

## **Partial Slip Effect of MHD Boundary Layer Flow of Nanofluids and Radiative Heat Transfer over a permeable Stretching Sheet**

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### **Abstract**

This paper analyzes the combined effects of magnetic field, velocity slip, radiative heating, Viscous dissipation, Brownian motion, thermophoresis parameters, Lewis number and Boundary Layer flow of heat transfer due to nanofluid flow towards a permeable stretching sheet. The partial differential equations are converted to ordinary differential equations which are solved numerically using the fourth-order Runge-Kutta method. Convergence of the derived solutions is studied. Effects of apposite parameters on the velocity, temperature and concentration profiles, skin friction, Nusselt number and Sherwood numbers are discussed. The physical explanations of incurred results are illustrated by graphs and tables. The results indicate that the skin friction coefficient increase and Sherwood number decrease with an increase in velocity slip parameter.

**Keywords:** MHD, Heat transfer, Nanofluids, Stretching sheet, Thermal radiation.

**Nomenclature:**

<b>List of variables:</b>	
$u, v$ :	Velocity components in the $x$ - and $y$ -axis, respectively ( $m/s$ )
$U_w$ :	Velocity of the wall along the $x$ -axis ( $m/s$ )
$x, y$ :	Cartesian coordinates measured along the stretching sheet ( $m$ )
$B(x)$ :	Magnetic field strength ( $A m^{-1}$ )
$C$ :	Nano particle concentration ( $mol m^{-3}$ )
$C_{fx}$ :	Skin-friction coefficient ( <i>Pascal</i> )
$Nu_x$ :	Nusselt number
$Sh_x$ :	Sherwood number
$C_w$ :	Nano particles concentration at the stretching surface ( $mol m^{-3}$ )
$C_\infty$ :	Nano particle concentration far from the sheet ( $mol m^{-3}$ )
$C_p$ :	Specific heat capacity at constant pressure ( $J Kg^{-1} K$ )
$D_T$ :	Brownian diffusion coefficient
$D_B$ :	Thermophoresis diffusion coefficient
$Ec$ :	Eckert number
$a$ :	Constant parameter
$R$ :	Thermal radiation parameter
$S$ :	Suction/Injection parameter
$f$ :	Dimensionless stream function
$Le$ :	Lewis number
$M$ :	Magnetic parameter
$Nb$ :	Brownian motion parameter
$Nt$ :	Thermophoresis parameter
$Pr$ :	Prandtl number
$Re_x$ :	Reynolds number
$T$ :	Fluid temperature ( $K$ )
$T_w$ :	Temperature at the surface ( $K$ )
$T_\infty$ :	Temperature of the fluid far away from the stretching sheet
$q_w$ :	Surface heat flux ( $W/m^2$ )
$q_m$ :	Surface mass flux
<b>Greek Symbols:</b>	
$\alpha$ :	Thermal diffusivity ( $m^2/s$ )

$\psi$ :	Stream function
$\eta$ :	Dimensionless similarity variable
$\mu$ :	Dynamic viscosity of the base fluid ( $kg/m.s$ )
$\nu$ :	Kinematic viscosity ( $m^2 s^{-1}$ )
$\rho_f$ :	Density of the fluid ( $Kg m^{-3}$ )
$\rho_p$ :	Density of the nanoparticle ( $Kg m^{-3}$ )
$\tau$ :	The ratio of the nanoparticle heat capacity and the base fluid heat capacity
$(\rho c)_f$ :	Heat capacity of the base fluid ( $kg/m.s^2$ )
$(\rho c)_p$ :	Heat capacity of the nano particle ( $kg/m.s^2$ )
$\theta$ :	Dimensionless temperature ( $K$ )
$p$ :	Pressure ( $N/m^2$ )
$\phi$ :	Nanoparticle volume fraction
$\phi_w$ :	Nanoparticle volume fraction at wall temperature
$\phi_\infty$ :	Ambient nanoparticle volume fraction
$\lambda$ :	Velocity slip parameter
<b>Sub Scripts:</b>	
$f$ :	Fluid
$W$ :	Condition on the sheet
$\infty$ :	Ambient Conditions
<b>Superscripts:</b>	
' :	Differentiation w.r.t $\eta$

## 1. INTRODUCTION:

Boundary layer behaviour over a permeable stretching surface is an important role in several engineering process. Some of the wide application areas we have polymer extrusion, cooling or drying of papers and in textile materials fabricated by extrusion and glass fiber production. P.S.Gupta and A.S.Gupta [1] examined that heat and mass transfer on a stretching sheet with suction or blowing. Heat transfer of a generalized stretching/shrinking wall problem with convective boundary conditions was investigated by Shanshan Yao et.al.[2]. Liancun Zheng et.al. [3] studied analytic solutions of unsteady boundary flow and heat transfer on a permeable stretching sheet with non-uniform heat source/sink. Numerical study of entropy generation for forced convection flow and heat transfer of a Jeffrey fluid over a stretching sheet was investigated by Nemat Dalir[4]. Exact analytical solutions for the flow and heat transfer near the stagnation point on a stretching/shrinking sheet in a Jeffrey fluid was studied by M. Turkyilmazoglu and I. Pop [5].

The study of magnetohydrodynamic has number of applications in engineering, agriculture and petroleum industries. The problem of natural convection under the effect of a magnetic field has also applications in geophysics and astrophysics. Hamad [6] studied analytical solution of natural convection flow of a nanofluid over a linearly stretching sheet in the presence of magnetic field. MHD stagnation-point flow and heat transfer towards stretching sheet with induced magnetic field was introduced by F. M. Ali et.al[7]. MHD flow of a dusty fluid near the stagnation point over a permeable stretching sheet with non-uniform source/sink was derived by G.K. Ramesh et.al[8]. N. S. Akbar et.al[9] analyzed dual solutions in MHD stagnation-point flow of Prandtl fluid impinging on shrinking sheet. Behrouz Raftari and K.Vajravelu [10] addressed homotopy analysis method for MHD viscoelastic fluid flow and heat transfer in a channel with a stretching wall.

