MHD Boundary Layer Flow of a VISCO-Elastic Fluid Past a Porous Plate with Varying Suction and Heat Source/Sink in the Presence of Thermal Radiation and Diffusion

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Abstract

This manuscript consists of the properties of natural convective flow of a viscous incompressible electrically conducting fluid past a vertical porous plate bounded by a porous medium in the presence of thermal radiation and variable permeability. A magnetic field of uniform strength is applied perpendicular to the plate and the presence of heat source is also considered. The novelty of the study is to investigate the effects of thermal radiation and Eckert number. The coupled dimensionless non-linear partial differential equations are solved by finite difference method. The numerical computations have been studied through graphs. The presence of thermal radiation decreases the temperature and an opposite nature is shown in the case of Eckert number. The influence of neat source leads to enhance the temperature.

Keywords: MHD, thermal radiation, variable suction, variable permeability, vertical porous plate, heat and mass transfer.

1. INTRODUCTION:

The studies related to MHD visco-elastic fluids with radiation effect past a porous media in the presence of heat source/sink plays significant role in many scientific, industrial and engineering applications. These flows were basically utilized in the fields of petroleum engineering concerned with the oil, gas and water through reservoir to the hydrologist in the analysis of the migration of underground water. To recover the water for drinking and irrigation purposes the principles of this flow are followed. In the recent days many researchers recognized the significance of these

We also have gone through the literature related to the present work and followed. Chandra Reddy et al. [14, 15] analyzed magnetohydrodynamic convective double diffusive laminar boundary layer flow past an accelerated vertical plate as well as Soret and Dufour effects on MHD free convection flow of Rivlin-Ericksen fluid past a semi infinite vertical plate. Further Chandra Reddy et al. [16] studied the properties of free convective magneto-nanofluid flow past a moving vertical plate in the presence of radiation and thermal diffusion. Mahdy et al. [17] studied thermosolutal marangoni boundary layer magnetohydrodynamic flow with the Soret and Dufour effects past a vertical flat plate. Kairi and Murthy [18] discussed the effect of melting and thermo-diffusion on natural convection heat mass transfer in a non-Newtonian fluid saturated non-Darcy porous medium. Swati & Murthy [19] analyzed Magnetohydrodynamic
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Nomenclature:

- \( C \) Non-dimensional Species concentration
- \( D \) Molecular diffusivity
- \( \text{Gr} \) Grashof number of heat transfer
- \( \sigma \) Electrical conductivity
- \( K \) Permeability of the medium
- \( k \) Thermal diffusivity
- \( \text{Re} \) Elastic parameter
- \( \text{Ec} \) Eckert parameter
- \( \text{Nu} \) Nusselt number
- \( S \) Heat source parameter
- \( \text{Sh} \) Sherwood number
- \( T \) Non-dimensional temperature
- \( T^1 \) Temperature of the field
- \( C^1 \) Species concentration
- \( \omega \) Non-dimensional frequency of oscillation
- \( G_c \) Grashof number for mass transfer
- \( G \) Acceleration due to gravity
- \( K_p \) Porosity parameter
- \( F \) Thermal radiation
- \( M \) Magnetic parameter
- \( B_0 \) Magnetic field of uniform strength
- \( \text{Pr} \) Prandtl number
- \( \text{Sc} \) Schmidt number
2. FORMULATION OF THE PROBLEM:
The unsteady free convection heat and mass transfer flow of a well-known non-Newtonian fluid, namely Walters B visco-elastic fluid past an infinite vertical porous plate, embedded in a porous medium in the presence of thermal radiation, oscillatory suction as well as variable permeability is considered. In addition to this the existence of heat generation / absorption is also considered. A uniform magnetic field of strength $B_0$ is applied perpendicular to the plate. Let $x^1$ axis be taken along with the plate in the direction of the flow and $y^1$ axis is normal to it. Let us consider the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison with the applied transverse magnetic field. The basic flow in the medium is, therefore, entirely due to the buoyancy force caused by the temperature difference between the wall and the medium. It is assumed that initially, at $t^1 \leq 0$, the plate as fluids are at the same temperature and concentration. When $t^1 > 0$, the temperature of the plate is instantaneously raised to $T_{w}^1$ and the concentration of the species is set to $C_{w}^1$. Under the above assumption with usual Boussinesq’s approximation, the governing equations and boundary conditions are given by

