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Radiation effects on boundary layer flow of nano-fluids Cu-water and Ag-water over a stretching plate with convective surface boundary condition

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Abstract

This paper deals with the steady laminar boundary layer flow of nano-fluids Cu-water and Ag-water past a stretching plate with convective surface boundary condition in the presence of thermal radiation. The present flow belongs to the category of boundary layer flow of Skiadis type. The closed form solution has been obtained for convective heat transfer under the given condition. The main thrust of our study is to read the following:

a) effect of radiation parameter on the convective heat transfer, and
b) the effect of volume fraction of nano-sized particles of Cu in Cu-water and Ag in Ag-water, nano-fluids.

The skin friction and Nusselt number both have been calculated and the possible effect of related parameters has been studied.

Keywords: Nano-fluids, boundary layer equations, Radiation flux and Nusselt number.

INTRODUCTION

Due to the number of applications of the boundary layer flow past a stretching sheet attracts the scholars to do researches in several variants. In particular, some of the possible applications of boundary layer flow past a stretching sheet are aerodynamic extrusion of plastic sheets, formation of boundary layer along liquid film in condensation process, the cooling of metallic plate in a cooling bath, drawing of polymer yarn in textile industry and manufacturing the glass sheet in glass industries.

In recent years, the conceptual birth of technology came to the existence due to legendry scientist R.P.Feynman. Feynman delivered a famous lecture 'There is plenty of room at the bottom' at the American Physical Society Meeting at Caltech on Dec.29, 1969. In 1980 the invention of scanning tunnelling microscope accelerated the growth of nanotechnology. The term technology was used by Drexler in 1986 book "engines of creation", [1]. Basically, Drexlers idea of nanotechnology was referred to molecular nanotechnology, [2]. Nano-fluids were a result of the experiments to increase the thermal conductivity of liquids. The birth of nano-fluids is attributed the revolutionary idea of adding solid particles in Heat Transfer Fluids (HTF) to increase thermal inductivity. This innovative idea put forth by Maxwell in 1973. There were issues to attend but uncertainty did linger on the practical utility and nature of nano-fluids. Till today nano-fluid are still in their early phase even through, the scientist are working, using nano-fluids in many variants. We smell that the nano-fluids is the future existing frontier in the technology. Referring review article "An overview of recent nano-fluid research [3]'',there are all around applications of nano-fluids such as in automotive engines to improve the efficiency of the heat transfer, cooling of microchips as applications in electronics where nano-fluids act as detergent. Seeing the utility of nano-fluids in science and technology, scholars paid their attention to do investigations on nano-fluids around the glob. In 2011, K. Vajravelu et al. [4] studied convective heat transfer in the flow of Cu-water and Ag-water nano-fluids over a stretching surface. Y.Yigra and B.Shankar [5] studied MHD flow and heat transfer of nano-fluids through a porous media due to a stretching sheet with viscous dissipation and chemical reaction effects. Kalidas Das [6] investigated Nano-fluids flow over a non-linear permeable stretching sheet with partial slip. Kalidas Das et al. [7] published numerical simulation of nano-fluid flow with convective boundary condition, Vasu Buddakkagari and Manish kumar solved transient boundary layer laminar free convective flow of a nano-fluid over a vertical cone/plate.

In this paper, we investigate the closed form solution for steady laminar boundary layer flow of nano-fluids Cu-water and Ag-water past a stretching plate with convective surface boundary condition in the presence of thermal radiation. We study the following:

a) the effect of radiation parameter on the convective heat transfer, and
b) the effect of volume fraction of nano-sized particles of Cu in Cu-water and Ag in Ag-water, nano-fluids.

The skin friction and Nusselt number both have been calculated and the possible effect of related parameters has been studied by table of numerical values.

MATHEMATICAL FORMULATION

Considering two dimensional boundary layer flow over a stretching sheet, we assume a coordinate system where x-axis is along the stretching sheet and y-axis is normal to the surface of the sheet in the positive direction. The figure-1 shows the geometry of the problem where the continuous stretching surface is governed by U(x) [=mx], m is a constant.