Nanofluids are categorized as a new class of nanotechnology based fluids which attract the attention of researchers and scientists due to enhance physical properties, especially in heat transfer. Nanofluid is a mixture of nanoparticles and the base fluid. Choi et.al. [11] first introduced the word nanofluid. Nanofluids are used to enhance the rate of heat transfer of microchips in computers, microelectronics, transportation, fuel cells, food processing, biomedicine, solid state lightening, and manufacturing. Almost of the liquids such as water, glycol, oil, ethylene, and they have low thermal conductivity. Umar Khan et al [12] analyzed MHD stagnation point flow towards a stretching sheet in nanofluid effects of Thermodiffusion using Runge-Kutta-Fehlberg method. Rana and Bhargava [13] investigated flow and heat transfer of a nanofluid over a nonlinearly stretching sheet using finite element and finite difference method. M. Govindaraju et.al.[14] addressed entropy generation analysis of magneto hydrodynamic flow of a nanofluid over a stretching sheet. Noreen Sher Akbar et.al.[15] derived a numerical study of magnetohydrodynamic transport of nanofluids over a vertical stretching sheet with exponential temperature-dependent viscosity and buoyancy effects. Lattice Boltzmann method for MHD natural convection heat transfer using nanofluid was studied by M. Sheikholeslami et.al.[16]. Mohsen Sheikholeslami and Davood Domiri Ganji[17] introduced nanofluid flow and heat transfer between parallel plates considering Brownian motion using DTM. Tasawar Hayat et.al.[18] analyzed viscous dissipation effect in flow of magnetonanofluid with variable properties. S. A. Shehzad et.al. [19] studied boundary layer flow of third grade nanofluid with Newtonian heating and viscous dissipation.

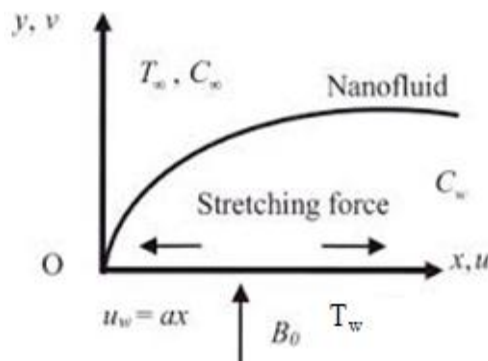
Thermal radiation has an important role in the overall surface heat transfer when the convection heat transfer coefficient is small. Several engineering processes occur due to high temperature, therefore, the study of radiation heat transfer plays an important role in the field of equipment designing. MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium was studied by Swati Mukhopadhyay[20]. Mohsen Sheikholeslami et.al [21] derived effect of thermal radiation on magneto hydrodynamics nanofluid flow and heat transfer by means of two phase model. M.M. Rashidi et.al. [22] studied buoyancy effect on MHD flow of nanofluid over a stretching sheet in the presence of thermal radiation. O.D. Makinde et.al.[23] introduced MHD variable viscosity reacting flow over a

convectively heated plate in a porous medium with thermophoresis and radiative heat transfer. Mohammad Mehdi Rashidi et.al.[24] analyzed free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation and buoyancy effects. Rafael Cortell[25] addressed that MHD (magneto-hydrodynamic) flow and radiative nonlinear heat transfer of a viscoelastic fluid over a stretching sheet with heat generation/absorption. Shravani et. al[26]investigated on the MHD boundary layer flow of nanofluids using Keller box method.

Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation was studied by Swati Mukhopadhyay [27].Wubshet Ibrahim and Bandari Shankar [28] derived MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions. T. Hayat et.al. [29] analyzed hall and ion slip effects on peristaltic flow of Jeffrey nanofluid with Joule heating . Jawad Raza et.al. [30] studied heat and mass transfer analysis of MHD nanofluid flow in a rotating channel with slip effects. M. Ramzan et.al [31] derived radiative and Joule heating effects in the MHD flow of a micropolar fluid with partial slip and convective boundary condition. Partial slip effect in flow of magnetite-Fe<sub>3</sub>O<sub>4</sub> nanoparticles between rotating stretchable disks was studied by Tasawar Hayat et.al.[32] . Aminreza Noghrehabadi et.al. [33] introduced effect of partial slip boundary condition on the flow and heat transfer of nanofluids past stretching sheet prescribed constant wall temperature. Ramya et.al[34], Ramya et.al[35] investigated on slip effect of nonlinear stretching sheet problems using nanofluids.

The objective of the present paper is therefore to extend the work of Meisam HABIBI MATIN and Pouyan JAHANGIRI [36] by taking steady MHD boundary-layer flow with permeable stretching sheet using radiative heat transfer in energy equation and slip condition at boundary. The problem is formulated and the similarity transformation is used to transform the partial differential equation in to a non linear ordinary differential equation and using RK method.

