Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 7 (2017), pp. 3033-3047

http://www.ripublication.com

Soft Minimal Continuous and Soft Maximal Continuous Maps in Soft Topological Spaces

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Abstract

In the paper we present a new class of soft space known as soft T_{min} space and soft T_{max} space and also a new class of soft maps known as soft minimal continuous and soft maximal continuous maps in soft topological spaces. The soft mapping g: A → B is soft minimal continuous if $g^{-1}((M,P))$ be a soft open set on A for every soft minimal open set (M, P) in B. The soft mapping g: A → B is soft maximal continuous if $g^{-1}((M, P))$ be a soft open set on A for every soft maximal open set (M, P) in B. Likewise few properties have been explored.

Keywords: Soft T_{min} space, soft T_{max} space, soft minimal continuous maps and soft maximal continuous maps.

2010 Mathematics classification: 54C05

1. INTRODUCTION AND PRELIMINARIES

In year 2001 and 2003, F.Nakaoka and N.Oda[1,2,3] presented and contemplated minimal open (resp. minimal closed) sets and maximal open (resp. maximal closed) sets, which are subclasses of open (resp. closed) sets. The complements of minimal open sets and maximal open sets are known as maximal closed sets and minimal closed sets respectively.

In year 1999, Russian specialist Molodtsov [4], started the idea on soft sets as another scientific instrument to manage vulnerabilities while demonstrating issues in building material science, software engineering, financial aspects, sociologies and restorative sciences. In Molodtsov [5], connected effectively in bearings, for example, smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability and hypothesis to estimation.The soft set is an accumulation of inexact portrayals of an article. Likewise he also indicated why soft set hypothesis is excluded from the parametrization deficiency disorder of fuzzy set hypothesis, rough set hypothesis, probability theory and game theory.

In year 2002 and 2003, Maji, Biswas and Roy [6], introduce few new statements on soft sets and displayed first practical use of soft sets in decision making problems that depends on the lessening of parameters to keep the ideal decision objects. In 2003, Maji, Biswas and Roy [7], examined the hypothesis of the soft sets started from Molodtsov. They characterized equity of two soft sets, subset and super set of the soft set, complement of the soft set, null soft set and absolute soft set along cases. The Soft binary operations like AND, OR furthermore the operations of union and the intersection were also characterized. In year 2005, D. Chen[8], introduced another meaning of the soft set parametrization lessening and correlation with property decrease on soft set hypothesis. In 2005 year, D. Pie, D. Miao [9], examined the difference between soft sets and data frameworks. They demonstrated soft sets are a class of unique data frameworks. In 2008, Z. Kong, L. Gao, L. Wong, S. Li [10], presented the thought of ordinary parameter decrease of soft sets and its utilization to explore the issue of imperfect decision and included a parameter set in soft sets.

As of late, specialists have contributed a lot towards fuzzification of Soft Set Theory. In 2001, Maji P. K., Biswas R and Roy A.R. [11], presented the idea of Fuzzy Soft Set and a few properties with respect to fuzzy soft union, intersection, supplement of a fuzzy soft set, De Morgan Law and so forth. In 2007, X. Yang, D. Yu, J. Yang, C. Wu [12], consolidated the interim esteemed fuzzy set and soft set models and presented the idea of interim esteemed fuzzy soft set.

Topological of soft set and fuzzy soft set of topological structures are studied by a few creators as of late. In 2011, Muhammad Shabir and Munazza Naz and Naim Cagman et al. started the investigation of soft topology and soft topological spaces independtly.Muhammad Shabir and Munazza Naz [13], presented the thought of soft topological spaces which are characterized over an underlying universe with a settled arrangement of parameters and demonstrated that a soft topological space gives a parameterized group of topological spaces. They presented the meanings of soft open sets, soft closed sets, soft interior, soft closure and soft separation axioms. Additionally they got few fascinating results for soft separation axioms, which are truly profitable for exploration in this field. N. Cagman, S. Karatas and S. Enginoglu [14], characterized the soft topology on a soft set, and displayed its related properties and establishments of the hypothesis of soft topological spaces. The thought of soft topology by Naim Cagman et al. is broad than that by Shabir and Naz.