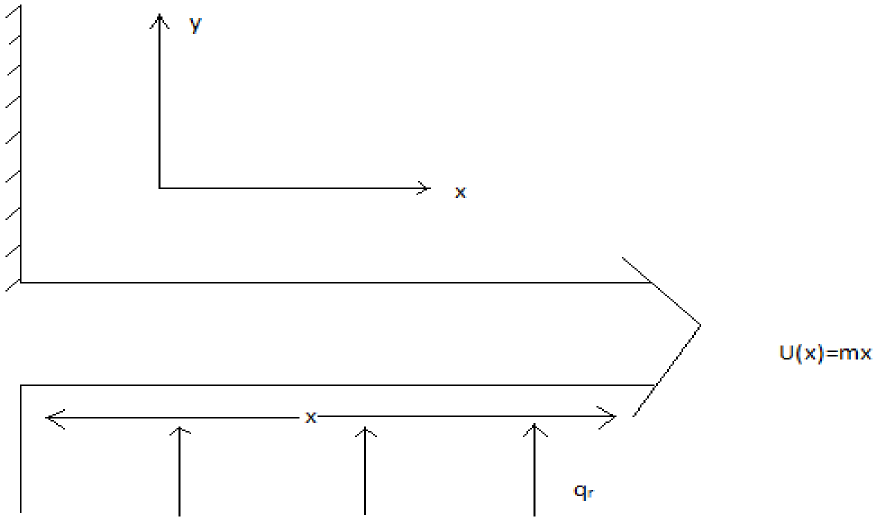

Figure-1

The fluids considered here are Cu-water and Au-water nano-fluids. We study the boundary layer flow and heat transfer problem here under the following assumptions

i. nano-fluids are incompressible
ii. there is no chemical reaction
iii. there is negligible viscous dissipation
iv. nano-sized solid particles and the base fluid both are in thermal equilibrium and no slip occurred between the nano-sized particles and the fluid.

(i) Boundary Layer Flow problem:

The governing equations for steady boundary layer flow of nano-fluids Cu-water and Ag-water past a stretching plate are:

Continuity equation $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Momentum equation $$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

where u and v are the velocity components along x and y axes, respectively, μ_{nf} and ρ_{nf} are dynamic viscosity, and density of Nano-fluids, respectively.

The appropriate boundary conditions for flow problem are:

$$u(x,0)=U(x)=mx,\ \ v(x,0)=0 \qquad (3a)$$

$$y\to\infty,\ \ u=0 \qquad (3b)$$

Introducing dimensionless variables

$$\bar{x} = \frac{x}{h}\ ,\ \ \bar{y} = \frac{y}{h}\ \ ,\qquad \bar{u} = \frac{uh}{\nu_{nf}}\ \ ,\qquad \bar{v} = \frac{vh}{\nu_{nf}}$$

Following the methodology of N.Ahmad and Ranivs[8] , we get the velocity distribution as:

$$u = mxe^{-ry} \quad \text{and} \quad v = -\frac{m}{r}(1 - e^{-ry}) \tag{4}$$

where $r = \sqrt{m}$

(ii) Heat Transfer Problem:

The energy equation with convective surface boundary condition is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y} \tag{5}$$

with relevant boundary conditions:

$$y = 0, -k_{nf}\frac{\partial T}{\partial y} = h_f(T_p - T_\infty) \tag{6a}$$

$$y \to \infty,\ \ T \to T_\infty \tag{6b}$$

where k_{nf} is the thermal conductivity of the nanofluid, , T_p is temperature of the plate and T_∞

is ambient fluid temperature, i.e. the temperature of the fluid far away from the plate, h_f is heat transfer coefficient Referring Rosseland, S and Siegel R, Howell JR [9,10], the radiative heat flux may be considered as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{7}$$

where σ^* and k^* are the Stefan-Bltzmann constant and the mean absorption coefficient, respectively. Here we use the approximation as it is being used by Battler [11,12], Pal [13], Mondal [14], Mukhopadhya and layek [15], Ishak [16] and very recently N.Ahmad and Ravins [8], as

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{8}$$

Using (7) and (8) we have,

$$\frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y} = \frac{1}{(\rho c_p)_{nf}}\frac{\partial}{\partial y}\left(\frac{-4\sigma^*}{3K^*}\frac{\partial}{\partial y}(4T_\infty^3 T - 3T_\infty^4)\right) = -\frac{16\sigma^* T_\infty^3}{3K^*(\rho c_p)_{nf}}\frac{\partial^2 T}{\partial y} \tag{9}$$

Thus, the equation (5) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\alpha_{nf} + \frac{16\sigma^* T_\infty^3}{3K^*(\rho c_p)_{nf}}\right)\frac{\partial^2 T}{\partial y} \tag{10}$$

