 0.0971286 | 0.630121 | 7.04979 | 166.155 |
| 125    | 0.597329 | 0.743518 | 10.3570 | 354.195 | 49     | 0.0975222 | 0.624199 | 6.98936 | 163.803 |

| $m$    | $t_1$   | $T$    | $Q$  | $TP$   | $C_2$  | $t_1$   | $T$    | $Q$  | $TP$   |
| 0.15   | 0.542476 | 0.618753 | 8.82465 | 391.297 | 10     | 0.0915244 | 0.625208 | 6.97339 | 199.633 |
| 0.20   | 0.543614 | 0.617835 | 8.81965 | 392.348 | 12     | 0.0928083 | 0.623948 | 6.96585 | 191.960 |
| 0.25   | 0.544954 | 0.617376 | 8.82029 | 393.263 | 14     | 0.0940888 | 0.622643 | 6.95384 | 184.309 |
| 0.30   | 0.546500 | 0.617380 | 8.82663 | 394.041 | 16     | 0.0953660 | 0.621294 | 6.94931 | 176.679 |
| 0.35   | 0.548249 | 0.617844 | 8.83862 | 394.682 | 18     | 0.0966398 | 0.619900 | 6.94031 | 169.071 |
EOQ Model with Time Induced Demand, Trade Credits and Price Discount.

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From the above discussion the inferences can be made:

- Increase in $t_1$, $TP$ and decrease in $T$, $Q$ with the increase in $s$.
- Decrease in $t_1$, $TP$, $T$ and $Q$ with increase in $h$ and $p$ respectively.
- Increase in $t_1$, and decrease in $TP$, $T$, $Q$ with increase in $C_1$ and $C_2$ respectively.
- Increase in $t_1$, $T$, $Q$ and decrease in $TP$ with increase in $A$.
- Decrease in $t_1$, $T$ and increase in $Q$, $TP$, with increase in $a$.

6. CONCLUSION

It is difficult for inventory manager to decide when and how many items are kept in the shop. It depends on the situation and public demand. In most of the cases the demand in dynamic state. In this study, we have considered the demand in time dependent and deterioration is constant. Shortages are allowed. Two different cases have been considered. Mathematical model is provided for finding optimal solution. Based on the optimal solution two properties have been obtained based on the optimal
solution. We have proved that the total profit in concave with cycle time. From managerial point of view the total profit have decrease with increase of unit holding cost, back order cost, lost sell cost and purchase cost, but increase with the increase of initial demand, selling price and ordering cost.

The possible extension of the model for including weibull distribution deterioration. We may also add the advertisements charges and freight charges.

**APPENDIX**

\[
\frac{\partial(SR/T)}{\partial T} = -\frac{st_1}{T^2} \left( a - a\alpha - bt_1 \right), \quad \frac{\partial(SR/T)}{\partial t_1} = \frac{s}{T} (a - a\alpha - bt_1), \quad \frac{\partial^2(SR/T)}{\partial T^2} = \frac{2st_1}{T^3} \left( a - a\alpha - \frac{bt_1}{2} \right)
\]

\[
\frac{\partial^2(SR/T)}{\partial t_1^2} = -\frac{sb}{T}, \quad \frac{\partial^2(SR/T)}{\partial T \partial t_1} = -\frac{s}{T^2} (a - a\alpha - bt_1), \quad \frac{\partial^2(HC/T)}{\partial T^2} = \frac{h(a - bt_1) (e^{bt_1} - 1)}{\lambda T^2}
\]

\[
\frac{\partial(HC/T)}{\partial t_1} = \frac{h(a - bt_1) (e^{bt_1} - 1)}{\lambda T}, \quad \frac{\partial^2(HC/T)}{\partial T \partial t_1} = \frac{h(a - bt_1) \lambda e^{bt_1} - b (e^{bt_1} - 1)}{\lambda T^2}
\]

\[
\frac{\partial(BO/T)}{\partial t_1} = -\frac{a\alpha C_t(T - t_1)}{T}, \quad \frac{\partial^2(BO/T)}{\partial t_1^2} = -\frac{a\alpha C_t}{T}, \quad \frac{\partial(BO/T)}{\partial T} = \frac{a\alpha C_t(T - t_1)}{2T^2} T^2 \quad \text{and} \quad \frac{\partial^2(BO/T)}{\partial T \partial t_1} = -\frac{a\alpha C_t t_1}{T^2}
\]