In the meantime, Abdulkadir Aygunoglu and Halis Aygun [15], presented soft topological spaces and soft continuity of soft mappings. They additionally explored starting soft topologies and soft compactness.In 2011, Sabir Hussain and Bashir Ahmad [16], examined the properties of soft open (closed), soft neighborhood and soft closure. Likewise characterized and examined the properties of soft interior, soft exterior and soft boundary which are essential for further research on soft topology and establishments of the hypothesis of soft topological spaces. In 2012, Bashir Ahmad and Sabir Hussain [17], characterized soft exterior and examined its essential properties and set up a few critical results relating soft interior, soft exterior, soft closure, and soft boundary in soft topological spaces. In addition, they described soft open sets, soft closed sets and soft clopen defines by means of soft boundary. In 2007, H.Hazra, P. Majumdar and S.K.Samanta [18], presented the thoughts of topology on soft subsets and soft topology. Some essential properties of these topologies are studied.In 2004, Metin Akdag and Alkan Ozkan[19], presented and examined the idea of soft α-Open sets and soft α-constant functions. In 2015, A. Selvi and I. Arockiarani[20,21], presented and contemplated the idea of soft almost g-continuous functions. In 2014, Metin Akdag and Alkan Ozkan[22], presented and considered the idea of soft β-Open Sets and soft β-Continuous functions .In 2010,Pinaki Majumdar and S.k.Samanta,[23] presented and examined the idea of soft mappings in soft topological spaces.In 2011, S. S. Benchalli, Basavaraj M.I and R. S. Wali[24] presented the idea On Minimal Open Sets and Maps in Topological Spaces.In 2015, Hai-Long Yang , Xiuwu Liao and Sheng-Gang Li[25] presented the idea On soft continuous mappings and soft connectedness of soft topological spaces.

We review the accompanying statements, which are requirements for present study.

1.1 Definition [5]: Let I be an initial universe and P be the set of parameters. Let P(I) denote the power set of I and A be a non-empty subset of P. A pair (M, A) is called a soft set over I, where M is a mapping given by M: A → P(I).

1.2. Definition [6]: For two soft sets (M, A) and (N,B) over a common universe U, we say that (M, A) is a soft subset of (N, B) if

i) $A \subseteq B$ and

ii) for all $p \in A$, M (p) and N(p) are identical approximations.

We write (M, A) $\tilde{\subset}$ (N, B). (M, A) is said to be a soft super set of (N, B), if (N, B) is a soft subset of (M, A). We denote it by (M, A) $\tilde{\supset}$ (N, B).

1.3. Definition [6]: Two soft sets (M, A) and (N, B) over a common universe U are said to be soft equal if (M, A) is a soft subset of (N, B) and (N, B) is a soft subset of (M, A).

1.4. Definition [6]: Let P = $\{p_1, p_2, p_3 \ldots, p_n\}$ be a set of parameters. The NOT set of $\rceil P$ denoted by $\rceil P$ is defined by $\rceil P=\{p_1, p_2, p_3, \ldots p_n\}$, where $\rceil p_i$ = not p_i for all i.

1.5. Definition [6]: The complement of a soft set (M, A) is denoted by $(M,A)^c=(M^c, \rceil A)$ where, $M^c: \rceil A \to P(I)$ is a mapping given by $M^c(\alpha) = I \backslash M(\rceil\alpha)$, for all $\alpha \in \rceil A$

Let us call M^c to be the soft complement function of M. Clearly $(M^c)^c$ is the same as M and $((M, A)^c)^c = (M , A)$.

1.6. Definition [6]: A soft set (M, A) over I is said to be a NULL soft set denoted by "ϕ" if $\forall\ \varepsilon \in A, M(\varepsilon) = \phi$, (null-set).