We define the dimensionless temperature T by $\theta(\eta) = \frac{T-T_\infty}{T_p-T_\infty}$ and assuming $\eta = ry$ substituting u and v in (10) we get

$$\theta'' + \frac{(\mathrm{Pr})_{nf}K_0 m}{r^2}(1-e^{-\eta})\theta' = 0 \tag{11}$$

with boundary conditions:

$$\theta'(0) = -\frac{h_f}{k_{nf}\sqrt{m}} \qquad \theta \to 0, \quad as \quad \eta \to \infty \tag{12}$$

where $(\mathrm{Pr})_{nf} = \frac{\nu_{nf}}{\alpha_{nf}}$ is the Prandlt number of nanofluid , $K_0 = \frac{3N}{3N+4}$ with $N = \frac{k_{nf}k^*}{4\sigma^* T_\infty^3}$, the radiation parameter.

A solution of the equation (11) together with boundary condition (12) is

$$\theta(\eta) = \frac{h_f}{k_{nf}\sqrt{m}} e^{(\mathrm{Pr})_{nf}K_0}\left((\mathrm{Pr})_{nf}K_0\right)^{-(\mathrm{Pr})_{nf}K_0} \boldsymbol{\gamma}((\mathbf{Pr})_{\boldsymbol{nf}}\boldsymbol{K_0},\ (\mathbf{Pr})_{\boldsymbol{nf}}\boldsymbol{K_0}\boldsymbol{e^{-\eta}}) \tag{13}$$

where $\boldsymbol{\gamma}(\boldsymbol{a},\boldsymbol{x})= \int_{\mathbf{0}}^{\boldsymbol{x}} \boldsymbol{e^{-t}}\, \boldsymbol{t^{a-1}}\boldsymbol{dt}$, is incomplete gamma function.

The effective density of nanofluid is given as

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s \tag{14}$$

where φ is the solid volume fraction of nanoparticles. Thus Thermal diffusivity of the nanofluid becomes

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \tag{15}$$

where the heat capacitance of the nanofluid is taken as

$$(\rho c_p)_{nf} = (1-\varphi)\rho c_{p_f} + \varphi\rho c_{p_s} \tag{16}$$

Brinkman [17], effective dynamic viscosity of the nanofluid is given by

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}} \tag{17}$$

we are having k_{nf}, the thermal conductivity of the nanofluid given by Maxwell [18] as

$$k_{nf} = k_f\left\{\frac{k_s+2k_f-2\varphi(k_f-k_s)}{k_s+2k_f+\varphi(k_f-k_s)}\right\} \tag{18}$$

Skin Friction

The skin friction coefficient is defined as

$$C_f = -\frac{\mu_{nf}}{\rho_f U^2}\left(\frac{\partial u}{\partial y}\right)_{at\ y=0} = \frac{(Re_x)^{-1/2}}{(1-\varphi)^{2.5}}\sqrt{\nu_f} \tag{19}$$

where, $Re_x = \frac{Ux}{\upsilon_f}$ is the local Reynolds number.

Table-1 Variation in Skin friction for different volume fraction φ of nano-particles

φ	$C_f {Re_x}^{1/2}$
0.0	0.001001998
0.1	0.001303948
0.2	0.001750418

Nusselt number

The coefficient of convectional heat transfer is called Nusselt number Nu and it is defined as

$$Nu = -\frac{\left(\frac{\partial T}{\partial y}\right)_{at\, y=0}}{T_{p-T_\infty}} = \frac{h_f}{k_{nf}} \tag{20}$$

Table-2 Variation in Nusselt number for different volume fraction φ of nano-particles

φ	$Nu(Cu - water)$	$Nu(Ag - water)$
0.0	21.37030995	21.37030995
0.1	16.0481002	16.04678311
0.2	12.24162966	12.23960335

DISCUSSION AND RESULTS

The nanofluids Cu-water and Ag-water have been considered for boundary layer flow past a stretching plate and heat transfer with convective surface boundary condition to read the radiation effect. The exact solution to this problem has been obtained. Skin friction and Nusselt number have also been derived. The effect of Radiation parameter N and the volume fraction of nano-sized particles φ have been studied on temperature field through graphs. We summarize the results in the following paragraphs:

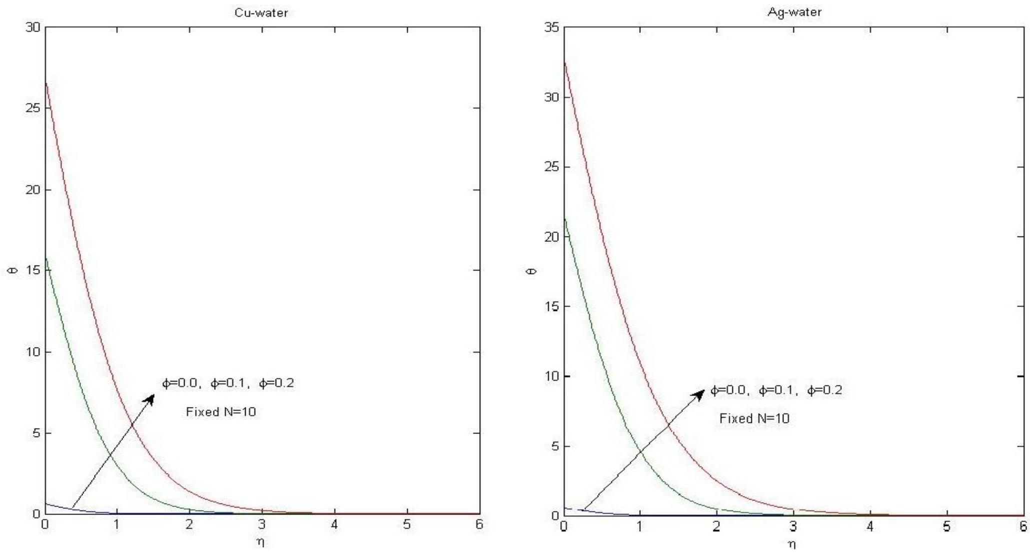

Figure-2 Dependence of temperature field on volume fraction of nano-particles for fixed Radiation parameter N=10.

The Figure -2 is the graph of temperature field versus spatial distance η from the slit for randomly chosen volume fraction of nanoparticle. It is observed that as volume fraction of nano-particles of Cu and Ag increases, the temperature of nano-fluids increases. Thermally, we express this fact as when solid particles get heated quickly in nano-fluid in turn the nano-fluid having more number of nano-particles becomes more warmer than the fluid having less number of nano-particles for same radiation parameter.

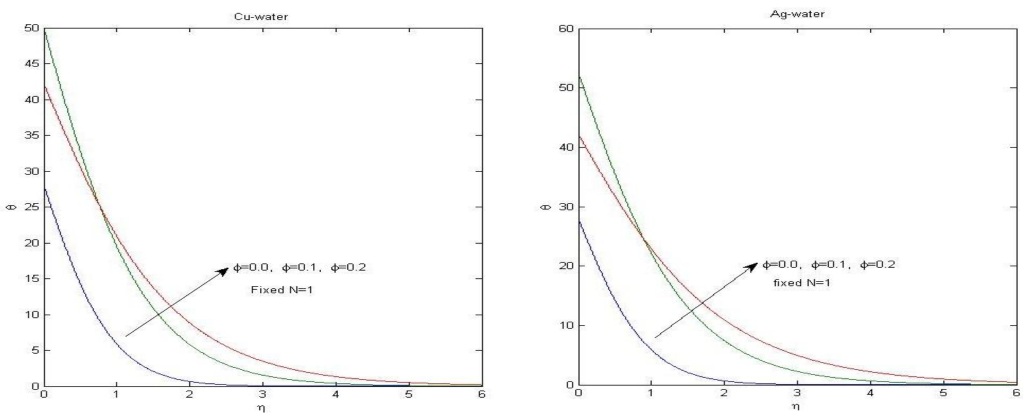

Figure-3 Dependence of nano-fluid temperature field on volume fraction of nano-particles in fluid for Radiation N=1

In figure 3, we observe that when the radiation is of unit 1, then the temperature of nano-fluids shoots up to 50 units in the immediate neighbourhood of stretching plate. This behaviour of nano-fluids advocate that the radiation of unit one has been well digested by nano-sized metallic particles, so that their temperature of nano-fluids shoot up to 50 units. This fact supports that nano-fluids enhance the heat transfer. As we move away from the plate the temperature profile asymptotically approaches to zero.