## 2. MATHEMATICAL FORMULATION:



**Figure 1.** Physical model and coordinate system.

We consider a steady state, two-dimensional laminar slip flow of an incompressible viscous fluid over a permeable stretching sheet with the surface temperature  $T_w$  and concentration  $C_w$ . Let  $T_\infty$  and  $C_\infty$  are the ambient temperature and concentration respectively. The stretching velocity of the sheet is assumed as  $u_w=ax$ , where  $a$  is a constant. The flow is subjected to the combined effects of transverse magnetic field of strength  $B_0$ , viscous dissipation and thermal radiation. The induced magnetic field is neglected for a small magnetic Reynolds number. The flow is assumed to be in the  $x$ -direction, which is chosen along the sheet and the  $y$ -axis perpendicular to it. Let  $u$  and  $v$  are the fluid tangential velocity and normal velocity, respectively.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \nabla^2 T + \frac{(\rho c)_p}{(\rho c)_{nf}} \left\{ D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right\} + \frac{\mu_f}{(\rho c)_{nf}} \left( \frac{\partial u}{\partial y} \right)^2 - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y}, \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \left( \frac{D_T}{T_\infty} \right) \frac{\partial^2 T}{\partial y^2} \quad (4)$$

Here  $u$  and  $v$  are the velocity components along the  $x$  and  $y$  directions, respectively,  $\rho_f$  the density of the base fluid,  $\alpha_m$  the thermal diffusivity,  $\nu$  the kinematic viscosity,  $a$  a positive constant,  $D_B$  the Brownian diffusion coefficient,  $D_T$  the thermophoretic diffusion coefficient,

$\tau = \frac{(\rho c)_p}{(\rho c)_f}$  the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid,  $c$  the volumetric volume expansion coefficient, and  $\rho$  the density of the particles.

Using Rosseland approximation for radiation we can write

$$q_r = -\frac{4\sigma^* \partial T^4}{3k^* \partial y} \quad (5)$$

Where  $\sigma^*$  is the Stefan–Boltzman constant,  $k^*$  is the absorption coefficient. Assuming that the temperature difference within the flow is such that  $T^4$  may be expanded in a Taylor series and expanding  $T^4$  about  $T_\infty$ , the free stream temperature and neglecting higher orders we get  $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$ .

We introduce subjective boundary conditions are

$$\begin{aligned} u &= u_w + K_1 \frac{\partial u}{\partial y}, v = v_w(x), T = T_w, C = C_w \text{ as } y = 0 \\ u &= ax, T \rightarrow T_\infty, \theta \rightarrow \theta_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \end{aligned} \quad (6)$$

Where  $K_1$  the velocity slip parameter, Where  $v_w$  is the variable velocity components in vertical direction at the stretching surface in which  $v_w < 0$  represents to the suction cases and  $v_w > 0$  represents to the injection ones. Following the similarity

transformation approach, we introduce the following new variables to transform the governing equations into the ordinary differential equations:

$$\psi = u_w x (Re_x)^{-0.5} f(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}, \eta = \frac{y}{x} (Re_x)^{0.5} \quad (7)$$

Where the stream function  $\psi(x, y)$  is defined as:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad (8)$$

Using the variables defined by eq. (7), the momentum equation, eq. (2), the energy equation, eq. (3), and the equation for the solid volume fraction of nanofluid, eq. (4) may be rewritten as:

$$f''' + ff'' - f'^2 - Mf' = 0 \quad (9)$$

$$(1 + \frac{4}{3}R) \theta'' + Pr(f\theta' + Nb\phi'\theta' + Nt\theta'^2 + Ec f'^2) = 0 \quad (10)$$

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta'' = 0 \quad (11)$$

with the transformed boundary condition

$$f(0) = S, f'(0) = 1 + \lambda f''(0), \theta(0) = 1, \phi(0) = 1 \quad (12)$$

$f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \phi(\eta) \rightarrow 0$  as  $\eta \rightarrow \infty$

where prime denotes differentiation with respect to  $\eta$ .  $S$  is the suction-injection parameter ( $S > 0$  for suction,  $S < 0$  for injection) and the dimensionless parameters  $Nb, Nt, Pr, Le, Ec, M, S, Re, R, \lambda$ , are the Brownian motion parameter, the thermophoresis parameter, Prandtl, Lewis, Eckert, the magnetic parameter, the suction-injection parameter, Reynolds, Thermal radiation parameter, velocity slip parameter, respectively. These parameters are defined as follows:

$$Nb = \frac{\rho c_p D_B (\phi_w - \phi_\infty)}{\rho c_f V}, \quad Nt = \frac{\rho c_p D_T (T_w - T_\infty)}{\rho c_f V T_\infty}, \quad Pr = \frac{V}{\alpha}, \quad Le = \frac{V}{D_B}, \quad Ec = \frac{u_\infty^2}{c_f \Delta T},$$

$$M = \frac{\sigma B_0^2 x}{\rho u_\infty}$$

$$S = -v_w(x) \sqrt{\frac{2x}{\nu u_\infty}}, \quad Re = \frac{u_w}{\nu} x, \quad R = \frac{4\sigma^* T_\infty^3}{KK^*}, \quad \lambda = K_1 \nu Re^{1/2} x^{-1/4} \quad (13)$$

The physical quantities of interest are the skinfrictions  $C_f$  and the local Nusselt number  $Nu_x$ , which are defined as

$$C_f = \frac{2\tau_w}{\rho u_w^2}, \quad Nu_x = \frac{xq_w}{K(T_w - T_\infty)}, \quad Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)} \quad (14)$$

Where  $\tau_w$  is the surface shear stress,  $q_w$  is the surface heat flux and  $q_m$  is the surface mass flux.

### 3. RESULTS AND DISCUSSIONS

In this section, the numerical results calculated for the velocity, temperature and concentration profiles are presented through graphs and tables. The computations are performed to study the effects of variation of magnetic parameter (M), Prandtl number(Pr), Eckert number(Ec), Brownian motion(Nb), thermophoresis parameters (Nt), Lewis number (Le), velocity slip parameter ( $\lambda$ ), suction and injection parameter(S), thermal radiation (R) on velocity, temperature and concentration profiles for water nanofluids. To calculate the present solution, comparisons have been made with previously published data in the literature for heat transfer ( $-\theta'(0)$ ) and mass transfer( $-\phi'(0)$ ) in Tables 1 and 2 and we found to be an excellent agreement with previous work. Effects of radiation and velocity slip parameters on skin friction factor, Nusselt and Sherwood numbers are presented in Table 3. From Table 3 it is clear that the Sherwood number is a decreasing function of increasing R and Sherwood number is found to be an increasing function of R.