1.7. Definition [6]: If (M, A) and (N, B) are two soft sets then (M, A) AND (N, B) denoted by $(M, A)\Lambda(N, B)$ is defined by $(M, A)\ \Lambda\ (N, B) = (H, A \times B)$, where $H((\alpha, \beta)) = M(\alpha) \cap N(\beta)$, for all $(\alpha, \beta) \cap A \times B$.

1.8. Definition [6]: If (M, A) and (N, B) are two soft sets then (M, A) OR (N, B) denoted by (M, A) V (N, B) is defined by $(M, A)\ V\ (N, B) = (O, A \times B)$ where, $O((\alpha, \beta)) = M(\alpha) \cap N(\beta)$ for all $(\alpha, \beta) \cap A \times B$.

1.9. Definition [6]: The union of two soft sets of (M, A) and (N, B) over the common universe I is the soft set (H, C), where $C = A \cup B$ and for all $p\ \varepsilon\ C$,

$$H(e) = \begin{cases} M(p) & \text{if } p \in A - B \\ N(p) & \text{if } p \in B - A \\ M(p) \cup N(p) & \text{if } p \in A \cap B. \end{cases}$$ We write $(M, A) \cup (N, B) = (H, C)$.

1.10. Definition [8]: The intersection (H, C) of two soft sets (M, A) and (N, B) over a common universe I, denoted $(M, A) \cap (N, B)$, is defined as $C=A\cap B$, and $H(p) = M(p) \cap N(p)$ for all $p \in C$

1.11. Definition [13]: Let τ be the collection of soft sets over A; then τ is called a soft topology on A if τ satisfies the following axioms:

i) Φ, A belong to τ.

ii) The union of any number of soft sets in τ belongs to τ.

iii) The intersection of any two soft sets in τ belongs to τ.

The triplet (A, τ, P) is called a soft topological space over A. The members of τ are said to be soft open in A. A soft set (M, P) over A is said to be soft closed in A if its relative complement $(M, P)^c$ belongs to τ .

1.12. Definition [13]: Let (M, P) be a soft set over A and a $\in$ A. We say that a $\in$ (M, P) read as 'a' belongs to the soft set (M, P), whenever a $\in$ M(α) for all $\alpha \in$ P.

Note that for a $\in$ A, a $\notin$ (M, P) if a $\notin$ M(α) for some $\alpha \in$ P.

1.13. Definition [13]: Let a $\in$ A; then (a, P) denotes the soft set over A for which a(α) = {a}, for all $\alpha \in$ P.

1.14. Definition [2]: Let (A, τ, P) be a soft topological space over A, (N, P) be a soft set over A and a $\in$ A. Then (N, P) is said to be a soft neighborhood of a if there exists a soft open set (M, P) such that a $\in$ (M, P) $\tilde{\subseteq}$ (N, P).

1.15. Definition [2]: Let (A, τ, P) be a soft topological space and (A, P) be a soft set over A.

i) The soft interior of (A, P) is the soft set sint(A, P) = $\cup$\{(O, P): (O, P) is soft open and (O, P) $\tilde{\subseteq}$ (A, P)\}.

ii) The soft closure of (A, P) is the soft set scl(A, P) = $\cap$\{(M, P) : (M, P) is soft closed and (A, P) $\tilde{\subseteq}$ (M, P)\}.

1.16 Definition [1]: A proper nonempty open subset I of a topological space A is said to be a minimal open set if any open set which is contained in I is ϕ or I.

1.17 Definition [2]: A proper nonempty open subset I of a topological space A is said to be maximal open set if any open set which contains I is A or I.

1.18 Definition [3]: A proper nonempty closed subset M of a topological space A is said to be a minimal closed set if any closed set which is contained in M is ϕ or M.

1.19 Definition [3]: A proper nonempty closed subset M of a topological space A is said to be maximal closed set if any closed set which contains M is A or M.

1.20 Definition [26]: A proper nonempty soft open subset (M, P) of (A, τ, P) is said to be a soft minimal open set if and only if any soft open set which is contained in (M, P) is either ϕ or (M, P) itself.

