MHD Heat and Mass Transfer of the Unsteady Flow of a Maxwell Fluid over a Stretching Surface with Navier Slip and Convective Boundary Conditions

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Abstract

The combined effect of Navier slip and convective boundary conditions on an unsteady flow of a Maxwell fluid over a stretching surface in the presence of magnetic field, thermal radiation, heat source and chemical reaction is investigated. The governing equations are reduced to nonlinear ordinary differential equations by means of similarity transformations. These equations are then solved numerically by applying Runge-Kutta fourth order method along with shooting technique. Due to engineering applications, the boundary layer flows of Maxwell fluids have been given considerable attention in the recent years. The velocity, temperature and concentration distributions are discussed graphically for different parameters. The results for local Skin friction, Nusselt number and Sherwood number are analyzed graphically.

Keywords – Maxwell fluid, unsteady flow, Navier slip parameter, Heat source, Convective boundary conditions.

INTRODUCTION:

Now days the unsteady flow of heat and mass transfer has gained attention due to its vast applications in science and engineering like polymer extrusion process, cooling of nuclear reactors, glass blowing, space technology, casting and spinning of fibers etc. In many practical problems mass transfer includes the molecular diffusion of species in the presence of chemical reactions like homogeneous and heterogeneous which affects the quality of finished products. Hence the study of heat and mass
transfer with the effect of chemical reaction and thermal radiation are important to engineers and scientists. The effect of thermal radiation plays a vital role in controlling heat and mass transfer as it increases the thermal diffusivity of the cooling liquid in stretching sheet problems.

Since the non-Newtonian fluids show more rheological behavior than Newtonian fluids different models were proposed. Among these the most important model is Maxwell fluid model which analyzes stress relaxation as well as its elasticity and viscosity behavior. This model is applicable in modeling behavior of polymers and geomaterials. Hayat et al [1] studied the unsteady MHD flow of a rotating Maxwell fluid in a porous medium. Choi et al [2] analyzed the two dimensional incompressible flow of upper convected Maxwell fluid. Nadeem et al [3] investigated effect of heat and mass transfer on MHD flow of Maxwell fluid. Recently, the unsteady boundary layer heat transfer of Maxwell viscoelastic fluid was studied by Zhao et al [4]. S. Mukhopadhyay [5] studied unsteady heat transfer flow of a Maxwell fluid over a stretching surface in the presence of heat source or sink.

The class of boundary conditions of convective flow for heat and mass transfer problems is Newtonian heating or convective boundary conditions. In this the surface heat transfer depends on surface temperature. The situation where the heat transported through a bounding surface having finite heat capacity is known as Newtonian heating. This arises in many engineering devices like heat exchangers where the conduction in solid tube is influenced by convection in the fluid past it. Makinde [6] illustrated the MHD flow of heat and mass transfer past a moving vertical plate with Newtonian heating conditions. Boundary layer flow of nanofluid over a non-linearly stretching sheet with convective boundary conditions was studied by Mustafa et al [7]. Navier [8] introduced a most general boundary condition, namely the velocity of the fluid tangential to the solid surface where the solid surface, is proportional to shear stress on the fluid solid interface. This proportionality is called the slip length that describes the slipperiness of the surface. Matthews and Hills [9] analyzed numerically the effects of slip on momentum boundary layer thickness on a flat plate. Makinde [10] investigated the unsteady MHD flow of heat transfer towards a flat plate with Navier slip and convective boundary conditions.

The main aim of this study is to determine the MHD unsteady flow of heat and mass transfer of a Maxwell fluid over a stretching surface with Navier slip and Convective boundary conditions. The governing equations are solved numerically using shooting technique with Runge-Kutta fourth order method and results are discussed graphically. The present results are in good agreement with S. Mukhopadhyay et al. [5] in the absence of slip and Newtonian heating.
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MATHEMATICAL FORMULATION:
Consider the unsteady two dimensional MHD flows of a Maxwell fluid over a stretching surface which coincides with the plane $y = 0$. The flow is in the region $y > 0$ and is subjected to magnetic field normal to the sheet with the velocity of free stream $U_\infty = \frac{x}{1-\alpha t}$, when $y$ tends towards infinity the ambient values of temperature and concentration of the plate $T_f$ and $C_f$, are denoted by $T_\infty$ and $C_\infty$ respectively. By applying the above assumptions the boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (1)

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \lambda \left( u \frac{\partial^2 u}{\partial x^2} + v \frac{\partial^2 u}{\partial y^2} + 2 \nu \frac{\partial^2 u}{\partial x \partial y} \right) = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} (u - U_\infty)$$  \hspace{1cm} (2)

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \frac{Q_p}{\rho c_p} (T - T_\infty) - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y}$$  \hspace{1cm} (3)

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K_1 (C - C_\infty)$$  \hspace{1cm} (4)

