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Total Bi-Magic Circulant Graphs with Generating Sets (1,2,3,4) and (1,2,3,4,5)

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Abstract

Let G = (V,E) be a simple , finite, connected and undirected graph with order p and size q .A total bi-magic labeling (TBML) of a graph G is defined as the one-to-one mapping taken from all the vertices and edges of G onto the set of integers {1,2,…..| V | + | E | }with the property that the sum of the label on a vertex and the labels on its incident edges is a bi-magic constant, independent of the choice of the element in V∪E.

Equivalently, A total bi-magic labeling of a (p,q) graph is defined as the bijection mapping f:V(G) ∪ E(G)→ {1,2,….p+q} such that for each element in V∪E , the value of f(u)+f(uv)+f(v) is a bi-magic constant either K_1 or K_2 .

Here (1,2,3,4) and (1,2,3,4,5) are the generators of G .

Keywords: Magic labeling, Total labeling , Bi-magic labeling, Total bi-magic labeling, Circulant graph .

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INTRODUCTION:

Magic labeling was introduced by J.Sedlacek in 1963. In general , for a magic- type labeling , we require the sum of the labels related to a vertex or to an edge to be constant all over the graph. Magic labelings have been studied by various authors:

For example, vertex-magic labelings by J.A.MacDougall, edge magic labeling by A.Kotzig and A.Rosa and by Baskaro.

In 2004, Babujee introduced the notion of bi-magic labeling in which there exists two constants K_1 and K_2 .

In this paper, we obtained the Total bi-magic labeling for the circulant graphs.

MAIN RESULTS:

Theorem 2.1:

For odd n ≥ 11, circulant graphs C_n (1,2,3,4,5) admits Total bi-magic labeling with the bi-magic constants 387 and 398.

Proof:

Let $C_n(1,2,3,4,5)$ be a subclass of circulant graphs with $n \geq 11$.

Let $\{V_i : i = 0,1\ldots.n-1\}$ be the vertices of $C_n(1,2,3,4,5)$.

Label all the vertices and edges as follows.

$\alpha_0(V_i) = 12-(i+1)$ for $i = 0,1,2\ldots$

$$\alpha_0(V_iV_{i+1}) = \begin{cases} 2n-10 & \text{for } i = 0 \\ \frac{3n+i}{2}+1 & \text{for } i = 1,3,\ldots n-2 \\ \frac{2n+i}{2}+1 & \text{for } i = 2,4,\ldots n-1 \end{cases}$$

$$\alpha_0(V_iV_{i+2}) = \begin{cases} 2n+3i+1 & \text{for } i = 0,1,2,3. \\ 2n+2 & \text{for } i = 4 \\ 2n+5 & \text{for } i = 5 \\ 2n+8 & \text{for } i = 6 \\ 2n+11 & \text{for } i = 7 \\ 2n+3 & \text{for } i = 8 \\ 2n+6 & \text{for } i = 9 \\ 2n+9 & \text{for } i = 10. \end{cases}$$

$\alpha_0(V_iV_{i+3}) = 3n+5+2i$ for $i = 0,1,2….n-1$.

$$\alpha_0(V_iV_{i+4}) = \begin{cases} 3n+i+13 & \text{for } i=0 \\ 4n+i+8 & \text{for } i=1 \\ 3n+i+14 & \text{for } i=2 \\ 3n+i+9 & \text{for } i=3 \\ 4n+i+4 & \text{for } i=4 \\ 3n+i+10 & \text{for } i=5 \\ 4n+i+5 & \text{for } i=6 \\ 4n+i & \text{for } i=7 \\ 3n+i+6 & \text{for } i=8 \\ 4n+i+1 & \text{for } i=9 \\ 3n+i+7 & \text{for } i=10. \end{cases}$$

$$\alpha_0(V_iV_{i+5}) = \begin{cases} 4n-i+18 & \text{for } i=0,1..6 \\ 6n-i+7 & \text{for } i=7,..10. \end{cases}$$

The vertex and edge labels under the labeling α_0 are $\alpha_0(v) = \{1,2,3…n\}$ and $\alpha_0(E) = \{n+1,n+2,…6n\}$.

It means that the labeling α_0 is a bijection from the set $V(C_n(1,2,3,4,5)) \cup E(C_n(1,2,3,4,5))$.onto the set $\{1,2,….6n\}$.