1.21 Definition [18]: A proper nonempty soft open subset (M,P) of (M,τ,P) is said to be a soft maximal open set if and only if any soft open set which is contains in (M,P) is either A or (M,P) itself.

1.22 Definition [22]: A soft set (M, P) of a soft topological space (A, τ, P) is called soft α-open set if (M, P)$\tilde{\subseteq}$ int(cl(int(M,P))). The complement of soft α-open set is called soft α-closed set.

1.23 Definition [22]: A soft set (M, P) is called soft preopen set [9] (resp., soft semiopen [5]) in a soft topological space X if (M, P)$\tilde{\subseteq}$ int(cl(M, P)) (resp., (M, P)$\tilde{\subseteq}$ cl(int(M, P))).

1.24 Definition [22]: A soft mapping g:A → B is said to be soft α- continuous if the inverse image of each soft open subset of B is a soft α-open set in A.

1.25 Definition[22]:A soft mapping g:A → B is called soft precontinuous(resp., soft semicontinuous) if the inverse image of each soft open set in B is soft preopen (resp., soft semiopen) in A.

1.26 Definition [20]: A function g:A → B is called soft almost open(Soft almost closed), if the image of every soft regular open subset of A is soft open(soft regular closed) subset of B.

1.27 Definition [22]: A subset (M, P) of a topological space A is called soft generalized-closed (soft g -closed), if cl(M,P) ⊂ (I,P) whenever (M,P) ⊂ (I,P) and (I,P) is soft open in A.

1.28 Definition [19]: A subset (M, P) of a topological space A is called soft regular closed, if cl(int(M,P)) = (M,P). The complement of soft regular closed set is soft regular open set.

1.29 Definition [19]: A soft set (M, P) of a soft topological space (A, τ, P) is said to be soft β-open if (M,P)$\tilde{\subset}$cl(int(cl(M,P))).

1.30 Definition [19]: Let A,B be two non-empty sets and P' be a parameter set.Then the mapping $M: P' \rightarrow U(B')$ is called a soft mapping from A to B under P',where B' is the collection of all mappings from A to B.

1.31 Definition[17]: A soft mapping g:A → B is called soft β-continuous (resp., soft α-continuous , soft precontinuous , and soft semicontinuous) if the inverse image of each soft open set in B is soft β-open (resp., soft α-open, soft preopen, and soft semiopen) set in A.

1.32 Definition[24]: Consider A and B are topological spaces.

The mapping g: A→B is known as

i) minimal continuous (min-continuous) if $g^{-1}(M)$ is an open set in A for each minimal open set M in B.

ii) maximal continuous (max-continuous) if $g^{-1}(M)$ is an open set in A for each maximal open set M in B.

1.33 Definition[25]: Let (A,τ_1,P) and (B,τ_2,P) be two soft topological spaces over A and B respectively,and g be a mapping from A to B.If $\forall(G,P)\in\tau_2$,we have mapping $f^{\leftarrow}(G,P)\in\tau_1$, then g is called a soft continuous mapping from (A,τ_1,P) to (B,τ_2,P).

2. SOFT MINIMAL CONTINUOUS AND SOFT MAXIMAL CONTINUOUS MAPS

2.1. Definition: The soft topological space (A,τ,P) is known as soft T_{min} space if for each nonempty proper soft open subset of A is soft minimal open set.

2.2. Definition: The soft topological space (A,τ,P) is known as soft T_{max} space if for each nonempty proper soft open subset of A is soft maximal open set.

2.3. Remark: The concepts of soft T_{min} and soft T_{max} spaces are not enough general. Soft T_{min} and soft T_{max} in soft topological space (A,τ,P) is either indiscrete space or $\{\phi,(M,P),A\}$ *or* $\{\phi,(M,P),(M,P)^c,A\}$.

2.4. Remark: The concepts of soft T_{min} and soft T_{max} spaces are identical.

2.5 Definition: Consider A and B are soft topological spaces.

The mapping g: A→B is known as

i) Soft minimal continuous (soft min-continuous) if $g^{-1}((M,P))$ is a soft open set in A for each soft minimal open set (M,P) in B.

ii)Soft maximal continuous (soft max-continuous) if $g^{-1}((M,P))$ is an soft open set in A for each soft maximal open set (M,P) in B.