\[
\frac{\partial u^1}{\partial t^1} + v \frac{\partial u^1}{\partial y^1} = \nu \frac{\partial^2 u^1}{\partial y^1^2} + g\beta(T^1 - T_w^1) + g\beta^1(C^1 - C_w^1) - \frac{\sigma B_0^2 u^1}{\rho}
\]

\[
- \frac{u u^1}{K'(t^1)} - k_0 \left[ \frac{\partial^2 u^1}{\partial y^1^2} + v \frac{\partial^3 u^1}{\partial y^1^3} \right]
\]

\[
\frac{\partial T^1}{\partial t^1} + v \frac{\partial T^1}{\partial y^1} = K \frac{\partial^2 T^1}{\partial y^1^2} + S'(T^1 - T_w^1) + \mu \left( \frac{\partial u^1}{\partial y^1} \right)^2 - \frac{\partial q^1_y}{\partial y^1}
\]

\[
\frac{\partial C^1}{\partial t^1} + v \frac{\partial C^1}{\partial y^1} = D \frac{\partial^2 C^1}{\partial y^1^2} + D_1 \frac{\partial^2 T^1}{\partial y^1^2}
\]
with the boundary conditions

\[ u = 0, T^1 = T_w + \varepsilon(T_w - T_\infty)e^{at_y}, \quad C^1 = C_w + \varepsilon(C_w - C_\infty)e^{at_y} \quad \text{at} \quad y = 0 \quad (4) \]

\[ u \to 0, T^1 \to T_\infty, C^1 \to C_\infty \quad \text{as} \quad y \to \infty \]

Let the permeability of the porous medium and the suction velocity be of the form

\[ K^1(t^1) = K_0^1(1 + \varepsilon e^{at_y}) \quad (5) \]

\[ v(t^1) = -v_0(1 + \varepsilon e^{at_y}) \quad (6) \]

where \( v_0 > 0 \) and \( \varepsilon \ll 1 \) are positive constants.

Introducing the non-dimensional quantities

\[ y = \frac{v_0^2 t^1}{\nu}, \quad \tau = \frac{v_0^2 t^1}{4\nu}, \quad w = \frac{4\nu w_1}{v_0^2}, \quad u = \frac{u_1}{v_0}, \quad T = \frac{T^1 - T_\infty}{T_w - T_\infty}, \quad C = \frac{C^1 - C_\infty}{C_w - C_\infty}, \]

\[ \rho = \frac{\nu S^1}{v_0^2}, \quad K_0 = \frac{v_0^2 K_0^1}{v_0^2}, \quad Pr = \frac{\nu}{K}, \quad Sc = \frac{\nu}{D}, \quad M^2 = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, \quad Rc = \frac{k_0 v_0^2}{\sigma v_0^2}, \]

\[ n = \frac{4\nu n_0^1}{v_0^3}, \quad Gc = \frac{\nu g \beta(C_w - C_\infty)}{v_0^3}, \quad Gr = \frac{\nu g \beta(T_w - T_\infty)}{v_0^3}, \quad So = \frac{D(T_w - T_\infty)}{\nu(C_w - C_\infty)}, \quad (7) \]

\[ \frac{\partial q^*}{\partial y^*} = 4(T - T_\infty) I^*, \quad F = \frac{4\nu I^*}{\rho C_\rho U_0^2}, \quad E = \frac{\mu U_0^2}{v \rho C_\rho (T_w - T_\infty)}. \]

The equations (1), (2), (3) reduce to the following non-dimensional form:

\[ \frac{1}{4} \frac{\partial u}{\partial t} - (1 + \varepsilon e^{\alpha t_y}) \frac{\partial u}{\partial y^2} = \frac{\partial^2 u}{\partial y^2} + GrT + GcC - M^2 u \]

\[ - \frac{u}{Kp(1 + \varepsilon e^{\alpha t_y})} - \frac{Rc}{4} \frac{\partial^3 u}{\partial t \partial y^2} \quad (8) \]

\[ \frac{1}{4} \frac{\partial T}{\partial t} - (1 + \varepsilon e^{\alpha t_y}) \frac{\partial T}{\partial y^2} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} + E \left( \frac{\partial u}{\partial y} \right)^2 + ST - FT \quad (9) \]

\[ \frac{1}{4} \frac{\partial C}{\partial t} - (1 + \varepsilon e^{\alpha t_y}) \frac{\partial C}{\partial y^2} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + So \frac{\partial^2 T}{\partial y^2} \quad (10) \]
with the boundary conditions
\[ u = 0, T = 1 + \varepsilon e^{\alpha t}, C = 1 + \varepsilon e^{\alpha t} \quad \text{at} \quad y = 0 \]
\[ u \to 0, T = 0, C = 0 \quad \text{as} \quad y \to \infty \] (11)