We considered the vertex – weight of C_n as follows .

Case (i) For i = 0,1,2,3,4.

a) For i = 0

$\mathbf{Wt}\boldsymbol{\alpha_0}(\boldsymbol{v_0})$ = 12-(i+1) +2n-10+2n+3i+1+3n+5+2i+3n+i+13+4n-i+18+4n-i+18+3n+i+13
+3n+5+2i+2n+3i+1+2n-10+12-(i+1).
= 12-1+2n-10+2n+0+1+3n+5+0+3n+0+13+4n-0+18+4n-(n+5)+18+3n+(n-6)
+13+3n+5+2(n-7)+2n+3(n-8)+1+2n-10+12-(3+1).

= 33n+24.

$(v_0\ )$ = 387.

b) For i = 1

$\mathbf{Wt}\boldsymbol{\alpha_0}(\boldsymbol{v_1})$ = 12-(i+1) + $\frac{3n+i}{2}$ +1+2n+3i+1+3n+5+2i+4n+i+8+4n-i+18+4n-i+18+4n-i+18
+4n+i+8+3n+5+2i+2n+3i+1+2n-10+12-(i+1).

= 12-0+17+1+2n+3+1+3n+5+2+4n+1+8+4n-1+18+4n-(n-3)+18+4n+(n-4)
+8+3n+5+2(n-5)+2n+3(n-6)+1+2n-10+12-(n+10)+1.
= 32n+46.
(v_1) = 398.

c) For i = 2

$\mathbf{Wt}\boldsymbol{\alpha_0}(\boldsymbol{v_2})$ = 12-(i+1)+ $\frac{2n+i}{2}$ +1+2n+3i+1+3n+5+2i+3n+i+14+4n-i+18+4n-i+18
+3n+i+14+3n+5+2i+2n+3i+1+2n-10+12-(i+1).
= 12-3+12+1+2n+6+1+3n+5+4+3n+2+14+4n-2+18+4n-(n-6)+18+3n+
2+14+3n+5+2(n-7)+2n+3(n-8)+1+2n-10+12-(n+1).
= 29n+79.

(v_2) = 398.

d) For i = 3

$\mathbf{Wt}\boldsymbol{\alpha_0}(\boldsymbol{v_3})$ = 12-(i+1) + $\frac{3n+i}{2}$ +1+2n+3i+1+3n+5+2i+3n+i+9+4n-i+18+4n-i+18
+3n+i+9+3n+5+2i+2n+3i+1+2n-10+12- (i+1).
= 12-(3+1) +18+1+2n+9+1+3n+5+6+3n+3+9+4n-3+18+4n-(n-6)+18
+3n+(n-7)+9+3n+5+2(n-8)+2n+3(n-9)+1+2n-10+12-n+1+1.

= 30n+68.

(v_3) = 398.

e) For i = 4

$\mathbf{Wt}\boldsymbol{\alpha}_{\mathbf{0}}(\boldsymbol{v}_{\mathbf{4}})$ = 12- (i+1) + $\frac{2n+i}{2}$ +1+2n+2+3n+5+2i+4n+i+4+4n-i+18+4n-i+18+4n+i+4

3n+5+2i+2n+2+2n-10+12-(i+1).

= 12- (4+1)+13+1+2n+2+3n+5+8+4n+4+4+4n-4+18+4n-(n-4)+18+4n +(n-5)+4+3n+5+2(n-6)+2n+2+2-10+12-n+9+1.

= 29n+68.

(v_4) = 387.

Case (ii) For i = 5,6….10 , for i = i,i-1,i-2…..

$\mathbf{Wt}\boldsymbol{\alpha}_{\mathbf{0}}(\boldsymbol{v}_{\boldsymbol{i}})$ = 12- (i+1) +2n-10+2n+9+3n+5+2i+3n+i+7+6n-i+7+6n-i+7+3n I+7+3n+5+2i+2n+9+2n-10+12-(i+1).

= 12-i-1+2n-10+2n+9+3n+5+2i+3n+i+7+6n-i+7+6n-(i-5)+7+3n+(i-6) +7+3n+5+2(i-7)+2n+9+2n-10+12- (2i+4)+1.

= 32n+i+41.

Thus we obtained Wt$\alpha_0(v_i)$ = 32n+i+41.