2.6 Theorem: Each soft continuous map is soft minimal continuous but not conversely.

Proof: Consider g: A→B is a soft continuous map. To prove that g is soft minimal continuous. Consider (M, P) be soft minimal open set in B. Since each soft minimal open set be soft open set, (M, P) be soft open set in B. Since g is soft continuous, $g^{-1}((M, P))$ be soft open set in A. Hence *f* be a soft minimal continuous.

2.7 Example: Consider $A = B = \{a,b,c\}, P = \{p_1, p_2\}, \tau = \{A, \phi, (M_1, P), (M_2, P)\}$ inwhich $M_1(p_1) = \{a\}, M_1(p_2) = \{b\}$; $M_2(p_1) = \{a,b\}, M_2(p_2) = \{b,c\}$ and $\mu = \{B, \phi, (M_1, P), (M_2, P)\}$ inwhich

$M_1(p_1) = \{a\}, M_1(p_2) = \{b\}$; $M_2(p_1) = \{a,c\}, M_2(p_2) = \{b,c\}$.

Consider g:A→B is an identity map. Then g be soft minimal continuous but it is not a soft continuous map, since for the soft open set (M_2, P) in B, $g^{-1}(M_2, P) = (M_2, P)$ which is not soft open set in A.

2.8 Theorem: Consider g: A→B is soft minimal continuous, onto map and B is the soft T_{min} space. Then g be a soft continuous.

Proof: Consider g is the soft minimal continuous, onto map. Note that the inverse image of ϕ and B are always soft open sets in A. Consider (M,P) be any nonempty proper soft open set in B. By theory B is soft T_{min} space, it continues that (M,P) be a soft minimal open set in B. Since g be soft minimal continuous,$g^{-1}((M,P))$ be soft open in A. Hence g be soft continuous.

2.9 Theorem: Each soft continuous map is soft maximal continuous but not conversely.

Proof: Similar to that of Theorem 2.8.

2.10Example: Consider
$A = B = \{a,b,c\}, P = \{p_1, p_2\}, \tau = \{A, \phi, (M_1, P), (M_2, P)\} where$
$M_1(p_1) = \{a\}, M_1(p_2) = \{b\};\ M_2(p_1) = \{a,b\}, M_2(p_2) = \{b,c\}$ and
$\mu = \{B, \phi, (M_1, P), (M_2, P)\}$ inwhich

$M_1(p_1) = \{b\}, M_1(p_2) = \{a\};\ M_2(p_1) = \{a,b\}, M_2(p_2) = \{a,c\}$.

Consider g:A→B be soft identity map.Then g be a soft maximal continuous but it is not a soft continuous map, since the soft open set (M_1, P) in B, $g^{-1}((M_1, P))$ $= (M_1, P)$ is not soft open set in A.

2.11 Theorem: Consider g: A→B is a soft maximal continuous, onto map and consider B be a soft T_{max} space. Then g be a soft continuous.

Proof: Same to that of Theorem 2.8.

2.12 Remark: soft Minimal continuous and soft maximal continuous maps are independent.

2.13 Example: In 2.7 Example,g be soft minimal continuous but it is not soft maximal continuous. In 2.10 Example, g be soft maximal continuous but it is not a soft minimal continuous.

