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Simultaneous Quadruple Series Equations Involving Laguerree Polynomials

Indu Shukla

*Department of Physical science, M.G.C.G.V., Chitrakoot,
Satna, M.P.(India)*

Abstract

Lowndes [2] [3] have obtained the solution of some dual series equations involving laguerre polynomials and then solved triple series equations involving Laguerre polynomials. Singh, Rokne and Dhaliwal [4] obtained closed from solution of triple series equations involving Laguerre polynomials and Srivastava [6] have also obtained the solutions of certain dual series equations involving Laguerree polynomials. In the present paper, an exact solution has been obtained for the simultaneous quadruple series equations involving Laguerre polynomials by Noble`s [5] modified multiplying factor techniques.

Key words: Integral equation, Series equation, Series equation, Laguerre polynomial.

Subject Classification: 45XX, 45F10, 15A52, 33C45, 42C05.

1. INTRODUCTION

We consider the following Quadruple series equations

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij}\frac{A_{nj}}{\Gamma(\alpha+1+ni+p)}L_{ni+p}^{\alpha}[(x+d)^k] = \emptyset_{1i}(x)\ \ ;\ 0<x<a \tag{1.1}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\frac{A_{nj}}{\Gamma(\alpha+1+ni+p)}L_{ni+p}^{\gamma}[(x+d)^k] = \emptyset_{2i}(x)\ \ ;\ a<x<b \tag{1.2}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\frac{A_{nj}}{\Gamma(\alpha+1+ni+p)}L_{ni+p}^{\gamma}[(x+d)^k] = \emptyset_{3i}(x)\ \ ;\ b<x<c \tag{1.3}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} c_{ij}\frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)}L_{ni+p}^{\sigma}[(x+d)^k] = \emptyset_{4i}(x)\ ;\ c < x < \infty \quad (1.4)$$

Where $\alpha+\beta+1 > \beta > 1-m$, $\sigma+1 > \alpha+\beta > 0$, m is a positive integer and $0 < h < \infty$, $0 \leq b < \infty$ and h and b are finite constants. $L_{ni+p}^{\sigma}[(x+d)^k]$ is a laguerre polynomial , p is a non- negative integer. A_{nj} are unknown coefficients to be determine and $\emptyset_{1i}(x)$, $\emptyset_{2i}(x)$, $\emptyset_{3i}(x)$ and $\emptyset_{4i}(x)$ are prescribed functions for *i = 1,2,….s.*

$$\sum_{n=0}^{\infty}\frac{A_n}{\Gamma\alpha+n+1}L_n^{\alpha}(x) = \emptyset_1(x)\ ; \qquad 0 < x < a \quad (1.5)$$

$$\sum_{n=0}^{\infty}\frac{A_n}{\Gamma\alpha+n+1}L_n^{\gamma}(x) = \emptyset_2(x)\ ; \qquad a < x < b \quad (1.6)$$

$$\sum_{n=0}^{\infty}\frac{A_n}{\Gamma\alpha+n+1}L_n^{\gamma}(x) = \emptyset_3(x)\ ; \qquad b < x < c \quad (1.7)$$

$$\sum_{n=0}^{\infty}\frac{A_n}{\Gamma\alpha+n+\beta}L_n^{\sigma}(x) = \emptyset_4(x)\ ; \qquad c < x < \infty \quad (1.8)$$

The Quadruple series equations (1.5), (1.6), (1.7) and (1.8) are a special case of simultaneous Quadruple series equations (1.1), (1.2),(1.3) and (1.4) when *p = 0, d = 0, k =1,* $a_{ij} = b_{ij} = c_{ij} = 1$, A_{nj} is replaced by A_n and n_i is replaced by n for *j = 1,2,….s* and *i= 1,2,….s* and *s = 0.*

$$L_n^{\alpha}(x) = \sum_{k=0}^{n}\binom{n+\alpha}{n-k}\frac{(-x)^k}{k}\ , \quad n = 0, 1, 2\ldots\ldots \quad (1.9)$$

is the laguerre polynomial of order α and degree n in x.

2. PRELIMINARY RESULTS

(i) The orthogonal property of the laguerre polynomials is given by Erdelyi (1953-54)

$$\int_0^{\infty} e^{-x}x^{\alpha}L_m^{\alpha}(x)L_n^{\alpha}(x)dx = \frac{\Gamma(\alpha+n+1)}{n!}\delta_{m,n}\ \ , \alpha > -1 \quad (2.1)$$

where $\delta_{m,n}$ is the kronecker delta.