Subject to the boundary conditions

$$u = U_\infty + L \frac{\partial u}{\partial y}, \hspace{0.5cm} v = 0, \hspace{0.5cm} -K \frac{\partial T}{\partial y} = h_f (T_f - T), \hspace{0.5cm} C = C_w \hspace{0.5cm} \text{at} \hspace{0.5cm} y = 0$$

$$u \to U_\infty, \hspace{0.5cm} T \to T_\infty, \hspace{0.5cm} C \to C_\infty \hspace{0.5cm} \text{as} \hspace{0.5cm} y \to \infty$$  \hspace{1cm} (5)

where $u$ and $v$ are the velocity components in the x and y-directions, respectively, $\rho$ is the fluid density, $\nu$ is the kinematic viscosity coefficient, $\lambda = \lambda_0 (1-\alpha t)$ is the relaxation time of the period, $\lambda_0$ is constant. $T$ and $C$ are the fluid temperature and concentration respectively, $h_f$ is the heat transfer coefficient, $k$ is the thermal diffusivity of the fluid, $B_0$ is the strength of the transverse magnetic field, $\sigma$ is the electrical conductivity, $c_p$ is the specific heat capacity at constant pressure and $K_1$ is the rate of chemical reaction. The radiative heat flux term is simplified by using Rosseland approximation as

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}$$  \hspace{1cm} (6)
Where $q_y$ represents the radiative heat flux in the $y$-direction, $\sigma^*$ is the Stefan-Boltzmann constant and $k^*$ is the mean absorption coefficient. We assume that the temperature difference within the flow is sufficiently small such that $T^4$ may be expressed as a linear function of the temperature. This is accomplished by expanding $T^4$ in a Taylor's series about $T_\infty$ and neglecting higher order terms so that

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^3$$  \hspace{1cm} (7)

Introducing the similarity variables

$$\eta = \frac{y}{\sqrt{v(1-\alpha^t)}}, \quad \psi = x \left( \frac{c v}{\sqrt{(1-\alpha^t)}} f(\eta) \right), \quad \theta(\eta) = \frac{T - T_\infty}{T_\infty - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_\infty - C_\infty},$$

$$M = \frac{\sigma B_2}{\rho c} (1-\alpha^t), \quad \beta = c \lambda_0, \quad A = \frac{\alpha}{c}, \quad Pr = \frac{v}{k}, \quad Q = \frac{Q_\infty}{\rho c}, \quad Sc = \frac{v}{D}, \quad Nr = \frac{4\sigma^* T_\infty^3}{k^* k_0}, \quad \gamma = \frac{k^*}{c}$$

(8)

Where $\eta$ is the stream function with $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$ and $\eta$ is the stream variable.

Using equation (8), the equations (2), (3), and (4) reduce to

$$f'' + ff' - f^2 - \beta(f^2 f' - 2ff' f') - A(\frac{f^2}{2} + f') - M(f - 1) = 0$$

$$\left(1 + \frac{4}{3} Nr\right) \theta' + Pr(\theta'^2 + f \theta - 2f \theta - A (\eta \theta + 3 \theta)) = 0$$

$$\phi' + Sc(f \phi - 2f \phi - A (\eta \phi + 3 \phi) - \gamma \phi) = 0$$

(9)

The corresponding boundary conditions are

$$f(0) = 0, \quad f'(0) = 1 + hf^2(0), \quad \theta(0) = -Bi(1 - \theta(0)), \quad \phi(0) = 1 \quad \text{at} \quad \eta = 0$$

$$f' \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad \eta \to \infty$$

Here $A$ is the unsteadiness parameter, $Pr$ is the Prandtl number, $Q$ is the heat source parameter, $Nr$ is the radiation parameter, $Sc$ is the Schmidt number, $h = \frac{L}{\sqrt{c/n(1-\alpha^t)}}$ is the local slip parameter, $Bi = \frac{hf}{k} \sqrt{\frac{v(1-\alpha^t)}{c}}$ is the local Biot number. Knowing the
velocity field, temperature field, concentration field the Skin-friction, Nusselt number and Sherwood number, can be calculated by

\[(\text{Re}_x)^{\frac{1}{2}} c = (1 + \beta)f''(0), \quad (\text{Re}_x)^{\frac{1}{2}} \text{Nu} = -(1 + \frac{4}{3} \text{Nr}) \theta'(0) \quad \text{and} \quad (\text{Re}_x)^{\frac{1}{2}} \text{Sh} = -\phi'(0)\]

Where \(\text{Re}_x = \frac{xU_\infty}{\nu}\) (stretching Reynold’s number).

**RESULTS AND DISCUSSIONS:**

The effect of various parameters such as magnetic parameter \(M\), slip parameter \(h\), Maxwell parameter \(\beta\), Prandtl number \(Pr\), Biot number \(Bi\), unsteady parameter \(A\), heat source parameter \(Q\), Schmidt parameter \(Sc\) and chemical reaction \(\gamma\) on velocity, temperature and concentration are discussed in figs. (1-19). The results corresponding to the skin-friction coefficient for the unsteady flow of a Maxwell fluid in the absence of slip parameter and Biot number are compared with the results of Swati Mukhopadhyay in Tables (1-2). The effect of unsteady parameter \(A\) and Maxwell parameter \(\beta\) on skin-friction, Nusselt number and Sherwood number are shown in Figs. (20-22).