\[
\frac{\partial(LS/T)}{\partial t_1} = -\frac{C_a(1 - \alpha)}{T}, \quad \frac{\partial^2(LS/T)}{\partial t_1^2} = 0, \quad \frac{\partial(LS/T)}{\partial T} = \frac{C_a(1 - \alpha)}{T^2}
\]

\[
\frac{\partial(I_E /T)}{\partial t_1} = \frac{sl_e (a - bt_1) (e^{bt_1} - 1)}{\lambda T}, \quad \frac{\partial^2(I_E /T)}{\partial t_1^2} = \frac{sl_e (a - bt_1) \lambda e^{bt_1} - b (e^{bt_1} - 1)}{\lambda T}
\]

\[
\frac{\partial(I_E /T)}{\partial T} = -\frac{sl_e}{\lambda T^2} \left[ a + \frac{b}{\lambda} \left( e^{bt_1} - 1 \right) - t_1 \left\{ t_1 \left( e^{bt_1} - 1 \right) - \frac{t_1^2}{2} \right\} \right]
\]
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\[
\frac{\partial^2 (IE_1/T)}{\partial T^2} = \frac{2sl_e}{\lambda T^2} \left[ \left( a + \frac{b}{\lambda} \right) \left( \frac{e^{\lambda t_i}}{\lambda} - t_i \right) - b \left[ \frac{t_i (e^{\lambda t_i} - 1)}{\lambda} - \frac{t_i^2}{2} \right] \right].
\]

\[
\frac{\partial^2 (IE_1/T)}{\partial T \partial t_i} = -\frac{sl_e}{\lambda T^2} (a - bt_i) \left( e^{\lambda t_i} - 1 \right).
\]

\[
\frac{\partial (IP_1/T)}{\partial t_i} = \frac{pl_e}{\lambda T} \left[ \left( a + \frac{b}{\lambda} \right) \left( e^{\lambda (t_i - m)} - 1 \right) - b \left[ \frac{t_i (e^{\lambda (t_i - m)} - 1)}{\lambda} + t_i e^{\lambda (t_i - m)} - t_i \right] \right].
\]

\[
\frac{\partial^2 (IP_1/T)}{\partial T \partial t_i} = \frac{pl_e}{\lambda T^2} \left( a \lambda e^{\lambda (t_i - m)} - b \left( e^{\lambda (t_i - m)} + \lambda t_i e^{\lambda (t_i - m)} - 1 \right) \right).
\]

\[
\frac{\partial^2 (IP_1/T)}{\partial T^2} = \frac{-pl_e}{\lambda T^2} \left[ \left( a + \frac{b}{\lambda} \right) \left( e^{\lambda (t_i - m)} - (t_i - m) \right) - b \left[ \frac{t_i \left( e^{\lambda (t_i - m)} - 1 \right) - t_i^2 - m^2}{\lambda} + \lambda a a (m - t_i) \right] \right].
\]

\[
\frac{\partial^2 (IE_2/T)}{\partial t_i^2} = \frac{sl_e}{\lambda T} \left[ \left( a + \frac{b}{\lambda} \right) \left( e^{\lambda t_i} - 1 \right) - b \left[ \frac{e^{\lambda t_i} - 1}{\lambda} + t_i e^{\lambda t_i} - t_i \right] - \lambda a a (m - t_i) \right].
\]

\[
\frac{\partial^2 (IE_2/T)}{\partial T \partial t_i} = \frac{-sl_e}{\lambda T^2} \left( a + \frac{b}{\lambda} \right) \left( e^{\lambda t_i} - 1 \right) - b \left[ \frac{e^{\lambda t_i} - 1}{\lambda} + t_i e^{\lambda t_i} - t_i \right] - \lambda a a (m - t_i) \right].
\]