3. SOLUTION OF THE PROBLEM:

Equations (8)-(10) are coupled non-linear partial differential equations and are to be solved by using the initial and boundary conditions (11). However, exact solution is not possible for this set of equations and hence we solve these equations by finite-difference method. For this, a rectangular region of the flow field is chosen, and the region is divided into a grid of lines parallel to X and Y-axes, where the X-axis is taken along the plate and the Y-axis is normal to the plate as shown in Fig.1.

![Finite difference space grid](image)

**Fig. 1** Finite difference space grid

The equivalent finite difference schemes of equations for (8)-(10) are as follows:

\[
\frac{1}{4} \frac{u_{i,j+1} - u_{i,j}}{\Delta t} - (1 + \varepsilon e^{\alpha t}) \frac{u_{i+1,j} - u_{i,j}}{\Delta y} = Gr T_{i,j} + Gc C_{i,j} + \frac{u_{i-1,j} - u_{i+1,j} - 4u_{i,j}}{(\Delta y)^2} - \frac{Rc}{4} \left( \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j} - u_{i,j} + 2u_{i,j} - u_{i+1,j}}{\Delta t} \right) - M^2 u_{i,j} \left( \frac{\Delta t}{\Delta y} \right)^2 \]

\[-\frac{1}{K_p (1 + \varepsilon e^{\alpha t})} u_{i,j} \] (12)
Here, the index $i$ refer to $y$ and $j$ to time. The mesh system is divided by taking $\Delta y = 0.1$. From equation (11), we have the following equivalent initial condition

$$u(i,0) = 0, T(i,0) = 0, C(i,0) = 0 \text{ for all } i$$

(15)

The boundary conditions from (11) are expressed in finite-difference form as follows

$$u(0,j) = 1, T(0,j) = 1, C_{i-1,j} - C_{i+1,j} = -2.\Delta y \text{ for all } j$$

$$u(i_{\text{max}},j) = 0, T(i_{\text{max}},j) = 0, C(i_{\text{max}},j) = 0 \text{ for all } j$$

(16)

(Here $i_{\text{max}}$ was taken as 20)

First the velocity at the end of time step viz, $u(i,j+1)$ ($i=1,20$) is calculated from (12) in terms of velocity, temperature and concentration at points on the earlier time-step. Then $T(i,j+1)$ is computed from (13) and $C(i,j+1)$ is computed from (14). The procedure is repeated until $t = 0.5$ (i.e. $j = 500$). During computation $\Delta t$ was chosen as 0.001.

4. RESULTS AND DISCUSSION:

The influence of different physical parameters like Grashof number modified Grashof number, magnetic parameter, thermal radiation, Prandtl number, Eckert number, Soret number and Schmidt number on velocity, temperature and concentration is discussed by using graphical representations. The general nature of the velocity profile is parabolic with picks near the plate. Figure 2 shows that the velocity enhances for rising values of magnetic parameter. The effect of Prandtl number on velocity is displayed in figure 3. The Prandtl number is a dimensionless number approximating the ratio of momentum diffusivity (kinematic viscosity) and thermal diffusivity. The fluid velocity decreases for increasing values of Prandtl number. This is due to the effect of transverse magnetic field, which has the nature of reducing the velocity. These results are similar to that of Mishra et al. [11]. Figures 4, 5 depict the velocity variations...
under the effect of Grashof number and modified Grashof number respectively. The velocity of the flow grows when the values of these parameters increases. Figure 6 represents the impact of porous medium on velocity. It is evident that the velocity enhances for increasing values of porosity parameter. The changes in velocity under the existence of heat source / sink are depicted in figure 7. It is noticed that the velocity enhances in the presence of heat source where as it falls down in the case of heat sink. The variation in velocity under the influence of Schmidt number is shown in figure 8. It is evident that velocity comes down when the values of Schmidt number are increased. The existence of thermal diffusion results in improving the flow velocity which is clear from figure 9. These results coincide with that of Chandra Reddy et al. [13]. The temperature decreases in the presence of thermal radiation which is shown in figure 10. The effect of Prandtl number on temperature is presented in figure 11. The temperature decreases for increasing values of Prandtl number. Figure 12 reveals the same nature as that of velocity under the influence of heat source/sink. Figure 13 depicts the influence of Eckert number on temperature. It is observed that the temperature rises with increasing values of Eckert number. The effect of Schmidt number on concentration is shown in figure 14. The concentration reduces with an increase in Schmidt number. Increasing values of Soret number leads to enhance the concentration which is clear from figure 15.