2.14 Remark: soft Minimal (resp. soft maximal) continuous and almost soft continuous maps are independent of each other.

2.15 Example: Consider $A = B = \{a,b,c\}, P = \{p_1, p_2\}, \tau_1 = \{A, \phi, (M_1, P), (M_2, P)\}$ in which

$M_1(p_1) = \{b\}, M_1(p_2) = \{a\};\ M_2(p_1) = \{a,c\}, M_2(p_2) = \{b,c\}$ and
$\tau_2 = \{B, \phi, (M_1, P), (M_2, P), (M_3, P)\}$in
which $M_1(p_1) = \{a\}, M_1(p_2) = \{c\};\ M_2(p_1) = \{b\}, M_2(p_2) = \{a\}$;
$M_3(p_1) = \{a,b\}, M_3(p_2) = \{a,c\}$ and $\tau_3 = \{C, \phi, (M_1, P), (M_2, P), (M_3, P)\}$in
which $M_1(p_1) = \{a\}, M_1(p_2) = \{c\};\ M_2(p_1) = \{a,b\}, M_2(p_2) = \{a,c\}$;
$F_3(e_1) = \{a,c\}, F_3(e_2) = \{b,c\}$ and

$\mu = \{A, \phi, (M_1, P), (M_2, P), (M_3, P), (M_4, P)\}$in which

$M_1(p_1) = \{a\}, M_1(p_2) = \{c\};\ M_2(p_1) = \{b\}, M_2(p_2) = \{a\}$;

$M_3(p_1) = \{a,b\}, F_3(p_2) = \{a,c\}\ M_3(p_1) = \{a,c\}, M_3(p_2) = \{b,c\}$.

Let g: (A, τ_1) $\rightarrow$ (B, μ) be an identity map. Then g is an almost soft continuous but it is not a soft minimal (resp. soft maximal) continuous, since the soft minimal open set (M_1, P) (resp. soft maximal open set (F_3, E)) in B, $g^{-1}((M_1,P))$ $=(M_1,P)$ (resp. $g^{-1}((M_3,P))=(M_3,P)$) which is not soft open set in A. Consider *h*: (A, τ_2) $\rightarrow$ (B,μ) is an identity map. Then *h* be the soft minimal continuous but it is not an almost soft continuous, since for the soft regular open set (M_4,P) in B, $h^{-1}((M_4,P))$ $=(M_4,P)$ which is not soft open set in A. Consider *i*: (A, τ_3) $\rightarrow$ (B,μ) is an identity map. Then *i* be the soft maximal continuous but it is not an almost soft continuous, since for the soft regular open set (M_2,P) in B, $i^{-1}((M_2,P))=(M_2,P)$ which is not soft open set in A.

2.16 Remark: i) Soft Minimal continuous and soft precontinuous (resp. Soft α-continuous, β-continuous) maps are independent of each other.

ii) Soft Maximal continuous and soft precontinuous (resp. soft α-continuous,β-continuous) maps are independent of each other.

2.17 Example: Consider $A=B=\{a,b,c\}, P=\{e_1,e_2\}, \tau=\{A,\phi,(M_1,P)\}$ in which

$M_1(p_1)=\{a\}, M_1(p_2)=\{b\}$; and

$\mu=\{A,\phi,(M_1,P)\}$ in which $M_1(p_1)=\{a,b\}, M_1(p_2)=\{b,c\}$;

Consider g : A$\rightarrow$B is the soft identity map. Then

i) g be the soft precontinuous (resp. soft α-continuous, β-continuous) but it is not a soft minimal continuous, since for the soft minimal open set (M_1,P) in B,$g^{-1}((M_1,P)$)= (M_1,P) which is not soft open set in A.

ii) g be the soft precontinuous (resp. soft α-continuous, β-continuous) but it is not a soft maximal continuous, since for the soft maximal open set (M_1,P) in B, $g^{-1}((M_1,P))$= (M_1,P) which is not soft open set in A.

2.18 Example:

Consider $A=B=\{a,b,c\}, P=\{p_1,p_2\}, \tau=\{A,\phi,(M_1,P),(M_2,P),(M_3,P),(M_4,P)\}$ in which

$M_1(p_1)=\{a\}, M_1(p_2)=\{c\}$; $M_2(p_1)=\{b\}, M_2(p_2)=\{a\}$;

$M_3(p_1)=\{a,b\}, M_3(p_2)=\{a,c\}$; $M_4(p_1)=\{a,c\}, M_4(p_2)=\{b,c\}$; and

$\mu_1=\{B,\phi,(M_1,P),(M_2,P)\}$ in which

$M_1(p_1)=\{b\}, M_1(p_2)=\{a\}$; $M_2(p_1)=\{b,c\}, M_2(p_2)=\{a,c\}$ and
$\mu_2=\{B,\phi,(M_1,P),(M_2,P)\}$ in which