**Table 1.** Comparison of Nusselt number and Sherwood number ( $-\theta'(0)$ ) and ( $-\phi'(0)$ ) when the values  $M = Ec = \lambda = 0$ .

Pr	Wang[37]	Khan&pop[38]	Mabood[39]	Matin&Jahangiri[36]	Present results
0.07	0.0656	0.0663	0.0665	0.0656	0.0656
0.2	0.1691	0.1691	0.1691	0.1695	0.1691
0.7	0.4539	0.4539	0.4539	0.4543	0.4539
2	0.9114	0.9113	0.9114	0.9112	0.9116
7	1.8905	1.8954	1.8954	1.8955	1.8994
20	3.3539	3.3539	3.3539	3.3545	3.3815
70	6.4622	6.4621	6.4622	6.4721	6.6917

**Table 2.** Comparison of ( $-\theta'(0)$ ) and ( $-\phi'(0)$ ) when the values of  $Nb = Nt = 0.1$ ,  $Le = 10$ ,  $Pr = Ec = 1.0$

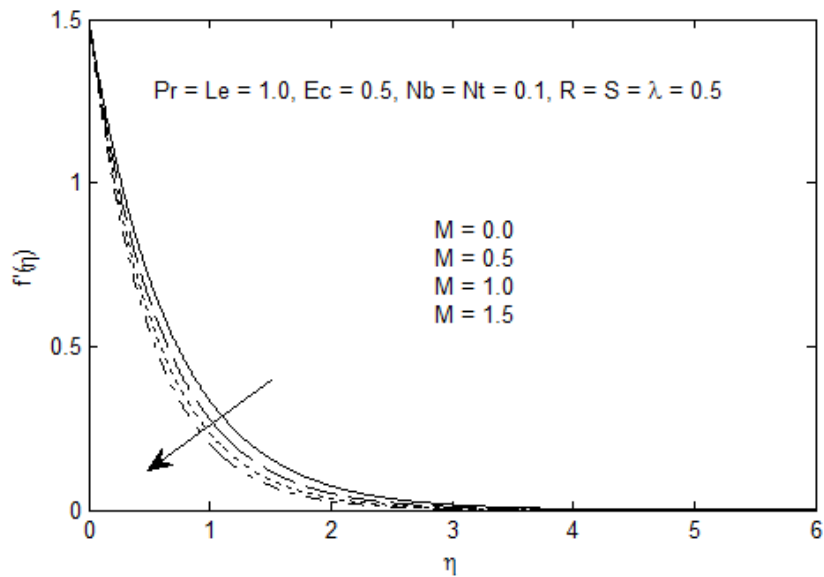
M	S	Matin&Jahangiri[36]		Present Results	
		$-\theta'(0)$	$-\phi'(0)$	$-\theta'(0)$	$-\phi'(0)$
0.0	-0.5	-0.0026	0.3422	-0.0028	0.3428
0.5		-0.1523	0.3503	-0.1529	0.3508
1.0		-0.2775	0.3643	-0.2783	0.3646
0.0	0	0.1185	2.4234	0.1184	2.4213
0.5		-0.0297	2.4618	-0.0299	2.4599
1.0		-0.1534	2.4995	-0.1539	2.4978



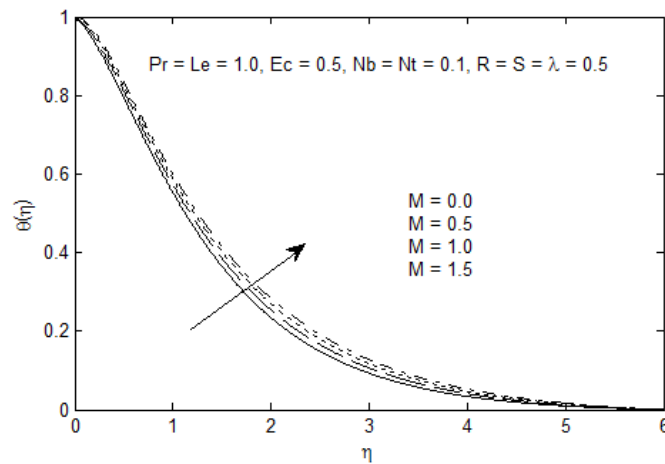
0.0	0.5	0.2800	6.1740	0.2800	6.1713
0.5		0.1471	6.2477	0.1470	6.2452
1.0		0.0353	6.3129	0.0352	6.3106

**Table 3.** Calculation of  $-f''(0)$ ,  $-\theta'(0)$  and  $-\phi'(0)$  when  $Nb = Nt = 0.1$ ,  $Le = Pr = Ec = 1.0$ ,  $M = 0.5$ ,  $S = 0.0$

R	$\lambda$	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.0	0.1	1.3914	-0.1282	0.8171
0.5		1.3914	-0.0244	0.7182
1.0		1.3914	-0.0227	0.6779
1.5		1.3914	-0.0507	0.6555
0.5	0.0	1.2248	0.0398	0.6317
	0.3	1.7442	-0.1876	0.9194
	0.5	2.1213	-0.3996	1.1608
	1.0	3.1623	-1.1626	1.9584