\[
\frac{\partial (IP_1/T)}{\partial t_i} = \frac{-st_i}{T^2} \left( a - aa - \frac{bt_i}{2} \right) + \frac{(h - st_i)}{\lambda T^2} \left[ \left( a + \frac{b}{\lambda} \right) \left( e^{\lambda t_i} - 1 \right) - b \left[ \frac{t_i (e^{\lambda t_i} - 1) - t_i^2}{\lambda} - \frac{t_i^2}{2} \right] \right] = \frac{a a C_i (T^2 - t_i^2)}{2T^2} - \frac{C_i a (1 - a) T_i}{T^2} + \frac{pl_e}{\lambda T^2} \left[ \left( a + \frac{b}{\lambda} \right) \left( e^{\lambda (t_i - m)} - (t_i - m) \right) - b \left[ \frac{t_i (e^{\lambda (t_i - m)} - 1) - t_i^2 - m^2}{\lambda} + \frac{t_i^2}{2} \right] \right].
\]

\[
\frac{\partial (IP_1/T)}{\partial T} = 0, \text{ gives}
\]
Differentiating (A12) w.r.t. \( t' \) two times, we get

\[
s(1-a\alpha) - \frac{(a-bt_1)}{\lambda} \left( (h-sI_x)(e^{\lambda t_1}-1) + pI_x \left( e^{\lambda (t_1)} \right) \right) + a\alpha C_1 \frac{dT}{dt_1} - a\alpha C_1 t_1 + C_2 a(1-\alpha) = 0
\]

(A13)

\[
-sb - (h-sI_x) \left\{ a\lambda e^{\lambda t_1} + b - b(1+\lambda t_1) e^{\lambda t_1} \right\}.
\]

\[
\frac{pI_x}{\lambda} \left\{ a\lambda e^{\lambda (t_1-m)} + b - b(1+\lambda t_1) e^{\lambda (t_1-m)} \right\} + a\alpha C_1 \left( \frac{dT}{dt_1} \right)^2 + a\alpha C_1 T \frac{d^2T}{dt_1^2} - a\alpha C_1 = 0.
\]

(A14)

From (A13) and (A14), we get

\[
dT^* = \frac{(a-bt_1) \left( (h-sI_x)(e^{\lambda t_1}-1) + pI_x \left( e^{\lambda (t_1)} \right) \right) - \lambda s(a-bt_1-a\alpha) - a\alpha C_1 t_1 + C_2 a(1-\alpha)}{a\alpha C_1 \lambda T}
\]

(A15)

and

\[
d^2T^* = \frac{(h-sI_x) \left\{ \lambda(a-bt_1) e^{\lambda t_1} - b(e^{\lambda t_1}-1) \right\} + pI_x \left\{ \lambda(a-bt_1) e^{\lambda (t_1-m)} - b(e^{\lambda (t_1-m)}-1) \right\}}{a\alpha C_1 \lambda T}
\]

\[
- \frac{\lambda(sb + a\alpha C_1) + a\alpha C_1 \lambda \left( \frac{dT}{dt_1} \right)^2}{a\alpha C_1 \lambda T} > 0
\]

(A16)

Again

\[
\frac{\partial(IP_1/T)}{\partial t_1} = \frac{s(a-bt_1-a\alpha)}{T} - \frac{(h-sI_x)(a-bt_1)(e^{\lambda t_1}-1)}{\lambda T} + a\alpha C_1 T - t_1 + C_2 a(1-\alpha)
\]

\[
- \frac{pI_x}{\lambda T} \left( a-bt_1 \right) \left( e^{\lambda (t_1-m)} - 1 \right)
\]

(A17)

\[
\frac{\partial(IP_1/T)}{\partial t_1} = 0,
\]

gives
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\[ s\lambda(a-bt_1-a\alpha)-(h-sI_c)(a-bt_1)\left(e^{\frac{t_1}{\lambda}}-1\right)+aaC_1(T-t_1)+C_2a(1-\alpha)-pl_r(a-bt_1)\left(e^{\frac{t_1}{(t_1-m)}}-1\right) = 0 \quad (A_3) \]