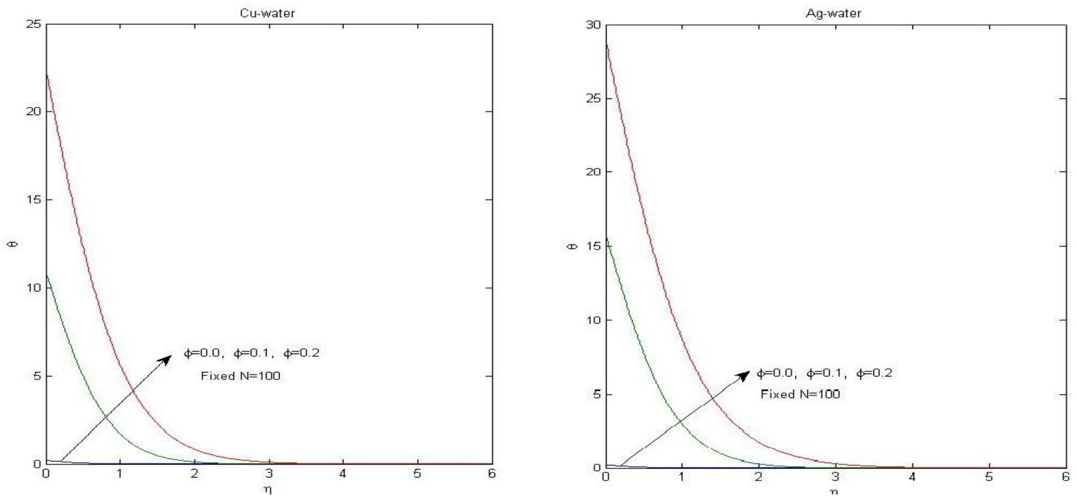

Figure-4 Variation in temperature profile for different volume fraction φ when RadiationN=100

Figure 4 is the graph of temperature field θ versus η for different volume fraction φ of nanoparticles when Radiation parameter N=100. We observe that the temperature profile increases as volume fraction increases but we observe $(\text{temperature})_{N=1} > (temperature)_{N=10} > (temperature)_{N=100}$. Thus, we conclude that the only one unit radiation has good impact on heat transfer.

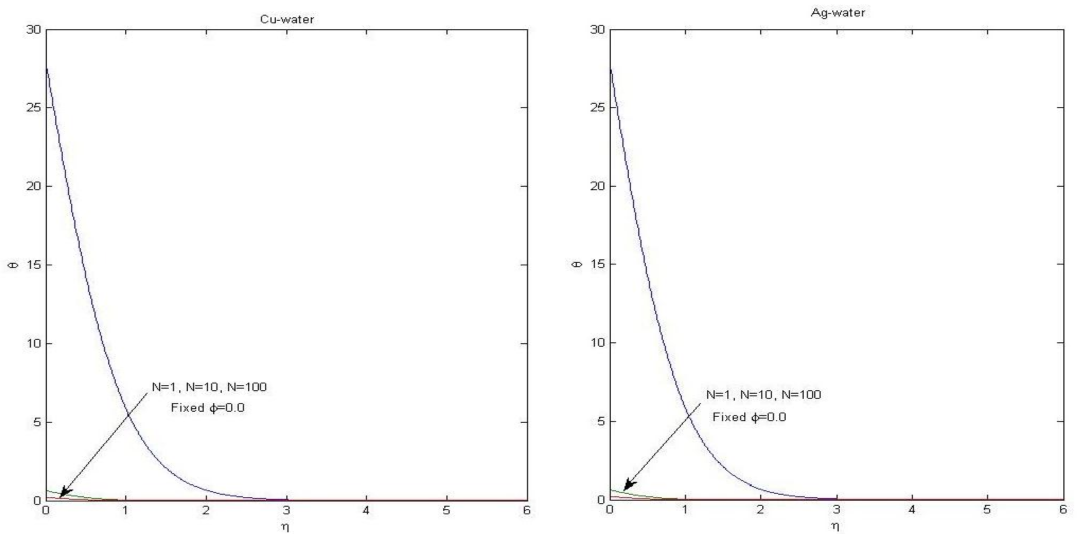

Figure-5 Variation in temperature field for different values of Radiation parameter N for fixed Volume fraction $\varphi = 0.0$

Figure 5 is the graph of temperature field for $\varphi = 0.0$ i.e for water only when radiation N=1, 10 and 100. For water where no nano-sized particles in water, there is no significant change in temperature but when volume fraction increases, we notice a significant change in temperature field by reading the graphs in figures 6 and 7.