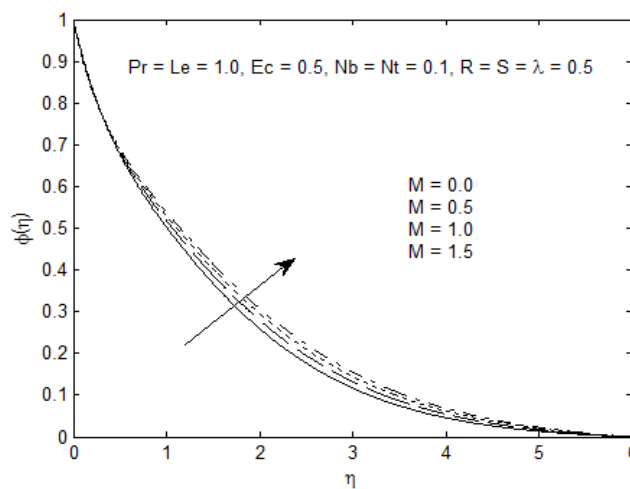


**Fig 2.** Effect of Magnetic parameter ( $M$ ) on velocity profile.

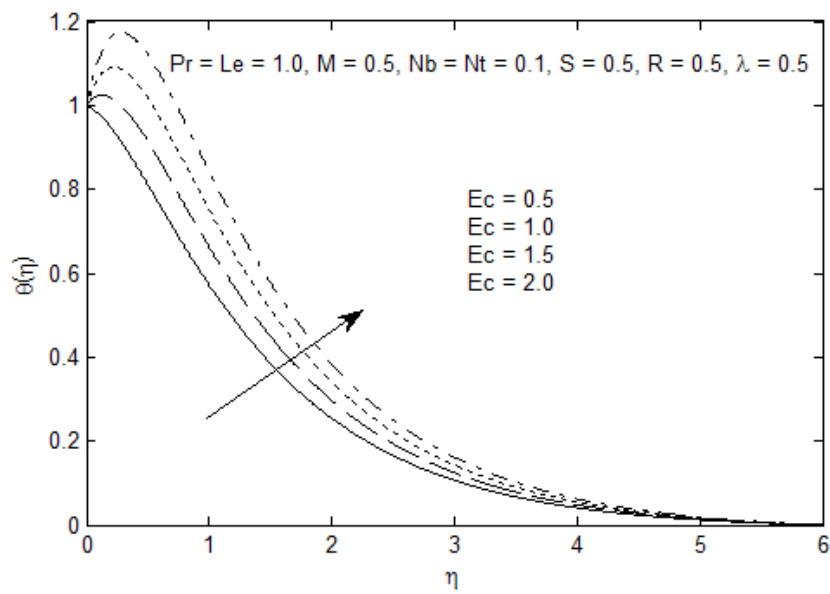


**Fig 3.** Effect of Magnetic parameter( $M$ ) on temperature.

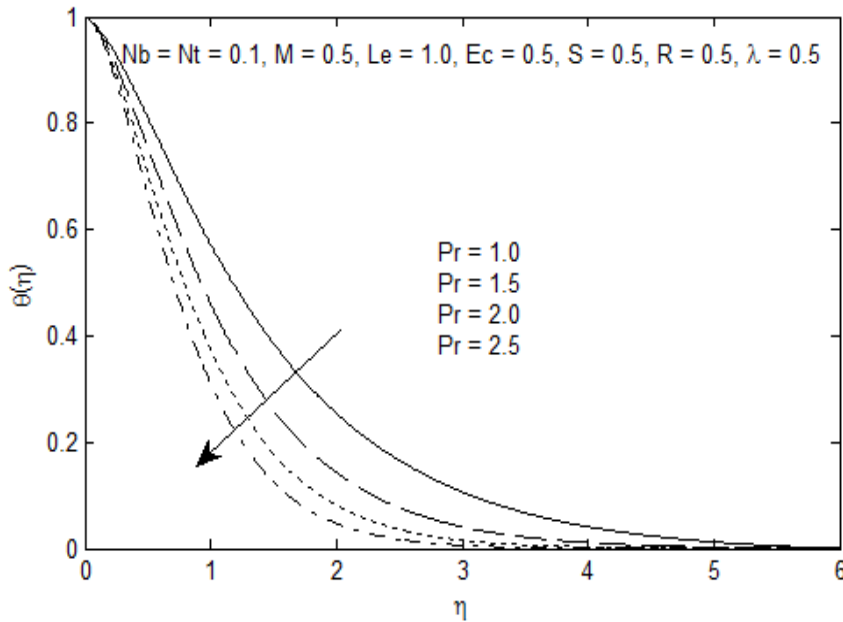
Fig.2 displays the effect of magnetic field ( $M$ ) on the velocity profiles. It is observed that the velocity of the fluid decreases as magnetic field increase. This is due to fact that retarding force was acted by Lorentz force. As the values of magnetic parameter  $M$  increase, the retarding body force enhances and consequently the velocity reduces. Figs.3 and 4 demonstrates the effect of magnetic field on temperature and concentration profiles. From these figures we observed that both the temperature and the concentration profiles demonstrated an increasing behavior for raising the values of  $M$ . Physical significance of this behavior is, As Lorentz force is a frictional resistive force opposes the fluid motion and then it is heat produced. According to result, the thermal boundary layer thickness and concentration boundary layer thickness become thicker for stronger magnetic field.



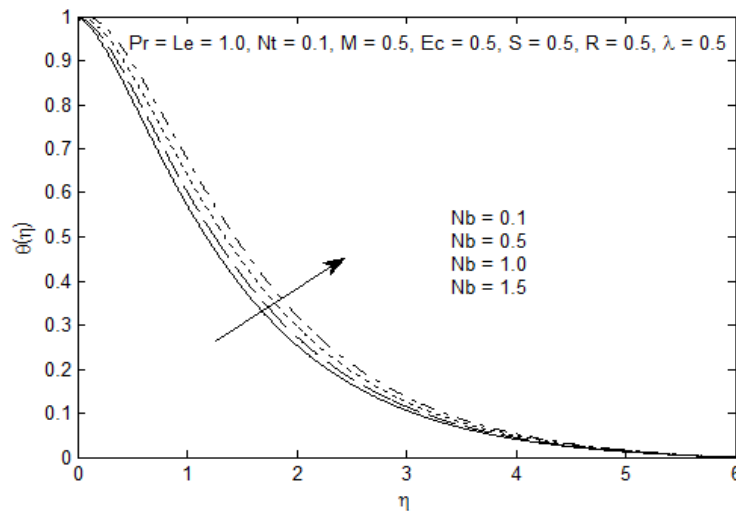
**Fig 4.** Effect of Magnetic parameter( $M$ ) on Concentration profiles.