Fig. 2: Effect of magnetic parameter on velocity

\[ \varepsilon = 0.01 \]

- \( \text{Sc}=0.22; \quad \text{Gc}=5; \)
- \( \text{Kp}=0.5; \quad \text{Rc}=0.1; \)
- \( \text{Ec}=0.01; \quad \text{Gr}=5; \)
- \( \text{F}=0.2; \quad \text{S}=0.1; \)
- \( \text{S0}=2; \quad \text{Pr}=0.71; \)
- \( \varepsilon=0.01; \quad n=1; \)
Fig. 3: Effect of Prandtl number on velocity

Fig. 4: Effect of Grashof number on velocity

Fig. 5: Effect of modified Grashof number on velocity
**Fig. 6:** Effect of porosity parameter on velocity

**Fig. 7:** Effect of heat source/sink on velocity

**Fig. 8:** Effect of Schmidt number on velocity
Fig. 9: Effect of Soret number on velocity

Fig. 10: Effect of radiation parameter on temperature

Fig. 11: Effect of Prandtl number on temperature
Fig. 12: Effect of heat source/sink on temperature

Fig. 13: Effect of Eckert number on temperature

Fig. 14: Effect of Schmidt number on concentration
The present work is extended to observe the changes in skin friction, Nusselt number and Sherwood number under the influence of thermal radiation, Eckert number, Schmidt number and magnetic parameter. This is done with the help of tabular values. Table 1 shows that the skin friction coefficient reduces for increasing values of Eckert number and Schmidt number. A reverse trend is shown in the case of radiation parameter and magnetic parameter. The rate of heat transfer increases under the influence of thermal radiation whereas it decreases in the case of Eckert number. Increasing values of Schmidt number leads to enhance the Sherwood number.

**Table 1.** Effect of various physical parameters on skin friction, Nusselt number and Sherwood number

<table>
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<tr>
<th>Ec</th>
<th>F</th>
<th>Sc</th>
<th>M</th>
<th>$\tau$</th>
<th>Nu</th>
<th>Sh</th>
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<td>0.2</td>
<td>1</td>
<td>0.22</td>
<td>5</td>
<td>2.0359</td>
<td>0.6742</td>
<td>0.2116</td>
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<tr>
<td>0.4</td>
<td>1</td>
<td>0.22</td>
<td>5</td>
<td>1.9848</td>
<td>0.6645</td>
<td>0.2116</td>
</tr>
<tr>
<td>0.6</td>
<td>1</td>
<td>0.22</td>
<td>5</td>
<td>1.5212</td>
<td>0.5932</td>
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</tr>
<tr>
<td>0.2</td>
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<td>5</td>
<td>2.0361</td>
<td>0.6742</td>
<td>0.2116</td>
</tr>
<tr>
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<td>5</td>
<td>2.0856</td>
<td>0.7356</td>
<td>0.2116</td>
</tr>
<tr>
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<td>5</td>
<td>2.1589</td>
<td>0.7931</td>
<td>0.2116</td>
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</tr>
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</table>
4. CONCLUSION:
In the present study the effect of thermal radiation due to natural convection on MHD flow of a visco-elastic fluid past a porous plate with variable suction and heat source/sink is analyzed. The governing equations for the velocity field, temperature and concentration by finite difference method. The main findings of this study are as follows.

- Velocity of the fluid reduces for increasing values of Prandtl number and magnetic parameter.
- Temperature of the fluid grows for rising values of Eckert number, but a reverse effect is noticed in the case of Prandtl number and radiation absorption parameter.
- The concentration reduces with an increase in Schmidt number.
- The existence of heat source leads to enhance the temperature and a reverse trend is observed in the presence of heat sink.
- Skin friction decreases with an increase of Eckert number and Schmidt number but a reverse effect is noticed in the case of radiation absorption parameter and magnetic parameter.
- Nusselt number increases as radiation absorption parameter increases but in the case of Eckert number it decreases.
- Sherwood number increases with an increase in Schmidt number.

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