(ii) Formula (27), pp.190 of Erdelyi (1953-54) in the forms;

$$\frac{d^m}{dx^m}\left\{x^{\alpha+m}L_n^{(\alpha+m)}(x)\right\} = \frac{\Gamma(\alpha+m+n+1)}{\Gamma(\alpha+n+1)}x^{\alpha}L_n^{\alpha}(x) \quad (2.2)$$

(iii) The following forms of the known results Erdelyi (1953-54)

$$\int_0^{\xi}(\xi - x)^{\beta-1} x^{\alpha} L_n^{\alpha}(x)dx = \frac{\Gamma(n+\alpha+1)\Gamma(\beta)}{\Gamma(n+\alpha+\beta+1)} \xi^{\alpha+\beta} L_n^{\alpha+\beta}(\xi) \quad (2.3)$$

when $\beta > 0, \alpha > -1$ and the second integral

$$\int_{\xi}^{\infty}(x - \xi)^{\beta-1} e^{-x} L_n^{\alpha}(x)dx = \Gamma(\beta) \,.\, e^{-\xi} L_n^{\alpha-\beta}(\xi) \quad (2.4)$$

where $\alpha + 1 > \beta > 0$.

3. SOLUTION OF QUADRUPLE SERIES EQUATIONS

We assume that

$$x + d = X^{\frac{1}{k}}, \emptyset_{1i}(X^{\frac{1}{k}} - d) = \emptyset'_{1i}(X), \emptyset_{2i}(X^{\frac{1}{k}} - d) = \emptyset'_{2i}(X), \emptyset_{3i}(X^{\frac{1}{k}} - d) = \emptyset'_{3i}(X)$$
$$, \emptyset_{4i}(X^{\frac{1}{k}} - d) = \emptyset'_{4i}(X), d^k = e, (a+d)^k = f, (b+d)^k = g, (c+d)^k = h. \quad (3.1)$$

Then the simultaneous quadruple series equation (1.1), (1.2),(1.3) and (1.4) can be written in the following form:

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij} \frac{A_{nj}}{\Gamma(\alpha+1+ni+p)} L_{ni+p}^{\alpha}(X) = \emptyset'_{1i}(X) \;\; ; e < X < f \quad (3.2)$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij} \frac{A_{nj}}{\Gamma(\alpha+1+ni+p)} L_{ni+p}^{\gamma}(X) = \emptyset'_{2i}(X) \;\; ; f < X < g \quad (3.3)$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij} \frac{A_{nj}}{\Gamma(\alpha+1+ni+p)} L_{ni+p}^{\gamma}(X) = \emptyset'_{3i}(X) \;\; ; g < X < h \quad (3.4)$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} c_{ij} \frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)} L_{ni+p}^{\sigma}(X) = \emptyset'_{4i}(X) \;\; ; h < X < \infty \quad (3.5)$$

We assume that

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij} \frac{A_{nj}}{\Gamma(\alpha+1+ni+p)} L_{ni+p}^{\alpha}(X) = \emptyset''_{1i}(X) \;\; ; 0 < X < e \quad (3.6)$$

Combining the series equations (3.6) and (3.2), we can write the simultaneous quadruple series equations (3.6) and (3.2) in the form

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij} \frac{A_{nj}}{\Gamma(\alpha+1+ni+p)} L_{ni+p}^{\alpha}(X) = \phi_{1i}(X) \;\; ; 0 < X < f \quad (3.7)$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij} \frac{A_{nj}}{\Gamma(\alpha+1+ni+p)} L_{ni+p}^{\gamma}(X) = \phi_{2i}(X) \;\; ; f < X < g \quad (3.8)$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij} \frac{A_{nj}}{\Gamma(\alpha+1+ni+p)} L_{ni+p}^{\gamma}(X) = \phi_{3i}(X) \;\; ; g < X < h \quad (3.9)$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} c_{ij}\frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)}L_{ni+p}^{\sigma}(X) = \phi_{4i}(X) \quad ; h < X < \infty \tag{3.10}$$

where, $$\phi_{1i}(X) = \begin{cases} \emptyset''_{1i}(X) \; ; & 0 < X < e \\ \emptyset'_{1i}(X) \; ; & e < X < f \end{cases} \tag{3.11}$$

$$\phi_{2i}(X) = \emptyset'_{2i}(X) \;, \phi_{3i}(X) = \emptyset'_{3i}(X) \,, \; \phi_{4i}(X) = \emptyset'_{4i}(X) \tag{3.12}$$

Multiplying equation (3.7) by $x^{\alpha}(\xi - x)^{\beta+m-2}$ and integrating with respect to X over (0, ξ) and first fractional integral formula (2.3) we get,

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij}\frac{A_{nj}}{\Gamma(ni+\beta+p+\alpha+m)}L_{ni+p}^{\beta+\alpha+m-1}(\xi)$$

$$= \frac{\xi^{-\beta-\alpha-m+1}}{\Gamma(\beta+m-1)}\int_0^{\xi} X^{\alpha}(\xi - X)^{\beta+m-2}\,\phi_{1i}(X)dX \tag{3.13}$$