**Fig 5.** Effect of Viscous dissipation ( $Ec$ ) on temperature profile.

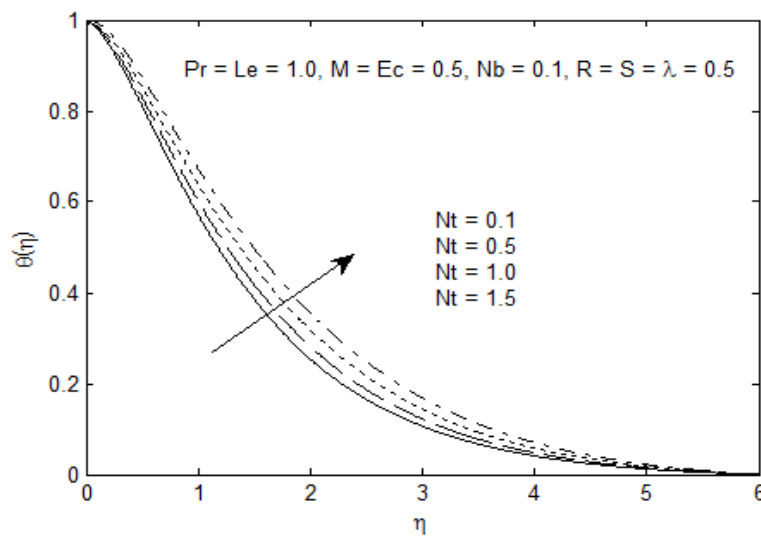


**Fig 6.** Effect of Prandtl number ( $Pr$ ) on temperature profile.

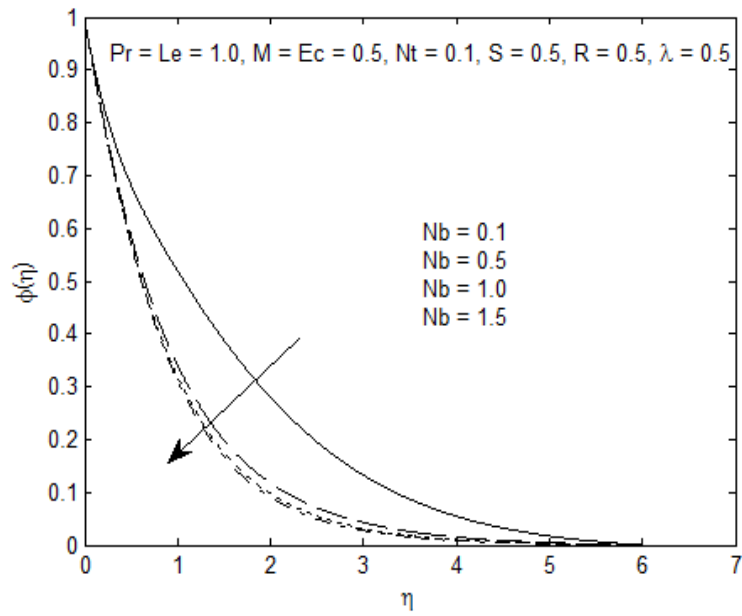


**Fig 7.** Effect of Brownian motion ( $Nb$ ) on temperature profile.

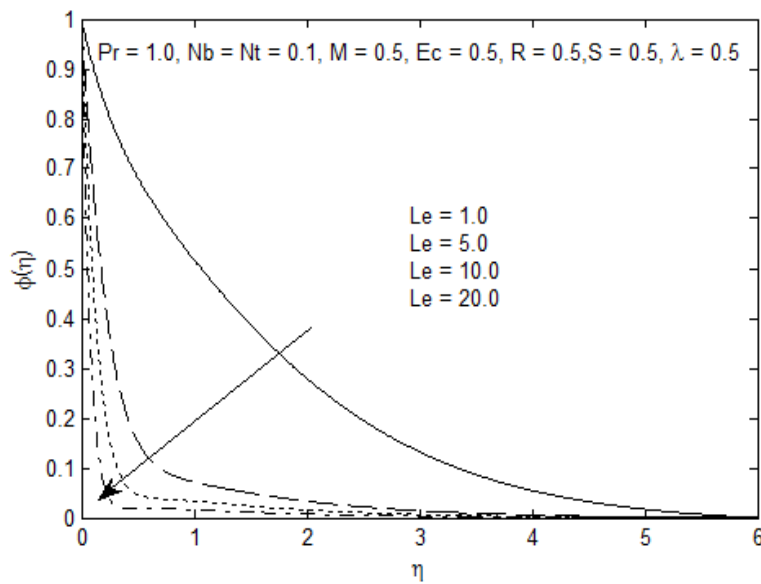
Fig 5 illustrates the influence of Eckert number  $Ec$  on temperature profile. Eckert number is defined as the ratio of kinetic energy to enthalpy. When the values of  $Ec$  increase, the thermal boundary layer thickness increases. Fig 6 indicates the effect of Prandtl number ( $Pr$ ) on temperature profile. The Prandtl number which is the ratio of the momentum diffusivity and the thermal diffusivity of base fluid on the dimensionless temperature distribution. It is clear that the effect of Prandtl number is to decelerate the temperature of the nanofluid, as it increases and also to decrease the thickness of thermal boundary layer. Thus the base fluid plays an important role in the heat transfer of the nanofluid.