$M_1(p_1)=\{c\}, M_1(p_2)=\{b\}$; $M_2(p_1)=\{a,c\}, M_2(p_2)=\{b,c\}$

i) Consider g: (A, τ) → (B, μ_1) is an identity map. Then g be the soft minimal continuous but it is not the soft precontinuous (resp. soft α-continuous, β-continuous) map, since for the soft open set (M_2,P) in B, $g^{-1}((M_2,P))=(M_2,P)$ which is not a soft preopen (resp. soft α-open, β-open) set in A.

ii) Consider *h*: (A,τ) → (B, μ_2) is the identity map. Then *g* is a soft maximal continuous but it is not the soft precontinuous (resp. soft α-continuous, β-continuous) map, since for the soft open set (M_1,P) in B, $h^{-1}((M_1,P))$ = (M_1,P) which is not a soft preopen (resp. soft α-open, β-open) set in A.

2.19 Remark: soft Minimal (resp. soft maximal) continuous and soft g-continuous maps are independent of each other.

2.20 Example:

Consider $A=B=\{a,b,c\}, P=\{p_1,p_2\}, \tau=\{A,\phi,(M_1,P),(M_2,P),(M_3,P)\}$

in which

$M_1(p_1)=\{a\}, M_1(p_2)=\{c\}$; $M_2(p_1)=\{b\}, M_2(p_2)=\{a\}$;
$M_3(p_1)=\{a,b\}, M_3(p_2)=\{a,c\}$; and $\mu_1=\{B,\phi,(M_1,P),(M_2,P)\}$ in which

$M_1(p_1)=\{a\}, M_1(p_2)=\{b\}$; $M_2(p_1)=\{a,c\}, M_2(p_2)=\{a,b\}$ and

$\mu_2=\{B,\phi,(M_1,P),(M_2,P)\}$ in which

$M_1(p_1)=\{c\}, M_1(p_2)=\{b\}$; $M_2(p_1)=\{a,c\}, M_2(p_2)=\{b,c\}$ .Consider g: (A, τ) → (B, μ_1) is an identity map. Then g be the soft minimal continuous but it is not a soft g-continuous(soft generalized continuous), since for the soft open set (M_2,P) in B, $g^{-1}((M_2,P))=(M_2,P)$ which is not a soft g-open set in A. Consider *h*: (A, τ) → (B, μ_2) be a soft map defined by *g*({a})=*g*({b})={a} and *g*({c})={c}. Then *h* is a soft maximal continuous but it is not a soft g-continuous, since for the soft open set (M_1,P) in B, $h^{-1}((M_1,P))=(M_1,P)$ which is not a soft g-open set in A. In Example 2.13, g is a soft g-continuous but it is not a soft minimal (resp. soft maximal) continuous.

2.22 Theorem: Consider A and B be the soft topological spaces. The map g: A→B is soft minimal continuous if and only if the inverse image of each soft maximal closed set in B is a soft closed set in A.

Proof: The proof follows from the definition and fact that the complement of soft minimal open set is soft maximal closed set.

2.23 Theorem: Consider A and B be the soft topological spaces and (D, P) be a nonempty soft subset of A. If g: A→B is soft minimal continuous then the restriction map g_D: (D,P)→B is a soft minimal continuous.

Proof: Consider g:A→B is soft minimal continuous map. To prove g_D: (D, P)→B is soft minimal continuous. Consider (N, P) be any soft minimal open set in B. Since g is soft minimal continuous, g^{-1}(N, P) is soft open set in A. But $g_D{}^{-1}$(N, P) =D$\cap f^{-1}$(N, P) and D$\cap f^{-1}$(N, P) is soft open set in D. Therefore g_D is a soft minimal continuous.