Differentiating (A_3) w.r.t. \( t_1 \) two times we get

\[-sb\lambda-(h-sI_c)\left\{b-be^{\frac{t_1}{\lambda}}+(a-bt_1)\lambda e^{\frac{t_1}{\lambda}}\right\}+aaC_1\frac{dT}{dt_1}-a\alpha C_i\]
\[=0 \quad (A_{3a})\]

\[(a-bt_1)\lambda\left\{(h-sI_c)e^{\frac{t_1}{\lambda}}+pl_re^{\frac{t_1}{(t_1-m)}}\right\}-2b\left\{(h-sI_c)e^{\frac{t_1}{\lambda}}+pl_re^{\frac{t_1}{(t_1-m)}}\right\}+a\alpha C_i\frac{d^2T}{dt_1^2} = 0 \quad (A_{3b})\]

From \( (A_{3a}) \) and \( (A_{3b}) \), we get

\[dT^* = \frac{(h-sI_c)\left\{a\lambda e^{\frac{t_1}{\lambda}}+b-b(1+\lambda t_1)e^{\frac{t_1}{\lambda}}\right\}+pl_r\left\{a\lambda e^{\frac{t_1}{(t_1-m)}}+b-b(1+\lambda t_1)e^{\frac{t_1}{(t_1-m)}}\right\}+\lambda(sb+a\alpha C_i)}{a\alpha C_i} \]

\[> 0 \quad (A_{3c})\]

and

\[\frac{d^2T^*}{dt_1^2} = \frac{\left\{(a-bt_1)\lambda-2b\right\}+\left\{(h-sI_c)e^{\frac{t_1}{\lambda}}+pl_re^{\frac{t_1}{(t_1-m)}}\right\}}{a\alpha C_i} > 0 \quad (A_{3d})\]

Again

\[\frac{\partial(TP/T)}{dT} = -\frac{st_1\left(a-bt_1-\alpha t_1\right)}{T^2} + \frac{(h-sI_c)}{\lambda T^2}\left[\left(a+b\lambda\right)\left(e^{\frac{t_1}{\lambda}}-1\right)-b\left(t_1\left(e^{\frac{t_1}{\lambda}}-1\right)\right)-t_1^2\right] \]
\[= \frac{aaC_1\left(T^2-t_1^2\right)}{2T^2} - \frac{aC_2(1-\alpha)t_1}{T^2} - \frac{sl_c a\alpha (m-t_1)^2}{2T^2} \]

\[\frac{\partial(TP/T)}{dT} = 0, \text{ gives} \]

\[st_1\left(a-bt_1-\alpha t_1\right) - \frac{(h-sI_c)}{\lambda}\left[\left(a+b\lambda\right)\left(e^{\frac{t_1}{\lambda}}-1\right)-bt_1\left(e^{\frac{t_1}{\lambda}}-1\right)-t_1\right] + \frac{aaC_1\left(T^2-t_1^2\right)}{2} \]
\[+C_2a(1-\alpha)t_1 + \frac{sl_c a\alpha (m-t_1)^2}{2} = 0 \quad (A_3)\]

Differentiating \( (A_3) \) two times with respect to \( t_1 \), we get

\[s\left(a-bt_1-a\alpha\right)-\frac{(h-sI_c)}{\lambda}(a-bt_1)\left(e^{\frac{t_1}{\lambda}}-1\right)+aaC_1 T\frac{dT}{dt_1}-a\alpha C_it_1 \]
\[+C_2a(1-\alpha) - sl_c a\alpha (m-t_1)^2 = 0 \quad (A_{3e})\]
From Equations (A31) and (A32), we get

\[
\frac{dT^*}{dt_1} = \left( h - sl_x \right) (a - bt_1)(e^{\lambda t_1} - 1) - \lambda \left\{ s(a - bt_1 - a\alpha) - a\alpha C_T t_1 + C_T a(1 - \alpha) + sl_x a\alpha (m - t_1) \right\} \gg 0 \quad (A33)
\]

and

\[
\frac{d^2T^*}{dt_1^2} = \left( h - sl_x \right) \left\{ a\lambda e^{\lambda t_1} + b(1 + \lambda t_1)e^{2\lambda t_1} \right\} + \lambda \left\{ sb + a\alpha C_T - sl_x a\alpha - a\alpha C_T \left( \frac{dT}{dt_1} \right)^2 \right\} \gg 0 \quad (A34)
\]

REFERENCES


EOQ Model with Time Induced Demand, Trade Credits and Price Discount

and Computation, 219, 2568-5282.