Now multiplying both sides of equation (3.13) by $\xi^{\beta+\alpha+m-1}$ and differentiating both sides 'm' times with respect to ξ and using the derivative formula (2.2) we get,

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\frac{A_{nj}}{\Gamma(ni+\beta+p+\alpha)}L_{ni+p}^{\beta+\alpha-1}(\xi) = \sum_{j=1}^{s} e_{ij}\frac{\xi^{-\beta-\alpha+1}}{\Gamma(\beta+m-1)}\frac{d^m}{d\xi^m}\int_0^{\xi} X^{\alpha}(\xi - X)^{\beta+m-2}\,\phi_{1i}(X)dX \tag{3.14}$$

where e_{ij} are the element of the matrix $[b_{ij}][a_{ij}].^{-1}$ and $0 < \xi < y\,, \alpha > -1, \beta + m > 1,\; i = 1,2,3 \ldots\ldots s.$

Equation (3.14) can be written as,

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\frac{A_{nj}}{\Gamma(ni+\beta+p+\alpha)}L_{ni+p}^{\beta+\alpha-1}(\xi) = \sum_{j=1}^{s} e_{ij}\frac{\xi^{-\alpha-\beta+1}}{\Gamma(\beta+m-1)}\phi_1(\xi) \tag{3.15}$$

Multiplying both sides of equation (3.10) by $e^{-X}(X - \xi)^{\sigma-\beta-\delta}$ and integrate with respect to X over (ξ, ∞) and using the second fractional integral formula (2.4) we get,

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\frac{A_{nj}}{\Gamma(ni+\alpha+p+\beta)}L_{ni+p}^{\alpha+\beta-1}(\xi) = \frac{e^{\xi}}{\Gamma(\sigma-\beta-\alpha+1)}\int_{\xi}^{\infty} e^{-X}(X - \xi)^{\sigma-\beta-\alpha}\,\phi_{4i}(X)dX \tag{3.16}$$

which can be written as,

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\frac{A_{nj}}{\Gamma(ni+\alpha+p+\beta)}L_{ni+p}^{\alpha+\beta-1}(\xi) = \frac{e^{\xi}}{\Gamma(\sigma-\beta-\alpha+1)}\phi_4(\xi) \tag{3.17}$$

Where $\xi < x < \infty$ and $\sigma + 1 > \beta + \delta > 0.$

Left hand sides of equation (3.15), (3.17), (3.8) and (3.9) are identical hence on using the orthogonal relation (2.1)

$$A_{nj} = \sum_{j=1}^{s} d_{ij} \left[\sum_{j=1}^{s} \frac{e_{ij}\ (ni+p)!}{\Gamma(\beta+m-1)} \int_0^a e^{-\xi} L_{ni+p}^{\beta+\alpha-1}(\xi)\phi_1(\xi)d\xi\right.$$

$$+\int_a^b e^{-\xi}\xi^{\beta+\alpha-1} L_{ni+p}^{\beta+\alpha-1}(\xi)\ \phi_2(\xi)d\xi + \int_b^c e^{-\xi}\xi^{\beta+\alpha-1} L_{ni+p}^{\beta+\alpha-1}(\xi)\phi_3(\xi)d\xi$$

$$\left.+\sum_{j=1}^{s} \frac{g_{ij}\,(ni+p)!}{\Gamma(\sigma-\beta-\alpha+1)} \int_c^\infty \xi^{\beta+\alpha-1} L_{ni+p}^{\beta+\alpha-1}(\xi)\phi_4(\xi)d\xi\right] \tag{3.18}$$

Where, $n = 0,1,2, \ldots \ldots \ldots$ and d_{ij} are the elements of the matrix $[b_{ij}]^{-1}$.

$$\phi_1(\xi) = \frac{d^m}{d\xi^m} \int_0^\xi X^\alpha (\xi - X)^{\beta+m-2}\ \phi_{1i}(X)dX\ ,\ \phi_2(\xi) = \phi_{2i}(X),\ \phi_3(\xi) = \phi_{3i}(X),$$

$$\phi_4(\xi) = \int_\xi^\infty e^{-X}(X-\xi)^{\sigma-\beta-\alpha}\ \phi_{4i}(X)dX \tag{3.19}$$

Provided that $\alpha + \beta + 1 > \beta > 1 - m,\ \sigma + 1 > \alpha + \beta > 0,\ m$ being a positive integer with the help of (3.11), (3.19) can be written in the form

$$\phi_1(\xi) = \frac{d^m}{d\xi^m}\left[\int_0^e X^\alpha(\xi - X)^{\beta+m-2}\emptyset''_{1i}\,(X)\,dX + \int_e^\xi X^\alpha(\xi - X)^{\beta+m-2}\emptyset'_{1i}\,(X)\,dX\right],$$

$$e < \xi \tag{3.20}$$

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