Consequently , We proved that α_0 is a Total bi-magic labeling for C_n(1,2,3,4,5) with the bi-magic constants 387 and 398.

Theorem 2.2:

For odd n≥ 13 , circulant graph C_n(1,2,3,4) admits Total bi-magic labeling with the bi-magic constants 329 and 316.

Proof :

Let C_n(1,2,3,4) be a subclass of a circulant graphs with n≥ 13 .

Let {V_i: i = 0,1,….n-1} be the vertices of C_n(1,2,3,4).

Label all the vertices and edges as follows :

$$\alpha_1(V_i) = \begin{cases} \frac{2n+i}{2} - 12 & \text{for } i = 0,2,4\ldots n-1 \\ \frac{3n+i}{2} - 12 & \text{for } i = 1,3,\ldots\ldots n-2 \end{cases}$$

$$\alpha_0(V_iV_{i+1}) = \begin{cases} 2n-1 & \text{for } i = 0 \\ \frac{3n+1}{2} - 2 & \text{for } i = 1 \\ 2n-2 & \text{for } i = 2 \\ \frac{3n+1}{2} - 3 & \text{for } i = 3 \\ 2n-3 & \text{for } i = 4 \\ \frac{3n+1}{2} - 4 & \text{for } i = 5 \\ 2n-4 & \text{for } i = 6 \\ \frac{3n+1}{2} - 5 & \text{for } i = 7 \\ 2n-5 & \text{for } i = 8 \\ \frac{3n+1}{2} - 6 & \text{for } i = 9 \\ 2n-6 & \text{for } i = 10 \\ 2n & \text{for } i = 11 \\ \frac{3n+1}{2} - 1 & \text{for } i = 12 \end{cases}$$

$$\alpha_0(V_iV_{i+2}) = \begin{cases} 3n-7+3i & \text{for } i = 0,1,2 \\ 2n+i-1 & \text{for } i = 3 \\ 2n+i+1 & \text{for } i = 4 \\ 2n+i+3 & \text{for } i = 5 \\ 2n+i+5 & \text{for } i = 6 \\ 2n+i-6 & \text{for } i = 7 \\ 2n+i-4 & \text{for } i = 8 \\ 2n+i-2 & \text{for } i = 9 \\ 2n+I & \text{for } i = 10 \end{cases}$$

$$\begin{cases} 2n+i+2 & \text{for } i=11 \\ 2n+i-9 & \text{for } i=12. \end{cases}$$

$$\alpha_0(V_iV_{i+3}) = \begin{cases} 4n-2+2i & \text{for } i=0,1 \\ 3n+i & \text{for } i=2 \\ 3n+i+1 & \text{for } i=3 \\ 3n+i+2 & \text{for } i=4 \\ 3n+i+3 & \text{for } i=5 \\ 3n+i+4 & \text{for } i=6 \\ 3n+i+5 & \text{for } i=7 \\ 3n+1 & \text{for } i=8 \\ 3n+3 & \text{for } i=9 \\ 3n+5 & \text{for } i=10 \\ 3n+7 & \text{for } i=11 \\ 3n+9 & \text{for } i=12 \end{cases}$$

$$\alpha_0(V_iV_{i+3}) = \begin{cases} 5n-5i-2 & \text{for } i=0 \\ 5n-5i-5 & \text{for } i=1 \\ 5n-5 & \text{for } i=2 \\ 5n & \text{for } i=3 \\ 5n-8 & \text{for } i=4 \\ 5n-3 & \text{for } i=5 \\ 5n-11 & \text{for } i=6 \\ 5n-6 & \text{for } i=7 \\ 5n-1 & \text{for } i=8 \\ 5n-9 & \text{for } i=9 \\ 5n-4 & \text{for } i=10 \\ 5n-12 & \text{for } i=11 \\ 5n-7 & \text{for } i=12 \end{cases}$$

The vertex and edge labels under the labeling α_1 are α_1(V) = {1,2…..n} and α_1(E) = {n+1,n+2 …..5n}. It means that the labeling α_1 is a bijection from the set V(C_n(1,2,3,4) ∪ E (C_n(1,2,3,4)) onto the set {1,2,…..5n}.