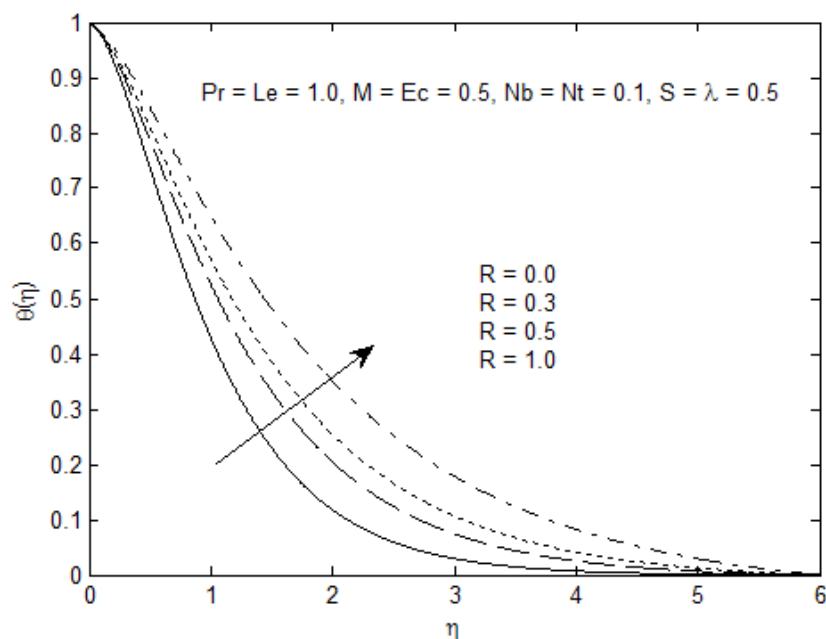
**Fig 8:** Effect of Thermophoresis ( $Nt$ ) on temperature profile.



**Fig 9:** Effect of Brownian motion ( $Nb$ ) on concentration profile.



**Fig 10:** Effect of Lewis number ( $Le$ ) on concentration profile.



**Fig 11:** Effect of Thermal Radiation ( $R$ ) on Temperature profile.

The effect of Brownian motion parameter  $Nb$  and thermophoresis parameter  $Nt$  on temperature profile for the fixed values of other parameters are shown in Figs. 7 and 8, respectively. It is observed that the temperature profile enhances with increasing values of Brownian motion parameter  $Nb$  and thermophoresis parameter  $Nt$ . This is due to the fact that the temperature gradient generates a thermophoretic force and it makes a fast flow away from the stretching sheet. In this way more heated fluid is moved away from the surface, and consequently, as  $Nt$  increases, the temperature within the boundary layer increases. As expected, an increase in the Brownian motion parameter  $Nb$  thickens the thermal boundary layer. Fig. 9 shows the effect of Brownian motion parameter  $Nb$  on concentration profile. The nanoparticle concentration profile decreases with the increase in the Brownian motion parameter  $Nb$ . Brownian motion attends to warm the boundary layer and simultaneously exasperates particle deposition away from the fluid regime or onto the surface, thereby describing for the reduced concentration magnitudes. For large particles and clearly Brownian motion does maintain a significant enhancing influence on concentration profiles. The converse is the case, for minor particles and Brownian motion is strong and the parameter  $Nb$  will have high values.

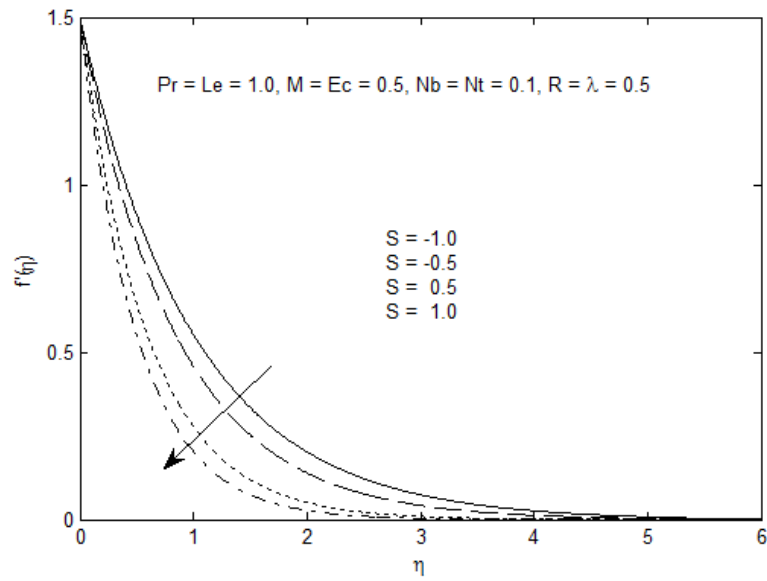


Fig 12: Effect of Suction Parameter( $S$ ) on velocity profile.