Figs. (1-3) analyzes the influence of velocity, temperature and concentration for different values of magnetic parameter \(M\). It is observed that with the increase of magnetic field, the velocity of the fluid decreases but the temperature and concentration increases in this case. The transverse magnetic field, applied normal to the flow direction has a tendency to create a dragging force known as Lorentz force which opposes the motion of the fluid. Figs.(4-6) exhibits the influence of slip parameter \(h\) with velocity, temperature and concentration. It is seen that the fluid velocity decreases with an increase in slip parameter \(h\) whereas the temperature and concentration increases. Fig.(8) shows the effect of Biot number \(Bi\) with temperature. It is clear that as \(Bi\) en, the heat transfer rate from the hot fluid at the lower side of the plate to the cold fluid at the upper side increases. This results in an elevation of the fluid temperature at the upper side. Figs.(10-12) shows the velocity, temperature and concentration profiles for several values of the unsteadiness parameter \(A\). It is seen that with the increase of unsteadiness parameter the velocity along the sheet decreases and this implies decrease in the thickness of momentum boundary layer near the wall. However, away from the wall the velocity of the fluid increases with increasing unsteadiness. The temperature and concentration of the fluid decreases with the increase of unsteadiness parameter \(A\). The rate of heat transfer increases with increasing \(A\), but the amount of heat transferred from the sheet to the fluid decreases, hence the temperature \(\theta(\eta)\) decreases. Here \(A = 0\) represents the steady flow and the rate of cooling is much faster for higher values of the unsteadiness parameter than in a
steady flow. Fig.(13) shows that with an increase in the Prandtl number Pr the temperature decreases due to decrease in the fluid thermal diffusivity. This is in agreement with the physical fact, at higher prandtl number, the fluid has a thinner thermal boundary layer and this increases the gradient of temperature. Fig.(14) present the effect of heat source Q on temperature. It is observed that temperature increases with the increase of heat source parameter. From Fig.(15) it is noted that the concentration boundary layer decreases with the increase of chemical reaction $\gamma$. Fig. (16) shows that the concentration boundary layer thickness decreases with the increase of Schmidt number Sc. The effect of velocity, temperature and concentration on Maxwell parameter $\beta$ are exhibited in figs. (17-19). The increasing values of $\beta$ reduces the velocity and hence the boundary layer thickness decreases. It can also be seen that with the increase of $\beta$ the temperature and concentration increases. Thus the mass transfer rate at the surface increases with increasing $\beta$. The effect of unsteadiness parameter A and the Maxwell parameter $\beta$ on skin friction $f''(0)$ is shown in Fig. (20). The value of $f''(0)$ decreases with the increase of A and $\beta$. Further it is clear that from Figs. (21-22) the Nusselt number and Sherwood number decreases for increasing values of unsteady parameter A and $\beta$.

CONCLUSIONS:

1. The fluid velocity decreases with the enhancement of slip parameter h but the temperature and concentration increases with the increasing values of slip parameter h.

2. The temperature enhances with the increasing values of Newtonian heating parameter (Bi).

3. An increase in the Maxwell parameter $\beta$ leads to decrease in velocity and increase in temperature and concentration.

4. It is observed that the fluid velocity at the sheet, the temperature and concentration depreciates due to enhancement of the unsteadiness parameter.

Table-1: Values of $f''''(0)$ for different values of unsteadiness parameter A with slip parameter h = 0 and Biot number Bi = 0.

<table>
<thead>
<tr>
<th>A</th>
<th>SwatiMukhopadhyay</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>-1.261479</td>
<td>-1.261522</td>
</tr>
<tr>
<td>1.2</td>
<td>-1.377850</td>
<td>-1.378154</td>
</tr>
</tbody>
</table>
Table-2: Values of $f''(0)$ for various values of Maxwell parameter $\beta$ when $M = 0.2$ with $h = 0$ and $Bi = 0$.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Swati Mukhopadhyay</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.095444</td>
<td>1.095378</td>
</tr>
<tr>
<td>0.2</td>
<td>1.188270</td>
<td>1.188243</td>
</tr>
<tr>
<td>0.4</td>
<td>1.275878</td>
<td>1.275810</td>
</tr>
<tr>
<td>0.6</td>
<td>1.358732</td>
<td>1.358698</td>
</tr>
</tbody>
</table>

Fig. (1) Variation of velocity with $M$

Fig. (2) Variation of temperature with $M$

Fig. (3) Variation of concentration with $M$

Fig. (4) Variation of velocity with slip parameter $h$
Fig. (5) Variation of temperature with slip parameter $h$

Fig. (6) Variation of concentration with slip parameter $h$

Fig. (7) Variation of velocity with $B_i$

Fig. (8) Variation of temperature with $B_i$

Fig. (9) Variation of concentration with $B_i$

Fig. (10) Variation of velocity with $A$
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Fig. (11) Variation of temperature with A

Fig. (12) Variation of concentration with A

Fig. (13) Variation of temperature with Pr

Fig. (14) Variation of temperature with Q

Fig. (15) variation of concentration with γ

Fig. (16) Variation of concentration with Sc
REFERENCES


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