We considered the vertex-weights of C_n(1,2,3,4) as follows :

Case (i) For i=0,1,2

a) **For i=0**

$Wt\alpha_1(v_0)$ = $\frac{2n+i}{2}$ -12+2n-1+3n-7+3i+4n-2+2i+5n-5i-2+5n-5i-2+4n-2+2i+3n-7+3i

+2n-1+ $\frac{2n+i}{2}$ - 12.

= 13-12+2n-1+3n-7+4n-2+5n-2+5n-5(n+3)-2+4n-2+2(n+4)+3n-7 3(n-2)+2n-1+13-12.

=28n-35.

(v_0) = 329.

b) **For i=1**

$Wt\alpha_1(v_1)$ = $\frac{3n+i}{2}$ -12 + $\frac{3n+1}{2}$ -2 +3n-7+3i+4n-2+2i+5n-5i-5+5n-5i-5+4n-2+2i

+3n-7+3i+2n-1+ $\frac{3n+i}{2}$ -12 .

= 20-12 +20-2+3n-7+3+4n-2+2+5n-5-5+5n-5(n)-5+4n-2+2(n-1)+3n

-7+3(n-4)+2n-1+20-12.

= 26n-9.

(v_1) = 329.

c) **For i=2**

$Wt\alpha_1(v_2)$ = $\frac{2n+i}{2}$ - 12 +2n-2+3n-7+3i+3n+i+5n-5+5n-5+3n+i+3n-7+3i+2n-2+ $\frac{2n+i}{2}$ -12.

= 14-12+2n-2+3n-7+6+3n+2+5n-5+5n-5+3n+(-n-2)+3n-7+3(n-10)+

2n-2+14-12.

= 28n-48.

(v_2) = 316.

d) For i=3

$\mathbf{Wt}\boldsymbol{\alpha_1}(\boldsymbol{v_3})$ = $\frac{3n+i}{2}$ -12 +$\frac{3n+1}{2}$ -3+2n+i-1+3n+i+1+5n+5n+3n+i+1+2n+i-1+2n-1+

$\frac{3n+i}{2}$ -12.

= 2o-12+20-3+2n+3-1+3n+3+1+5n+5n+3n+(n-10)+1+2n+(n-12)

-1+2n-1+21-12.

= 24n+17.

(v_3) = 329.

e) For i=4

$\mathbf{Wt}\boldsymbol{\alpha_1}(\boldsymbol{v_4})$ = $\frac{2n+i}{2}$ -12+2n-3+2n+i+1+3n+i+2+5n-8+5n-8+3n+i+2+2n+i+1+2n-3

$\frac{2n+i}{2}$ -12.

= 15-12+2n-3+2n+4+1+3n+4+2+5n-8+5n-8+3n+(n-5)+2+2n+(n-2)

+1+2n-3+15-12.

= 26n-9.

(v_4) = 329.

Case(ii) For i=5,6,8,9,10 i=i,i-1,i-2,….

$\mathbf{Wt}\boldsymbol{\alpha_1}(\boldsymbol{v_i})$ = $\frac{3n+i}{2}$ -12 +$\frac{3n+1}{2}$ -4 +2n+i+3+3n+i+3+5n-3+5n-3+3n+i+3+2n+i+3+2n

-1+ $\frac{3n+i}{2}$ -12.

= 22-12+20-4+2n+i+3+3n+i+3+5n-3+5n-3+3n+(i-9)+3+2n+(-2i-7)

+3+2n-1+22-12.

= 22n+i+25.

Case(iii) For i=7,11,12 i=i.i-3,i-4…..

$$\mathbf{Wt}\boldsymbol{\alpha_1}(\boldsymbol{v_i}) = \frac{3n+i}{2} -12+\frac{3n+1}{2} -5+2n+i-6+3n+i+5+5n-6+5n-6+3n+7+5+2n+i -6+2n-1+\frac{3n+i}{2} -12.$$

$$= 23-12+20-5+2n+i-6+3n+i+5+5n-6+5n-6+3n+(i+5)+5+2n +(-2i+9)-6+2n-1+23-12.$$

$$= 22n+i+36.$$

Thus we obtained weight $Wt\alpha_1(v_i)$ = 22n+i+25 and 22n+i+36 .Consequently it is proved that α_1 is a Total bi-magic labeling for $C_n(1,2,3,4)$ with the bi-magic constants 316 and 329.

CONCLUSION:

In this paper, we obtained Total bi-magic labeling on circulant graphs with generating sets (1,2,3,4) and (1,2,3,4,5).

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