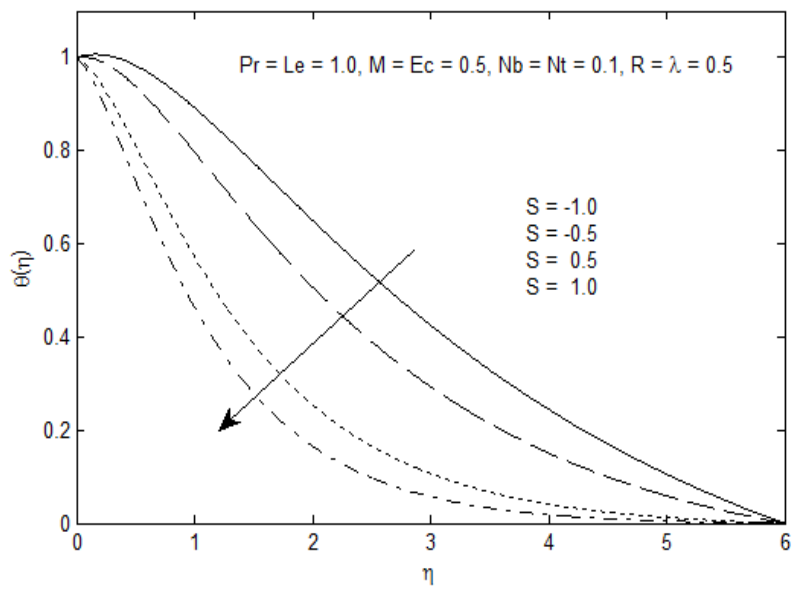
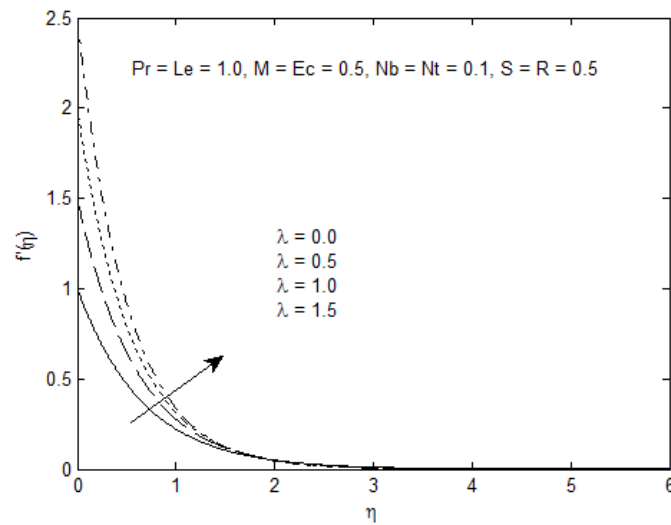
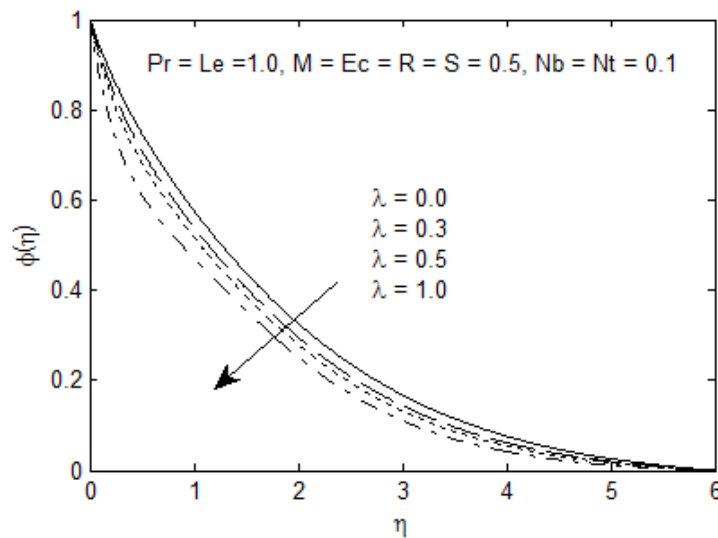


Fig 13: Effect of Suction parameter ( $S$ ) on temperature profile.



**Fig 14:** Effect of Velocity Slip Parameter ( $\lambda$ ) on velocity profile.



**Fig 15:** Effect of Velocity Slip Parameter ( $\lambda$ ) on concentration profile

Fig 10 displays the effect of Lewis number ( $Le$ ) on the nanoparticle volume fraction profiles. It is observed that the concentration profiles decrease as  $Le$  increase. Fig. 11 shows the temperature profiles for the different time step and different values of thermal radiation parameter. It can be observed that the increasing value of  $R$  caused the rising effects on temperature profile. As the value of thermal radiation  $R$  increases, temperature increase. Moreover, temperature gradient and thermal boundary layer thickness decrease. Fig. 12 and 13 demonstrates the effect of the suction or injection parameter on the velocity and temperature profiles. On observing these figures, as the values of 'S' increase, both the velocity and



temperature profiles decreases. Fig. 14 shows the effect of velocity slip parameter ( $\lambda$ ) on velocity profile. Fluid velocity was found to be increase with increasing values of  $\lambda$ . With the increase in  $\lambda$ , such slip velocity increases and consequently fluid velocity increases because under the slip condition, the pulling of the stretching sheet can be only partly transmitted to the fluid. Fig. 15 displays the effect of velocity slip parameter ( $\lambda$ ) on concentration profile. It is observed that the concentration profile reduces with increasing values of velocity slip parameter.

#### 4. CONCLUSIONS

The Magnetohydrodynamics boundary layer viscous flow and heat transfer of a nanofluid over a permeable stretching sheet with the effect of thermal radiation and slip condition have been studied numerically. The influence of the governing parameters: magnetic parameter ( $M$ ), Eckert number ( $Ec$ ), Lewis number ( $Le$ ), Brownian motion parameter ( $Nb$ ), Thermophoresis parameter ( $Nt$ ), Thermal radiation ( $R$ ), Velocity slip parameter( $\lambda$ ) on velocity, temperature and concentration profiles. It is observed that the present results are equalized with the previous work done by Matin&Jahangiri [36]. The results are as follows:

1. When the magnetic parameter  $M$  increases, then it reduces the velocity profiles and enhances the temperature and concentration profiles due to the effect of Lorentz force.
2. As the value of  $Ec$  increases, the temperature profiles increases.
3. As the value of  $Pr$  increases, the temperature profiles decreases.
4. By increasing the Brownian motion parameter  $Nb$ , the temperature increases and the concentration decreases.
5. By increasing the values of the thermophoresis Parameter  $Nt$ , the temperature distribution increase. This is due to the fact that thermophoretic is produced by the temperature gradient. In this case the fluid is more heated and passes away from the stretching sheet.
6. As  $Le$  increases, the concentration profiles decreases.
7. As the value of  $S$  increases, the velocity and temperature profile decreases
8. By increasing the Radiation parameter ( $R$ ), the temperature increases.
9. When the velocity slip parameter increases, then it enhances the velocity profile and reduces the concentration profiles.

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