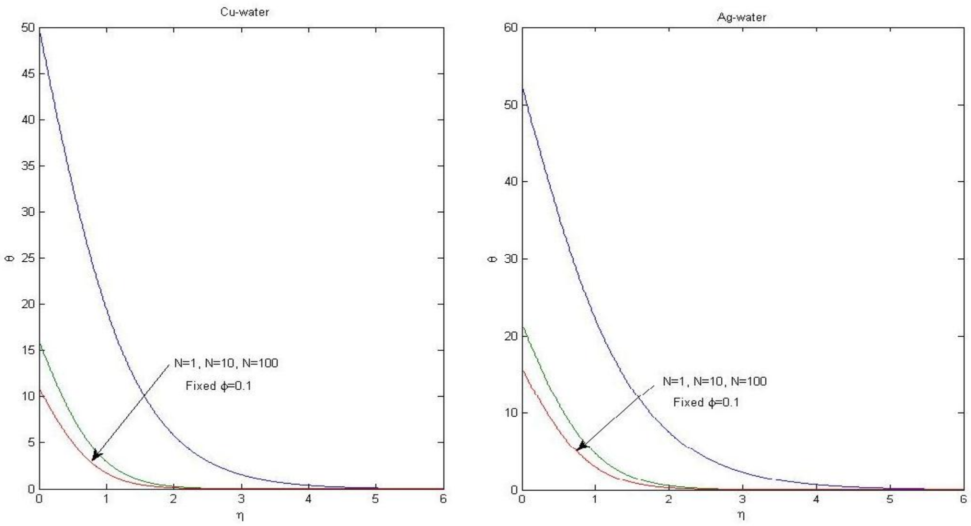

Figure-6 Temperature distribution for different radiation parameter N for fixed volume fraction $\varphi = 0.1$.

Looking at figures 6 and 7, we observe that there is in general increase in temperature field due to presence of nano-sized Cu or Ag particles in the fluid. These particles receive the heat by radiation and get heated. So that the temperature of fluids increases.

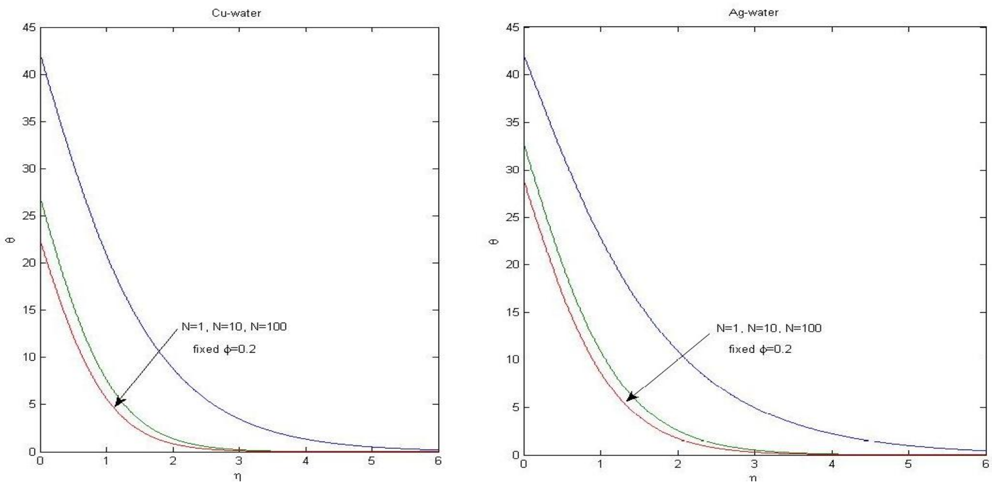

Figure-7 Variation in temperature field for different values of radiation parameter N when $\varphi = 0.2$.

When we compare figures 5, 6 and 7, we observe that radiation N=1 is well digested by nano-fluid with $\varphi = 0.1$. Hence the nanofluid with volume fraction $\varphi = 0.1$ is a very practical fluid to enhance the temperature upto 50 unit

By reading the table 1 for Skin friction, we conclude that as the volume fraction φ increases, the Skin friction increases. In this case, a force called semi-frictional force increases between nanofluid and stretching plate in turn skin friction increases. Looking at table 2, we conclude that the coefficient of convective heat transfer is independent of radiation and the Nusselt number decreases as volume fraction increases.

CONCLUSION:

We conclude the following by our analysis:

1. We obtained the closed form solution to the heat transfer problem
2. The radiation of unit one is very much useful to increase the temperature of nanofluids. Comparatively we observe that the temperature profile increases as volume fraction increases but we observe $(\text{temperature})_{\text{N}=1} > (temperature)_{N=10} > (temperature)_{N=100}$.
3. Nusselt number is independent of Radiation parameter but it is a fraction of φ, the volume fraction parameter.

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