2.24 Remark: The composition of soft minimal continuous maps need not be a soft minimal continuous map.

2.25 Example: Consider $A = B = \{a,b,c,d\}, P = \{p_1, p_2\}, \tau = \{A, \phi, (M_1, P), (M_2, P)\}$ in which

$M_1(p_1) = \{a\}, M_1(p_2) = \{b\}$; $M_2(p_1) = \{a,c\}, M_2(p_2) = \{b,a\}$ and
$\mu = \{B, \phi, (M_1, P), (M_2, P)\}$in which

$M_1(p_1) = \{a\}, M_1(p_2) = \{b\}$; $M_2(p_1) = \{a,b\}, M_2(p_2) = \{b,c\}$
$M_3(p_1) = \{a,b,c\}, M_3(p_2) = \{b,c,d\}$ and

$\eta = \{C, \phi, (M_1, P), (M_2, P)\}$in which

$M_1(p_1) = \{a,b\}, M_1(p_2) = \{c,d\}$; $M_2(p_1) = \{a,b,c\}, M_2(p_2) = \{b,c,d\}$.

Consider g: A→B and *h*: B→C be the identity maps. Then g and *h* are soft minimal continuous maps but *h*og: A→C is not a soft minimal continuous, since for the soft minimal open set (M_1, P)in C, $(h\text{og})^{-1}((M_1, P)) = (M_1, P)$ which is not a soft open set in A.

2.26 Theorem: Let g: A→B be soft continuous map and *h*: B→C is soft minimal continuous maps. Then *h*og: A→C is a soft minimal continuous.

Proof: Let (N, P) be any soft minimal open set in C. Since *h* is soft minimal continuous, h^{-1}(N, P) be soft open set in B. Again since g is soft continuous,

$g^{-1}(h^{-1}(N,P)) = (hog)^{-1}(N,P)$ is soft open set in A. Hence *h*og be the soft minimal continuous.

2.27 Theorem: Consider A and B be the soft topological spaces.A map g: A→B is soft maximal continuous if and only if the inverse image of each soft minimal closed set in B is a soft closed set in A.

Proof: The proof follows from the definition and fact that the complement of soft maximal open set is soft minimal closed set.

2.28 Theorem: Consider A and B be the soft topological spaces and let (D,P) be a nonempty soft subset of A. If g: A→B is soft maximal continuous then the restriction map g_D:(D,P)→B is a soft maximal continuous.

Proof: Similar to that of Theorem 2.23.

2.29 Remark: The composition of soft maximal continuous maps need not be a soft maximal continuous map.

2.30 Example: Consider $A = B = \{a,b,c,d\}, P = \{p_1, p_2\}, \tau = \{A, \phi, (M_1,P), (M_2,P)\}$ in which $M_1(p_1) = \{a\}, M_1(p_2) = \{b\}$; $M_2(p_1) = \{a,b,c\}, M_2(p_2) = \{b,c,d\}$ and $\mu = \{B, \phi, (M_1,P), (M_2,P), (M_3,P)\}$ in which $M_1(p_1) = \{a\}, M_1(p_2) = \{b\}$; $M_2(p_1) = \{a,b\}, M_2(p_2) = \{b,c\}$; $M_3(p_1) = \{a,b,c\}, M_3(p_2) = \{b,c,d\}$, $\eta = \{C, \phi, (M_1,P), (M_2,P)\}$ in which $M_1(p_1) = \{a\}, M_1(p_2) = \{b\}$; $M_2(p_1) = \{a,b\}, M_2(p_2) = \{b,c\}$.Consider g: A→B and *h*: B→C be the soft identity maps. Then g and *h* are soft maximal continuous maps but *h*og: A→C is not a soft maximal continuous, since for the soft maximal open set (M_2,P) in C, $(goh)^{-1}((M_2,P)) = (M_2,P)$ which is not soft open set in A.

2.31 Theorem: Let g: A→B is soft continuous map and *h*: B→C is soft maximal continuous maps, then *h*og: A→C is a soft maximal continuous.

Proof: Similar to that of Theorem 2.26.

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